

Journey to the Heart of Music

Philip Perry

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Example M *Johann Jakob Froberger - Courante*

MUTABLE NUMBER ANALYSIS

Johann Jakob Froberger (1616 - 1667) occupies an important position in the evolution of the baroque suite, in that during its formative stages he was travelling about Europe, from his positions with the court chapels in Stuttgart and later Vienna (studying in Rome with Frescobaldi and travelling to Paris, Brussels, London and widely in Germany) accumulating the various innovations made in Italy and France and creating a synthesis of these new elements which he communicated to his German contemporaries and successors. The dance suite form played a role in the baroque era much like that of the sonata in the classical period. This courante forms one part of a keyboard dance suite consisting of the sequence: allemande, courante, sarabande and gigue.

The mutable number analysis is set rather lower than most of the previous examples: both in the choice of fundamental H1 frequency of 0.5 hertz and in the general closeness of the conjunction pitches to those of the upper part of the courante -- in places they coincide, for example measures 5, 7 and 8. The relatively low level of the absolute fundamental tone (0.5 Hz) is in part a consequence of the piece being in a minor key which necessitates some addition 'headroom' for the nesting of the minor tonic, dominant and subdominant chords in groups of five elements of the underlying nested series. In addition to which, another factor bearing upon the choice of the fundamental is the use of 'pedal' like figures (Froberger was an organist) where the E-minor tonic and the dominant (both B-minor and B-major chords) are connected by the bass pedal note B -- scale degree V. This particular arrangement implies an underlying nested series carrying the minor tonic chord aggregated in groups of five elements, with the pedal note 'B' only becoming available at the relatively high position of B-h15*. This feature is particularly evident in measures 14 to 16.

However, despite these two reasons for extending the fundamental frequency down to 0.5 hertz, it would be possible with a little ingenuity to rearrange the mutable number analysis in a way that would allow

a doubling of the absolute fundamental to 1Hz, without altering the conjunctions, resulting in a halving of the mutable number values. This is evident from the second level of digits mostly being divisible by two -- those directly above the bottom '1' in the MBN stacked format. Some re-working of the exchanges involving the subdominant would be required, as the A-minor chord is normally carried within a nested series which cannot be reduced below F-H21 of the fundamental series. And an even more drastic re-assignment of the E-minor chord, to a nested series aggregated in groups of three and built on A-H13 or A-H26 of the fundamental series would be necessary. Though quite feasible, such a strategy would perhaps yield a somewhat less 'natural' mutable number analysis for a piece in the key of E-minor.

Finally, some consideration should be given to the question of what is the most appropriate choice for the fundamental nesting series for pieces in the minor mode. Essentially this fundamental level of structure comes into existence through the motions of the higher level aggregated and nested series, in particular the positions adopted by the nested fundamental 'h1/Hn' of the middle level nested harmonic series. (This is the lowest element in the written-out columns of harmonic series shown in the analysis below.) Conceptually the system is being driven by the harmonic motion of the chords in the top level aggregated series (marked by asterisks: hn*), the motion of which necessitates the middle level nested series to change configuration in order to accommodate this movement. These changes in the middle level nested series describe a sequence of ratios, relationships, for example on the first page of this analysis: H48, H32, H24, H26, H21, H18 (with the actual frequencies written in brackets below). The question is how best to encapsulate the ratios and what do these relationships describe, what do they mean? Taking the latter point first. As the ratios are the lowest and most fundamental relationships in the system, it is perhaps not unreasonable to ascribe to them the meaning of tonal center or (whole movement) key. In other words these generally simple low order ratios provide a regular structure upon which a 'sense of key' can be founded and sustained. In other words, the topography of the sequence provides a 'surface' upon which the perception of a tonal center can obtain some 'traction'. As to the question of encapsulating the relationships, naturally a harmonic series is the answer. But which harmonic series, which absolute fundamental unit: C-H1, E-H1 or maybe F-H1?

Theoretically any scale step series could just about work, it would be a matter of extending the more awkward choices far enough to gain necessary approximations. However, the principles of simplicity and elegance dictate a more constrained choice. Although a series built on E-H1 might at first seem most logical for a piece in the key of E-minor, consider that the middle level nested series which carries the objective notes of the E-minor chord is most naturally built on 'C-h1/Hn' and aggregated in groups of five. Unfortunately, this means that the tonic minor chord, set in its most natural way, would only be available to an E-H1 based fundamental series at the extended position of 'C-h1/H25'. This would place the tonic minor configuration at the most distant position from the fundamental as it could possibly be, and would also have the effect of forcing other relations up to a similarly high level. In contrast an F-H1 based fundamental series could produce a nested fundamental down as low as 'C-h1/H3' to carry the tonic minor chord and 'G-h1/H9' to hold the dominant minor chord, with the subdominant minor found nicely from 'F-h1/H5'. The F-H1 based series is very attractive in this regard, it provides low order ratios to underlie the main chords of the minor mode. But there is, I feel, something inelegant in an arrangement that links the key of E-minor, to an absolute fundamental of F-H1, two notes with a scale ratio of 15:16?

The compromise candidate is C-H1, not logical but awkward like E-H1 neither illogical and simple as

F-H1. The C-H1 based series, though not as simple overall as the F-H1 based series -- 'C-h1/H1' and 'G-h1/H3' for the tonic and dominant minor but the subdominant minor is found most naturally through 'F-h1/H21' -- notwithstanding it does also have a kind of logic to it, in that rather than mirroring the (tonic minor) top level aggregated series, it mirrors the tonic's middle level nested series. However, I must admit to a certain uneasiness about all the options, none (E-H1, F-H1, C-H1, still less any other) has both the economy and self-evident rightness of the arrangement of harmonic series found in the major mode.

One solution to this apparent infelicity in the MOS analysis of pieces in the minor mode is provided by the conceptual approach pioneered by Heinrich Schenker, where a composition is envisaged as ultimately being derived from the extension in time of a single chord through a process of drawing out fundamental scale lines and bass arpeggiation.

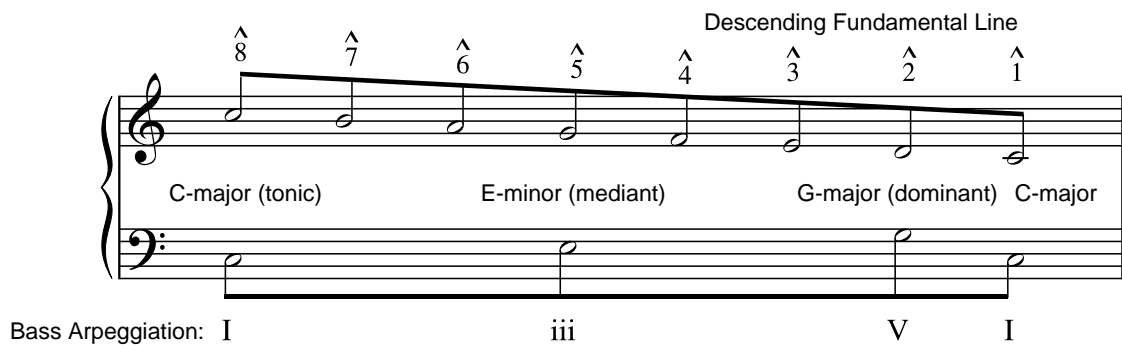


Figure M.1 The interpolation of the minor principle within a Schenkerian representation of the major 'Ursatz', the basic structure of a tonal composition.

Although Schenker applied his idea of extending the 'chord of nature' to both the major and minor triads separately, it is but a small conceptual step to combine the two modes into a single entity by subsuming the minor principle within the major. The outcome of such an amalgamation, as illustrated above, is a linking and embedding of the E-minor mode within that of 'C-major' as is the natural outcome for aggregating nested series in groups of five elements. Forearmed with this conceptual approach, the final step in reconciling the minor principle with an absolute fundamental of a different scale step (i.e. the key of E-minor built on a scale step fundamental C-H1) is the recognition of the minor mode as being a phenomenon deriving from the harmonic series, no less than the major, but with a more complex and interesting relationship to its absolute fundamental 'H1', than its sibling the major mode. As a consequence of this more intricate relationship with its absolute fundamental, the harmonic series carrying the minor chord breaks down to produce a perceived root different from, rather than the same as, its ultimate foundation; and through this 'broken' symmetry creates the separate, yet ultimately dependent, domain of the minor mode.

M.4 - COURANTE, J. J. FROBERGER

Value:	960-B (decimal)	960-B	960-B	960-B	960-B	640-E	640-E	640-E	576-D
Mutable	4	6	8	10	12	8	6	9	8
Digit	5	5	5	4	3	3	5	4	4
Sequence	48 } Tonal 1 } Center e	32	24	24	26	26	21	18	18
	C H1 = 0.5 hertz		1	1	1.026	1.026	1.016	0.988	1

- Comb. Tone/
Harmonic:
- Objective
Notes:
- Aggregates: *
- Mid.C=256Hz
- Conjunction:>
- Root: R
- Dif. Tone:

480.0Hz	B -h20*>	B -h30*>	B -h40*>	B -h40*>	B -h36*				
	A#h19	- h29	- h35*	A-h36*	- h33*				
	A-h18	A#h28	F#h30*	G-h32*	G#h30*				
	G#h17	- h27	- h29	F-h28*	- h27*				
	G-h16	A-h26	F-h28	- h27	F#h26				
	F#h15*	G#h25*	- h27	E-h26	F-h25				
	F-h14	G-h24	E-h26	D#h25	E -h24*>	E -h30*>	E -h36*	320.0Hz	
	E-h13	- h23	D#h25*	D -h24*	- h23	- h29	D#h34		
	D -h12	F #h22	D -h24	- h23	D#h22	D#h28	D -h32*>	284.4Hz	
	C #h11	F-h21	- h23	C#h22	D -h21*	- h27	C#h30		
	B -h10*	E -h20*	C-h22	C-h21	C#h20	D-h26	C -h28*		
	A- h9	D#h19	C-h21	B -h20*	C-h19	C #h25*	- h27		
	G- h8	D-h18	B -h20*	A#h19	B -h18*	C -h24	B -h26		
	F- h7	C#h17	A#h19	A-h18	A#h17	- h23	A#h25		
	D- h6	C-h16	A-h18	G#h17	A-h16	B -h22	A -h24*		
	B- h5*R	B -h15*	G#h17	G -h16*	G #h15*	A#h21	- h23		
	G- h4	A#h14	G-h16	F#h15	G -h14	A-h20*	G#h22		
	D- h3	A-h13	F #h15*	F-h14	F#h13	G#h19	G-h21		
	G- h2	G -h12	F-h14	E-h13	E -h12*	G -h18	F #h20*		
	G-h1/H48 (24.000Hz)	F#h11	E -h13	D -h12*	D#h11	F#h17	F-h19		
		E -h10*	D -h12	C#h11	C#h10	F-h16	E-h18		
		D- h9	C#h11	B -h10*	B -h10	E -h15*	D#h17		
		C- h8	B -h10*	A- h9	A- h9	D#h14	D -h16*		
		A# h7	A- h9	G- h8*	G- h7	D-h13	C#h15		
		G- h6	G- h8	F- h7	E - h6*	C-h12	C-h14		
		E - h5*R	F- h7	D- h6	C# h5	B -h11	B-h13		
		C- h4	D- h6	B- h5	A- h4	A -h10*	A -h12*		
		G - h3	B h5*R	G - h4*R	E h3*R	G- h9	G#h11		
		C h2	G- h4	D- h3	A- h2	F- h8	F#h10		
		C-h1/H32 (16.000Hz)	D h3	G- h2	A- h1/H26	D# h7	E- h9		
			G h2	G-h1/H24 (12.000Hz)		C- h6	D - h8*		
						A- h5*R	C- h7		
						F- h4	A- h6		
						C- h3	F# h5		
						F- h2	D h4*R		
						F- h1/H21 (10.666Hz)	A- h3		
							D- h2		
							D- h1/H18 (8.888Hz)		

Aggregated

Series: (4 groups of 5) 3:2 (6 groups of 5) 4:3 (8 groups 5) 5:4 (10 groups 4) 6:5 (12 groups 3)
(-4 groups 3)

(8 groups 3) 3:4 (6 grps 5) 3:2 (9 groups of 4)
(-1 group of 4)

Nested Series: (10 x two) -3:2-> (10 x three) --4:3--> (10 x four) -----> (4 x ten) 9:10 (4 x nine)
(-12 x one)

(6 x four) 5:4 (6 x five) -6:5-> (6 x six)
(-4 x one)

(8 groups of 4) -3:4->

M.5 - COURANTE, J. J. FROBERGER

576-D	960-B	960-B	960-B	960-B	960-B	1280-E	1280-E	1280-E	960-B	960-B
6	10	5	6	8	6	8	10	20	15	16
4	4	8	5	4	5	5	4	2	2	2
24	24	24	32	30	32	32	32	32	32	30
1	1	1	1	1	1	1	1	1	1	1

				[E]-h40*> 632.1Hz	[E]-h40**
				- h35*	D-h36**
[B]-h40**>	[B]-h30*> 474.1Hz	[B]-h32*>	[B]-h30*	C-h32**	
A-h36*	- h29	- h31	- h29	[B]-h30*> 474.1Hz	[B]-h32**>
G-h32**	A#h28	A#h30	A#h28	A#h28**	- h31
F-h28*	- h27	- h29	- h27	- h27	A#h30*
- h27	[A]-h26	A-h28*	[A]-h26	A-h26*	- h29
E-h26	G#h25*	- h27	G#h25*	G#h25	A-h28**
[D#]h25	[G]h24	G#h26	[G]h24	[G]h24**	- h27
284.4Hz D-h24**	- h23	G-h25	- h23	- h23	G#h26*
- h23	[F#]h22	[F#]h24*	[F#]h22	[F#]h22*	G-h25
[C#]h22	F-h21	- h23	F-h21	F-h21	[F#]h24**
C-h21	[E]h20*	F-h22	E-h20*	[E]h20**	- h23
[B]h20*	D#h19	E-h21	D#h19	D#h19	F-h22*
A#h19	D-h18	D#h20*	D-h18	D-h18*	[E]h21
A-h18	C#h17	D-h19	C#h17	C#h17	[D#]h20**
G#h17	C-h16	C#h18	C-h16	C-h16**	D-h19
[G]h16**	[B]h15*	C-h17	[B]h15*	B-h15	C#h18*
F#h15	A#h14	[B]h16*	A#h14	A#h14*	C-h17
F-h14	[A]h13	A#h15	[A]h13	A-h13	[B]h16**
E-h13	[G]h12	[A]h14	[G]h12	G-h12**	A#h15
D-h12*	[F#]h11	G#h13	[F#]h11	F#h11	[A]h14*
C#h11	[E]h10*	[F#]h12*	[E]h10*	E-h10*	G#h13
B-h10	D- h9	F-h11	[D]h9	D- h9	[F#]h12**
A- h9	C- h8	[D#]h10	C- h8	[C]h8**	F-h11
[G]h8**	A# h7	C# h9	A# h7	A# h7	D#h10*
F- h7	G- h6	B- h8*	G- h6	G- h6*	C# h9
D- h6	[E]h5*R	A- h7	[E]h5*R	E- h5	[B]h8**
B- h5	C- h4	F# h6	C- h4	[C]h4**	A- h7
[G]h4*R	G- h3	D# h5	G- h3	G- h3	F# h6*
D- h3	[C]h2	B- h4*R	[C]h2	C- h2*R	D# h5
G- h2	C-h1/H32	F# h3	C-h1/H32	C-h1/H32	B- h4**
G-h1/H24	(15.802Hz)	[B]h2	(15.802Hz)	(15.802Hz)	F# h3
(11.851Hz)		B-h1/H30			B- h2*R
		(14.814Hz)			B-h1/H30
					(14.814Hz)

----> (6 groups of 4)

(+4 groups of 4)

1:2 (5 groups of 8) -6:5-> (6 groups of 5) ---4:3---> (8 groups of 4) ---3:4--> (6 groups of 5)

(+2 groups of 5)

(8 groups of 5) -5:4-> (10 groups of 4)

-2:1->(20 groups of 2)

(-5 groups of 2)

(15 groups of 2) 16:15 (16 groups of 2)

-1:2->(8 groups of 4) 5:4->

----> (8 x three)

(+16 x one)

(10 x four) -3:4-> (10 x three)

(2 x fifteen) -----16:15-----> (2 x sixteen) ---15:16--> (2 x fifteen)

(+10 x one) -----> (-10 x one)

(2 x fifteen) -16:15-> (2 x sixteen)

(8 x four) ----5:4->

M.6 - COURANTE, J. J. FROBERGER

960-B	960-B	960-B	960-B	960-B	960-B	960-B	960-B	960-B	960-B
10	8	12	6	8	8	9	8	6	
4	5	5	5	4	4	4	4	5	
24	24	16	32	30	30	26	30	32	
1	1	1	1	1	1	1.026	1	1	

Ⓑ- h40 >	Ⓑ- h40 >	Ⓑ- h60 >	Ⓑ- h30 >	Ⓑ- h32 >	474.1Hz	Ⓑ- h32 >	Ⓑ- h36 >	Ⓑ- h32 >	Ⓑ- h30 >
A-h36*	Ⓐ-h36	- h55*	- h29	- h31		- h31	A#h34	- h31	- h29
G-h32*	- h35*	A-h52	A#h28	A#h30		A#h30	A-h32*	A#h30	A#h28
F-h28*	F#h30*	G#h50*	- h27	- h29		- h29	Ⓒ#h30	- h29	- h27
- h27	F-h28	Ⓒ-h48	Ⓐ-h26	A-h28*		Ⓐ-h28*	G-h28*	Ⓐ-h28*	A-h26
E-h26	E-h26	- h45*	G#h25*	- h27		- h27	Ⓒ-h27	- h27	G#h25*
D#h25	D#h25*	Ⓕ#h44	Ⓒ-h24	G#h26		G#h26	F#h26	G#h26	Ⓒ-h24
D-h24*	D-h24	E-h40*	- h23	G-h25		Ⓒ-h25	F-h25	G-h25	- h23
- h23	- h23	- h35*	Ⓕ#h22	Ⓕ#h24*		Ⓕ#h24*	E-h24*	Ⓕ#h24*	Ⓕ#h22
C#h22	C#h22	B-h30*	F-h21	- h23		- h23	- h23	- h23	F-h21
C-h21	C-h21	G#h25*	Ⓔ-h20*	F-h22		F-h22	D#h22	F-h22	E-h20*
B-h20*	B-h20*	G-h24	D#h19	E-h21		E-h21	D-h21	E-h21	D#h19
A#h19	A#h19	- h23	D-h18	Ⓓ#h20*		Ⓓ#h20*	C#h20*	D#h20*	D-h18
A-h18	A-h18	F#h22	C#h17	D-h19		D-h19	C-h19	D-h19	C#h17
G#h17	G#h17	F-h21	C-h16	C#h18		C#h18	Ⓕ-h18	C#h18	C-h16
Ⓒ-h16*	G-h16	Ⓔ-h20*	B-h15*	C-h17		C-h17	A#h17	C-h17	Ⓔ-h15*
Ⓕ#h15	Ⓕ#h15*	D#h19	A#h14	Ⓑ- h16 *		Ⓑ- h16 *	A-h16*	Ⓑ- h16 *	A#h14
F-h14	F-h14	D-h18	A-h13	A#h15		A#h15	G#h15	A#h15	A-h13
E-h13	E-h13	C#h17	G-h12	A-h14		A-h14	G-h14	A-h14	G-h12
D-h12*	D-h12	C-h16	F#h11	G#h13		G#h13	F#h13	G#h13	F#h11
C#h11	C#h11	Ⓑ- h15 *	Ⓔ- h10 *	Ⓕ#h12*		F#h12*	E-h12*	F#h12*	Ⓔ- h10 *
Ⓑ- h10	Ⓑ- h10 *	A#h14	D- h9	F-h11		F-h11	D#h11	F-h11	D- h9
A- h9	A- h9	A-h13	C- h8	D#h10		Ⓓ#h10	Ⓒ-h10	Ⓓ#h10	C- h8
G- h8*	G- h8	G-h12	A# h7	C# h9		Ⓒ# h9	B- h9	C# h9	A# h7
F- h7	F- h7	F#h11	G- h6	Ⓑ- h8 *		Ⓑ- h8 *	A- h8*	B- h8*	G- h6
D- h6	D- h6	E-h10*	E- h5*R	A- h7		A- h7	G- h7	A- h7	Ⓔ- h5 *R
B- h5	Ⓔ- h5 *R	D- h9	C- h4	F# h6		F# h6	E- h6	F# h6	C- h4
G- h4*R	G- h4	C- h8	G- h3	D# h5		D# h5	C# h5	D# h5	G- h3
D- h3	D- h3	A# h7	C- h2	Ⓑ- h4 *R		Ⓑ- h4 *R	Ⓐ- h4 *R	Ⓑ- h4 *R	C- h2
G- h2	G- h2	G- h6	C-h1/H32	F# h3		F# h3	E- h3	F# h3	C-h1/H32
G-h1/H24	G-h1/H24	Ⓔ- h5 *R	(15.802Hz)	B- h2		Ⓔ- h5 *R	A- h2	B- h2	(15.802Hz)
(11.851Hz)	(11.851Hz)	C- h4		B-h1/H30		Ⓔ- h5 *R	A- h1/H26	B-h1/H30	
		G- h3		(14.814Hz)		Ⓔ- h5 *R	(13.168Hz)	(14.814Hz)	
		C- h2				Ⓔ- h5 *R			
		C-h1/H16				Ⓔ- h5 *R			
		(7.901Hz)				Ⓔ- h5 *R			

5:4(10 grps 4)4:5(8grps 5)3:2(12 grps 5)1:2(6 grps 5) 4:3 (8 groups of 4) -----> (8 groups of 4) 9:8 (9 grps 4) 8:9 (8 grps 4) 3:4 (6 grps 5)

--5:4--> (8 x five)

(20 x two) ---3:2--> (20 x three) 1:2 (10 x three)

(2 x fifteen) 16:15 (2 x sixteen) -----> (4 x eight) -9:8-> (4 x nine) 8:9 (4 x eight)

← (2 x sixteen) 16:15 (2 x fifteen)

M.7 - COURANTE, J. J. FROBERGER

960-B 1920-B

8 16
4 4
30 30
1 1

1920-B

15
4
32
1

1920-B 1280-E

12 8
4 4
40 40
1 1

1280-E 832-A

12 8
4 4
26 26
1.026 1

832-A 1280-E

12 18
4 4
18 18
0.963 0.988

1280-E

16
5
16
1

[B]-h64*> 948.1Hz

[B]-h60*>

[B]-h48*

474.1Hz [B]-h32*

- h31
A#h30
- h29
A-h28*
- h27
G#h26
(G)-h25
(F#)h24*
- h23
F-h22
E-h21
(D)h20*
D-h19
C#h18
C-h17
(B)h16*
A#h15
A-h14
G#h13
F#h12*
F-h11
D#h10
C# h9
(B) h8*
A- h7
F# h6
D# h5
(D) h4*R
F# h3
B- h2
B-h1/H30
(14.814Hz)

B-h30
- h29
A#h28*
A-h26
G#h25
G-h24*
- h23
F#h22
F-h21
(E)h20*
D#h19
D-h18
C#h17
C-h16*
B-h15
A#h14
A-h13
G-h12*
F#h11
(E)h10
D- h9
(C) h8*
A# h7
G- h6
E- h5
C- h4*R
G- h3
(C) h2
C-h1/H32
(15.802Hz)

[E]-h32*> 632.1Hz

A#h44*

G#h40*

F#h36*

[E]-h28*

- h27

C#h26

C-h25

B-h24*

- h23

A#h22

A-h21

(G#)h20*

G-h19

(F#)h18

C#h17

F-h17

(E)h16*

D#h15

D-h14

C#h13

B-h12*

A#h11

G#h10

F# h9

(E) h8*

D- h7

(B) h6

G# h5

(E) h4*R

B- h3

(E) h2

E- h1/H40
(19.753Hz)

[E]-h48*

C#h40*

B-h36*

(A)-h32*> 421.4Hz

G#h30

(G)-h28*

F#h26

F-h25

E-h24*

- h23

D#h22

(D)-h21

(C#)h20*

C-h19

(B)-h18

A#h17

(A)-h16*

G#h15

G-h14

F#h13

E-h12*

D#h11

C#h10

B- h9

(A)- h8*

G- h7

E- h6

C# h5

(A) h4*R

E- h3

A- h2

A- h1/H26
(13.168Hz)

[E]-h72*> 632.1Hz

D-h64*

C-h56*

(A)-h48*

G#h44*

(F#)h40*

E-h36*>

D-h32*

C#h30

C-h28*

- h27

B-h26

A#h25

A-h24*

- h23

G#h22

G-h21

F#h20*

F-h19

E-h18

D#h17

(D)-h16*

C#h15

C-h14

B-h13

A-h12*

G#h11

F#h10

E- h9

(D) h8*

C- h7

A- h6

F# h5

D- h4*R

A- h3

D- h2

D- h1/H18
(8.779Hz)

[E]-h80*>

B-h60*

G#h50*

(G)h48

- h45*

(F#)h44

E-h40*

D#h38

D-h36

- h35*

C#h34

C-h32

B-h30*

A#h28

A-h26

G#h25*

G-h24

- h23

F#h22

F-h21

(E)h20*

D#h19

D-h18

C#h17

C-h16

B-h15*

A#h14

A-h13

G-h12

F#h11

E-h10*

(D) h9

C- h8

A# h7

G- h6

E- h5*R

C- h4

G- h3

C- h2

C-h1/H16
(7.901Hz)

-4:3->(8 groups of 4)
(+8 groups of 4)
(16 groups of 4) -15:16-> (15 groups of 4) 4:5 (12 groups of 4)
(-4 groups of 4)

16:15 (2 x sixteen)
(+32 x one)
(4 x sixteen) ----15:16----> (4 x fifteen)
(12 x five) ---4:5--> (12 x four)
(-16 x one)
(16 x two) ---3:2--> (16 x three)
(-16 x one)
(16 x two) -----3:2-----> (16 x three)

(8 groups of 4) 3:2 (12 groups of 4)
(-4 groups of 4)
(8 groups of 4) -----3:2-----> (12 groups of 4)
(+6 groups of 4)
(18 groups of 4) -8:9-> (16 grps of 5) 9:8
(+24 x one)
(8 x nine) ----10:9----> (8 x ten) 9:10

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1280-E	1664-A	1664-A	1664-A	1920-B	1920-B	1664-A	1664-A	1664-A	1664-A
18	24	16	14	16	20	18	16	24	18
4	4	5	5	5	4	4	5	4	4
18	18	21	24	24	24	24	21	18	24
0.988	0.963	0.990	0.990	1	1	0.963	0.990	0.963	0.963

[B]-h80* > 963.2Hz	[B]-h80*	[A]-h96* > 842.8Hz	[A]-h80* >	[A]-h70*	[A]-h72* >	[A]-h80* > 866.9Hz	[A]-h96* >	[A]-h72* >
A-h72	A#h76*	G#h88*	- h75*	- h65*	G#h68*	- h75*	G#h88*	G-h64*
F#h80*	- h70*	F#h60*	- h65*	F#h60*	G-h64*	- h70*	F#h80*	F-h56*
E-h72* (632.1Hz)	- h65*	- h55*	- h55*	F#h60*	- h65*	- h65*	E-h72*	D-h48*
D-h64*	E-h60*	D#h50*	- h48*	F-h56*	E-h60*	- h55*	D-h64*	B-h40*
C-h56*	- h55*	D-h48	- h45*	E-h52*	- h55*	- h55*	C-h56*	A-h36*
B-h52*	C#h50*	- h45*	- h35*	D-h48*	C#h50*	- h35*	A-h48*	G-h32*
A-h48*	C-h48	B-h40*	- h35*	C#h44*	C-h48	- h35*	F#h40*	F#h30
G#h44*	- h45*	- h35*	- h35*	B-h40*	B-h40*	- h35*	E-h36*	F-h28*
F#h40*	A-h40*	F#h30*	- h35*	A-h36*	A-h40*	- h35*	D-h32*	E-h26
E-h36*	- h35*	E-h26	- h35*	G-h32*	- h35*	- h35*	C-h28*	D#h25
D-h32*	E-h30*	D#h25*	- h23	F-h28*	E-h30*	- h27	A#h25	D-h24*
C-h28*	D#h28	D-h24	- h23	D-h24*	D#h28	- h23	A-h24*	- h23
A#h25	- h27	- h23	- h23	- h23	- h27	- h23	- h23	C#h22
A-h24*	D-h26	C#h22	C#h22	C#h22	D-h26	- h23	G#h22	C-h21
- h23	C#h25*	C-h21	C-h21	C-h21	C#h25*	- h23	G-h21	B-h20*
G#h22	C-h24	B-h20*	B-h20*	B-h20*	C-h24	- h23	F#h20*	A#h19
G-h21	- h23	A#h19	A#h19	A#h19	- h23	B-h22	F-h19	A-h18
F#h20*	B-h22	A-h18	A-h18	A-h18	B-h22	A#h21	E-h18	G#h17
F-h19	A#h21	G#h17	G#h17	G#h17	A#h21	D#h17	D#h17	G-h16*
E-h18	A-h20*	G-h16	G-h16	G-h16	A-h20*	D-h16*	F#h15	F-h14
D#h17	G#h19	F#h15*	F#h15	F#h15	G#h19	C#h15	C#h15	E-h13
D-h16*	G-h18	F-h14	F-h14	F-h14	G-h18	C-h14	C-h14	D-h12*
C#h15	F#h17	E-h13	E-h13	E-h13	F#h17	B-h13	B-h13	C#h11
C-h14	F-h16	D-h12	D-h12	D-h12	F-h16	A-h12*	A-h12*	B-h10
B-h13	E-h15*	C#h11	C#h11	C#h11	E-h15*	G#h11	G#h11	A-h9
A-h12*	D#h14	B-h10*	B-h10	B-h10	D#h14	F#h10	F#h10	G-h8*
G#h11	D-h13	A-h9	A-h9	A-h9	D-h13	E-h9	E-h9	F-h7
F#h10	C-h12	G-h8	G-h8	G-h8	C-h12	D-h8*	D-h8*	D-h6
E-h9	B-h11	F-h7	F-h7	F-h7	B-h11	C-h7	C-h7	B-h5
D-h8*	A-h10*	D-h6	D-h6	D-h6	A-h10*	A-h6	A-h6	G-h4*R
C-h7	G-h9	B-h5*R	B-h5	B-h5	G-h9	F#h5	F#h5	D-h3
A-h6	F-h8	G-h4	G-h4	G-h4	F-h8	D-h4*R	D-h4*R	G-h2
F#h5	D#h7	D-h3	D-h3	D-h3	D#h7	A-h3	A-h3	G-h1/H24
D-h4*R	C-h6	G-h2	G-h2	G-h2	C-h6	D-h2	D-h2	(12.039Hz)
A-h3	A-h5*R	G-h1/H24	G-h1/H24	G-h1/H24	A-h5*R	D-h1/H18	D-h1/H18	(9.029Hz)
D-h2	F-h4	(12.039Hz)	(12.039Hz)	(12.039Hz)	F-h4	(9.029Hz)	(9.029Hz)	
D-h1/H18	C-h3				C-h3			
(8.779Hz)	F-h2				F-h2			
	F-h1/H21				F-h1/H21			
	(10.534Hz)				(10.835Hz)			

-->(18 grps of 4)
 (+6 grps of 4)
 (24 grps of 4) 2:3 (16 grps of 5) 7:8 (14 grps of 5)
 (+2 grps of 5)
 (16 grps of 5) 5:4 (20 grps of 4)
 (-2 grps of 4)
 (18 grps of 4) 8:9 (16 grps of 5) 3:2 (24 grps of 4) 3:4 (18 groups of 4) -8:9->
 -->(8 x nine)
 (+24 x one)
 (16 x six) --5:6--> (16 x five)
 (10 x eight) 7:8 (10 x seven)
 (+10 x one) ----> (8 x ten)
 (-8 x one)
 (8 x nine) 10:9 (8 x ten)
 (16 x five) ---6:5---> (16 x six)
 (24 x four) -3:4-> (24 x three)
 (8 x nine) --10:9-->

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1664-A	1664-A	1408F#	1408F#	960-B	960-B	960-B	960-B	960-B	960-B	960-B
16	24	20	24	16	960-B	960-B	960-B	960-B	960-B	960-B
5	4	4	4	4	12	16	12	16	12	12
21	18	18	15	15	5	4	5	4	5	4
0.990	0.963	0.978	0.978	1	16	15	16	15	16	15
					1	1	1	1	1	1

[A]-h80*>	[A]-h96* 866.9Hz								
- h75*	G#h88*								
- h70*	[F#h80*>	[F#h96* 722.4Hz							
- h65*	E-h72*	F-h88*							
E-h60*	D-h64*	D#h80*							
- h55*	C-h56*	C#h72*							
C#h50*	[A]-h48*	[B]-h64*	[B]-h60*>	[B]-h64*>	[B]-h60*>	[B]-h64*>	[B]-h48* 481.6Hz		
[C]-h48	G#h44*	A-h56*	- h55*	A-h56*	- h55*	A-h56*	A#h44*		
- h45*	[F#h40*	F#h48*	[A]-h52	F#h48*	[A]-h52	[F#h48*	G#h40*		
[A]-h40*	E-h36*	D#h40*	G#h50*	D#h40*	G#h50*	D#h40*	F#h36*		
[G]-h35*	D-h32*	C#h36*	[G]-h48	C#h36*	[G]-h48	C#h36*	[E]-h32*		
[E]-h30*	C-h28*	B-h32*	- h45*	[B]-h32*	- h45*	B-h32*	D-h28*		
D#h28	B-h26	A#h30	[F#h44	A#h30	F#h44	A#h30	- h27		
- h27	A#h25	A-h28*	[E]-h40*	[A]-h28*	[E]-h40*	A-h28*	C#h26		
D-h26	[A]-h24*	- h27	- h35*	- h27	- h35*	- h27	C-h25		
C#h25*	- h23	G#h26	B-h30*	G#h26	B-h30*	G#h26	[B]-h24*		
C-h24	G#h22	G-h25	A#h28	G-h25	A#h28	G-h25	- h23		
- h23	G-h21	F#h24*	A-h26	[F#h24*	A-h26	F#h24*	A#h22		
B-h22	F#h20*	- h23	G#h25*	- h23	G#h25*	- h23	A-h21		
A#h21	F-h19	F-h22	G-h24	F-h22	[G]-h24	F-h22	[C#h20*		
[A]-h20*	E-h18	E-h21	- h23	E-h21	- h23	E-h21	G-h19		
G#h19	D#h17	D#h20*	F#h22	D#h20*	F#h22	D#h20*	F#h18		
G-h18	D-h16*	D-h19	F-h21	D-h19	F-h21	D-h19	F-h17		
F#h17	C#h15	C#h18	E-h20*	C#h18	[E]-h20*	C#h18	[E]-h16*		
F-h16	C-h14	C-h17	D#h19	C-h17	D#h19	C-h17	D#h15		
E-h15*	B-h13	[B]-h16*	D-h18	[B]-h16*	D-h18	[B]-h16*	D-h14		
D#h14	[A]-h12*	A#h15	C#h17	A#h15	C#h17	A#h15	C#h13		
D-h13	G#h11	A-h14	C-h16	A-h14	C-h16	A-h14	[B]-h12*		
C-h12	F#h10	G#h13	[B]-h15*	G#h13	[B]-h15*	G#h13	A#h11		
B-h11	E- h9	F#h12*	A#h14	F#h12*	A#h14	F#h12*	G#h10		
[A]-h10*	D- h8*	F-h11	A-h13	F-h11	A-h13	F-h11	F# h9		
G- h9	C- h7	D#h10	G-h12	D#h10	G-h12	D#h10	[E]- h8*		
F- h8	A- h6	C# h9	F#h11	C# h9	F#h11	C# h9	D- h7		
D# h7	F# h5	B- h8*	E-h10*	[B]- h8*	E-h10*	B- h8*	B- h6		
C- h6	[D]- h4*R	A- h7	D- h9	A- h7	D- h9	A- h7	G# h5		
[A]- h5*R	A- h3	F# h6	[C]- h8	F# h6	C- h8	F# h6	[D]- h4*R		
F- h4	D- h2	D# h5	A# h7	D# h5	A# h7	D# h5	B- h3		
C- h3	D- h1/H18	B- h4*R	G- h6	B- h4*R	G- h6	B- h4*R	E- h2		
F- h2	(9.029Hz)	F# h3	E- h5*R	F# h3	[E]- h5*R	F# h3	E- h1/H20		
F- h1/H21		B- h2	C- h4	B- h2	C- h4	B- h2	(10.033Hz)		
(10.835Hz)		B-h1/H15	G- h3	B-h1/H15	G- h3	B-h1/H15			
		(7.524Hz)	C- h2	(7.524Hz)	C- h2	(7.524Hz)			
		C-h1/H16	C- h1/H16	C- h1/H16	C- h1/H16	C- h1/H16			
		(8.026Hz)		(8.026Hz)					

-->(8 x ten)
 (16 x five) 6:5 (16 x six)
 (-16 x one)
 (16 x five) -6:5-> (16 x six)
 (4xsixteen)15:16(4xfifteen)16:15(4xsixteen)15:16(4xfifteen)16:15(4xsixteen)
 (16 x four) -3:4-> (16 x three)