

Journey to the Heart of Music

Philip Perry

Copyright P.J. Perry © 2003, 2006, 2009. This document may be reproduced and used for non-commercial purposes only. Reproduction must include this copyright notice and the document may not be changed in any way. The right of Philip J. Perry to be identified as the author of this work has been asserted by him in accordance with the UK Copyright, Designs and Patents Act, 1988.

Example K *L. van Beethoven – Waldstein Sonata, Op53*

MUTABLE NUMBER ANALYSIS

Beethoven's piano sonata in C major opus 53 dating from 1804, is dedicated to Count Ferdinand von Waldstein one of Beethoven's many patrons; it is a noble, spacious, elevated work indicating something of the path the composer was to take in the middle period of his artistic development. The detailed MOS analysis given below is from the first movement, covering the exposition of the first and second subjects, measures 1 to 87.

The movement begins quietly with a mysterious and agitated repeated C-major tonic chordal motive in the bass register which opens out into a first inversion dominant chord and melodic figure, almost like asking a question (I-ii⁷-V-?). The question is repeated and extended in an abrupt transposition to B-flat beginning at measure 5, this leads back via an F-minor chord to a cadence on the dominant – C-minor to bare G octaves at measures 12 and 13 (i-V). Having approached the dominant by an interesting 'flat-side' path, at measure 14 the quiet tonic C-major motive reappears in an arpeggiated Aberti bass figuration, this time developing via D-minor, E-major and A-minor chords into a transitional section focused on E-minor, before arriving at the E-major second subject in measure 35. Beethoven's choice of the mediant key, E-major, rather than the dominant G for the second subject is unusual and marks something of a departure from accepted classical practice. From the point of view of a mutable number analysis this is an interesting feature because the nested series which would normally host an aggregated E-minor chord is one built upon a C-fundamental, the tonic. Indeed this is particularly revealing, for an aggregated series in groups of three (on a C-fundamental nested series) would produce the standard dominant chord but when the tension is ratcheted up to the level of groups of five an E-based series emerges. The alternation between aggregated E-series in

groups of five and the B-based series in measures 23 to 34 generates a particular tension in the approach to the second subject at measure 35 which is thus ushered in with serene intensity and great import. The C-based nested series both connects the first and second subjects and creates the intensity of effect felt in the unusual choice of key for second subject. The mutable number analysis reveals a close affinity between the keys of C-major and E while also accounting for the impact of the second subject, due to its relatively high level of excitation. After introducing the second subject, Beethoven continues on in E-major moulding the motivic clay for an extended time – measures 35 to 79. In measure 80 the composition returns to E-minor as it begins to make a rapid descent from the intensity of E towards the tonic C-major at measure 84 and the repeat of the first subject at measure 86. Again a C-based nested series makes the link back from second subject to first.

There is often more than one way of construing a mutable number digit exchange, that is, more than one arrangement of nested and aggregated series which will fit any particular circumstance. And while ultimately the MOS model provides a mathematical interpretation of the harmonic structure of the score, if mutable numbers are to shed any useful light upon tonal compositions, the exercise should obviously be conducted in a manner sensitive to the nature and essence of listeners' appreciation of the music. While mathematically precise modulation exchanges will always remain the norm, perhaps, occasionally, the tolerance of the ear might allow for a simpler ratio to be understood from a given situation than the actual arithmetic demands. Of course usually one construction out of the choices available will be self-evidently the most satisfactory both mathematically and musically, however, where doubt arises as to which arrangement to choose, as a general rule it seems appropriate, that precedence should be yielded to the ratio of exchange in the upper aggregated series. The aggregated series, positioned at the top of the system of nested harmonic series, holds the most significant objective notes and normally the majority of objective notes too; and as these objective notes are what drives the whole system, it is probably reasonable to prefer the aggregated series to move by exchanges which most nearly fit the harmonic motion of the actual chords as written, played and most importantly, heard. After all the aim of the arithmetic in the MOS model, when applied to tonal compositions, is to explain something of the musical fact, and not to mangle the music in the interests of mathematical convenience.

An example can be found in the exchanges between the second and third chords of the second subject (B-major to C#-minor) which is repeated at measures 35, 39 and 43. Here, respectively, the ratios of exchange between the roots B and C# are 1) 9:10, 2) 8:9 and 3) 7:8 – that is narrow, medium and wide whole tones.

1) For conformity with the just scale, perhaps 9:10 should be chosen, as B and C# are the fifth and sixth degrees of the E-major scale ($3/2$ and $5/3$ equal $9/6$ and $10/6$, i.e. 9:10); E-major being the current key at measure 35. Here the sesquinona 9:10 exchange can be achieved in concert with a clean sesquitercia 3:4 exchange in the nested series.

2) At measure 39 a standard just whole step B to C# 8:9 between the roots of the aggregated series is illustrated. This interval of exchange produces a narrow step in the nested series from 10:9. B to A. Basically the consequence of allowing the aggregated series to rule the ratios of exchange is to force the underlying series into occasional accommodations. Given that all levels of series in a nested system of harmonic series

must share the same conjunction in whatever differing exchanges they execute, this rule restricts considerably the range of ratios which are mutually consistent.

3) Now to be entirely inconsistent for a moment, at measure 43 the exchange breaks the rule of a single shared conjunction – but only by a small margin. The aggregated exchange of 7:8 between B and C# is normal: eight groups (of six) are exchanged for seven groups (of five). In contrast the exchange of the mid-level nested series is improper in that the twelve-fold sesquitercia 3:4 exchange requires the conjunction to align B-h48 with B-h36 (rather than B-h48 with B-h35 used by the aggregated exchange). Is this acceptable? Well mathematically it is not, the figures simply don't add up. However, from the perspective of aural cognition one could invoke the tolerance of the ear and argue that the leeway provided by physiology and psychology allows this exchange a legitimacy that arithmetic would deny. The B-natural (h35/h36) conjunction of the C#minor chord is very roughly 1000Hz, and, stretching over the region of the ear's greatest sensitivity, the conjunction is echoed at the higher frequencies B-2000Hz, F#3000Hz and B-4000Hz and beyond. Given the adjustments of pitch due to equal temperament, the variability of fundamental and harmonic pitches arising from the violent and chaotic excitement of three pianos strings by hammer blow and the effects of distortion due to slightly off-true resonances within the instrument and in the acoustic environment, it would not be unreasonable to suggest that the ear detects a spectrum of harmonic maxima varying dynamically around the frequencies of these conjunctions:

C#minor Chord	B-1000Hz Conjunction	B-2000Hz Conjunction	F#3000Hz Conjunction	B-4000Hz Conjunction
Note G#	-----	-----	7th harmonic	-----
Note E	3rd harmonic	6th harmonic	9th harmonic	12th harmonic
Note C#	-----	7th harmonic	11th harmonic	14th harmonic

As can be seen from the above table, the ear would detect these (and higher) conjunctions as agglomerations of harmonics generated by the objective note and the very fact of the dynamic spread of frequencies around these maxima would tend to mask the slight mathematical imprecision of this improper exchange.

Also illustrative of the choices which have to be taken when constructing a mutable number analysis of a non-trivial tonal composition is the exchange at measure 19, stepping from the chord of D-minor to that of E⁷-major. Here the standard whole tone exchange (8:9) cannot be obtained from a G to E based mid-level nested series and so a factor other than the closeness with which the upper aggregated series mirrors the objective notes must be sought. This other factor could be the choice of conjunction. Firstly in the preceding measure 18, a downward octave leap is made in preparation for the relatively extended series needed to carry the A-minor chord which appears in measure 20. Such octave or even multi-octave resetting of the harmonic coordinates, in either direction, appears to be a feature of aural processing: While octave harmonic shifts are noticed they are not identified as a harmonic 'change', as a shift of a fifth or fourth would be. Indeed perhaps the processes of aural cognition maintain a set of parallel, nested, octave frames of reference? In mutable number terms this would be represented as the addition or deletion of a '0₂' column within the digit sequence. Also the ear's ready facility in identifying of the roots of chords as an attribute detached from the single unique fundamental of the chord, lends some support for this conjecture.

Secondly having gained the required register, addressing measure 19 directly, the problem is which

combination of aggregated and nested exchanges to choose? Two pairs of exchanges will work: a foreground 7:8 plus 7:6 in the background with the conjunction frequency D h96 to h112; or 9:10 in the foreground aggregated series and 6:5 for the background nested series using the conjunction frequency F# h60 to h72. The former yields a wide step (7:8) between the root tones D to E and a narrow minor-third between the nested fundamental frequencies of G and E (7:6). The other choice yields the opposite, a narrow step (9:10) between the chord roots D and E with a wider minor-third (6:5) between the fundamentals of the underlying nested series, G-h1/H24 and E-h1/H20. Either alternative is acceptable, in the sense that the composition is being treated an entirely autonomous and isolated set of relationships – an ideal ‘little world’ or model aural universe – without any external points of reference (like for example a fixed-note scale). Providing the internal relationships ‘compute’, that is to say they are commensurable, all is well. At any point in the composition, the notes *are* whatever the current state of the system compute them to be, no matter how much they may have strayed from any fixed external standard. So the choice comes down to the conjunction. Which of the two conjunctions D or F# best fits the harmonic context? Here D-h96 to D-h112 has been chosen on the basis of inertia: it was the conjunction prior to measure 19 and so would be established in the mind and ear. In contrast to the D conjunction, F# is rather antipathetic to the F-natural in a D-minor chord. However, it is perfectly possible to argue in favor of F#. The ear detects and processes much more than we are consciously aware of, and perhaps F#h60 (h5 of objective note D-h12) and F#h72 (h3 of objective note B-h24), has a reasonable claim along with the conjunction D-h96 to D-h112. And for certain there is no one unchallengeable path for a mutable number analysis through a complex and sophisticated piece of music. (Compare measure 2 with 87, which interpretation is preferable?) There is an element of art, as well as mathematics and science, in making a mutable number analysis; and yet on the other hand, most sequences of exchanges do have one clear interpretation and where there is room for doubt, as in measure 19, this does not change the interpretation of subsequent exchanges/relationships. (Though it will usually change all subsequent, externally measured, frequencies.) Though paths may diverge at times, quite soon they reunite.

Another form of divergent paths is illustrated at measures 21, 24 and 38: that is the possibility of using *forked mutable numbers* in a MOS analysis. Forked mutable numbers are single structures which express more than one magnitude. Such multi-value objects are discussed in Chapter 15 of *Journey to the Heart of Music* where they were (speculatively) applied to physical objects ranging from the fundamental particles of nature up to the arrangements of celestial mechanics exhibited in our solar system! On the much more down to earth level of music analysis these forked number structures can encapsulate the use of differing registers in a composition. Beethoven in the opening of the Waldstien sonata, dramatically contrasts a soft ‘rumbling-bass’ motive and a related ‘descending-sweep’ in a treble register two to three octaves higher. Harmonically the descending-sweep outlines the chord rumbling away below it and so is for all intents a register displaced echo of the bass harmony. A displacement of one, two or three octaves could be thought of as nesting one motive/harmony within another and might find written expressed in the form of a branching stacked (factor format) mutable number digit sequence. For example:

$$\begin{array}{r}
 2560\text{-E-}640 \\
 \hline
 24 \text{ -- } 6 \\
 \quad 5 \\
 \quad 21 \\
 \quad 1.016
 \end{array}$$

At measure 21 the melodic motive of measure 20 is catapulted up an octave and more into a high register, from its position an octave above the rumbling-bass figure. There is two octaves of 'clear water' between the bass and treble in measure 21 and these separate, yet essentially the same figures, harmonically, can have their separate-sameness expressed by the two values of a forked mutable number. The figures share a harmonic unity as shown by the common stem of the digit sequence but they occupy very different registers as evinced by their different values: decimal 2560 for the treble motive and 640 for the bass figure. Another example of forked mutable numbers is given at measure 24 (see score). Here the forking is extended over two chords (B-major and E-minor) which implies two conjunctions, one for each register. The lower level conjunction is illustrated by a dashed-line band. Finally a deeper level of forking is illustrated at measure 38, where a 6-4 to 5-3 cadential resolution on the dominant in E-minor is encapsulated by a two digit fork of the stem of a mutable number representing a single value. This is a different type of forking, at measures 21 and 24 two values in one number structure captured different simultaneous registers, while at measure 38 the forked digits mirror the temporal sequence of tension and resolution at a cadence.

$$\begin{array}{r}
 1920\text{-B-}1920 \\
 \hline
 12 \text{ -- } 16 \\
 4 \text{ -- } 3 \\
 40 \\
 1
 \end{array}$$

Perhaps the most difficult chord progression in the analysis for the MOS model to digest comes in measures 9 and 10, where the harmony alternates between F-minor and G-major, the subdominant minor and major dominant-seventh. The aggregated and nested series are necessarily extended so as to cover the extremes of register employed by Beethoven, which at measure 9 sees four octaves of empty space between left and right hands! (Using a forked digit sequence here would complicate the issue and not be germane to the point, because the crux of the difficulty lies in the awkward exchange of a diminished fifth/augmented fourth between a G-based and C#based nested series. Remarkable though the ear no doubt is, it perhaps asks too much of it, and the processes of aural cognition, to expect the mechanisms of hearing to be able to differentiate the fine graduations of frequency and extended harmonic series which would be needed to make a just augmented fourth exchange from C# to G of ratio 45:32. Though this exchange does work using the conjunction G-h180 to G-h128, with a 8:9 aggregated series exchange of 36 groups (of five) for 32 groups (of four) combined with a fourfold nested series exchange of forty-five for thirty-two (illustrated in square brackets at measure 11). Rather, given the ear's tolerance, some simpler approximation would perhaps be more likely, here 10:7 is used for the nested series exchange as it rests on the top F conjunction also highlighted in Beethoven's figuration.

Other alternatives are possible, for example the absence of an interval of a third in the dominant-seventh chord at measure 9 allows a nested series built on D#h1/H19 to be aggregated in groups of five, with the interval of a seventh in the chord (F-h9) paying the role normally taken by the minor-third in a minor chord, that is, occupying a position in the middle level nested series. Overall, whatever the hurdles the MOS model encounters it will almost always find the resources to vault the difficulties, however, as far as possible these varied resources should be deployed in a manner that reflects the apprehended tonal structure, hopefully thereby, enhancing our understanding of the compositions innermost mechanism.

Finally, the metrical aspect of a composition is accounted for with a separate and independent MOS structure, as shown in the box below and discussed in some detail in Chapter 10. The reasoning for this separation being that given the frequency gulf between the domains of pitch/timbre and duration (illustrated in Figure 4.19), and their different characters (deriving from the fixed units of pitch and the variability of tempo), in practice, the two cannot be bound within a single system.

meter	
(Metrical MOS)	
MBN: $4_4 0_1$	
h4...:sixteenthnote figuration.:measures 14 through 29, etc.	MBN: $4_4 0_1$
h3...:triplet figuration.....:measures 42 through 56, etc.	MBN: $3_4 0_1$
h2...:eighthnote figuration....:measures 1 through 12, etc.	MBN: $2_4 0_1$
h1/H4:quarternote pulse.....:measures 35 through 41, etc.	MBN: $1_4 0_1$
H3	
H2	
H1:meter 4/4 time.....:measures 13, 169, etc.	MBN: 4_1
First Movement	

Casually looking through the first movement of the Waldstein Piano Sonata, a range of mutable numbers are evident in the meter, pulse and variety of figuration used by Beethoven. The meter is 4/4 common time throughout and the movement opens with a very downright eighthnote figuration (MBN: $2_4 0_1$) which extends to measure 12. After one measure of semibreve closure with pause (MBN: 4_1), the figuration onward from measure 14 is dominated by sixteenthnotes (MBN: $4_4 0_1$) up to measure 30, where it reverts to eighthnotes (MBN: $2_4 0_1$) for the four measures 31 through 34. From measure 35 to 41 the figuration is in halfnotes and quarternotes (MBN: $1_4 0_1$) before, at measure 42, Beethoven breaks into sixteen measures of triplets (MBN: $3_4 0_1$), taking the piece through to measure 57 where sixteenthnote figuration emerges again (MBN: $4_4 0_1$). And so the sonata proceeds, locked into the one 4/4 meter but constantly moving back and forth amongst the range of digit sequences allowed by the basic pulse. Leaving aside the effect of tempo variation (discussed in Chapter 10), the range of values produced in the first movement of the sonata by the 4/4 meter modulating oscillatory system through changes in the level of figuration is, in decimal: 4, 8, 12 and 16 (plus somewhat theoretically 1 in measure 13 and 2 in measure 40).

K.7 - WALDSTEIN SONATA, L.v.BEETHOVEN

Allegro con brio

Value: 512-C (decimal) 512-C 512-C 576-D 576-D 576-D 2304-D

Mutable { 4
Digit { 2
Sequence { 64 } Tonal
1 } Center C

C-H1 = 0.5Hz

D-h96*> 1152.0Hz

D-h48*

D-288Hz(h36)*> D-h32*> D-h24*

- 280Hz(h35) - h31 - h23
C#272Hz(h34) C#h30 C#h22
- 264Hz(h33) - h29 C-h21

Conjunction	C- h8*>	C-h16*>	C-256Hz(h32)*	C-h28*	B-h20*
256.0Hz	A# h7	B-h15	- 248Hz(h31)	- h27	A#h19
	G- h6*	A#h14	B-240Hz(h30)	B-h26	A-h18
	(E) h5	A-h13	- 232Hz(h29)	A#h25	G#h17
	(C) h4*	G-h12*	A#224Hz(h28)*	A-h24*	(G) h16*
	(G) h3	F#h11	- 216Hz(h27)	- h23	F#h15
	(C) h2*R	(E) h10	A-208Hz(h26)	G#h22	F-h14
	C-h1/H64 (32.000Hz)	D- h9	G#200Hz(h25)	G-h21	E-h13
		(C) h8*	G-192Hz(h24)*	(F#)h20*	(D) h12*
		A# h7	- 184Hz(h23)	F-h19	C#h11
		(G) h6	F#176Hz(h22)	E-h18	B-h10
		E- h5	F-168Hz(h21)	D#h17	A- h9
		(C) h4*R	(E) 160Hz(h20)*	(D) h16*	(G) h8*
		G- h3	D#152Hz(h19)	C#h15	F- h7
		C- h2	D-144Hz(h18)	C-h14	D- h6
		C-h1/H32 (16.000Hz)	C#136Hz(h17)	B-h13	(B) h5
			(C) 128Hz(h16)*	(A) h12*	G- h4*R
			B-120Hz(h15)	G#h11	D- h3
			A#112Hz(h14)	F#h10	G- h2
			A-104Hz(h13)	E- h9	G-h1/H24 (12.000Hz)
			(G) 96Hz(h12)*	D- h8*	
			F# 88Hz(h11)	(C) h7	
			E- 80Hz(h10)	A- h6	
			D- 72Hz(h9)	F# h5	
			(C) 64Hz(h8)*	D- h4*R	
			A# 56Hz(h7)	A- h3	
			G- 48Hz(h6)	D- h2	
			E- 40Hz(h5)	D- h1/H18 (9.000Hz)	
			C- 32Hz(h4)*R		
			G- 24Hz(h3)		
			C- 16Hz(h2)		
			C- 8Hz(h1/H16) (8.000Hz)		

KEY

Objective

Notes: ○

Aggregates: *

Mid.C=256Hz

Conjunction:>

Root: R

Aggregated

Series: (4 groups of 2) --> (4 groups of 4) -2:1-> (8 groups of 4)

(+1 group of 4)

(9 groups of 4) --8:9--> (8 groups of 4) -3:4-> (6 groups of 4)

(+18 group of 4)

Nested

Series: (4 x two) --2:1--> (4 x four) ---2:1---> (4 x eight)

(+4 x one)

(4x nine) -----8:9-----> (4 x eight)

(8 x four) ---3:4---> (8 x three)

(+72 x one)

(32 x three) ----->

K.8 - WALDSTEIN SONATA, L.v.BEETHOVEN

CM: V

VII $\flat$

I 7

2304-D

2304-D 448A#

448A 512-C

24
4
24
1

40 8
4 4
14 14
1.029 1

7 8
4 4
16 16
1 1

D-h96*> 1152.0Hz

D-h160*

C#-h88*

C-h144*

C-h84*

A#-h128*

B-h80*

G#-h112*

A-h72*

F-h96*

G-h64*

G-h88*

D-h48*

D-h80*

263.3Hz C-h32*>

- h31

D-h24*

D-h40*

B-h30

- h23

C-h36*

- h29

C#-h22

A#-h32*> 234.4Hz

A#-h28*

C-h21

G#-h28*

- h27

B-h20*

- h27

A-h26

A#-h19

G-h26

G#-h25

A-h18

F#-h25

G-h24*

G#-h17

F-h24*

- h23

G-h16*

- h23

F#-h22

F#-h15

E-h22

F-h21

F-h14

D#-h21

E-h20*

E-h13

D-h20*

D#-h19

D-h12*

C#-h19

D-h18

C#-h11

C-h18

C#-h17

B-h10

B-h17

C-h16*

A-h9

A#-h16*

B-h15

G-h8*

A-h15

A#-h14

F-h7

G#-h14

A-h13

D-h6

G-h13

G-h12*

B-h5

F-h12*

F#-h11

G-h4*R

E-h11

E-h10

D-h3

D-h10

D-h9

G-h2

C-h9

C-h8*

G-h1/H24
(12.000Hz)

A#-h8*

A#-h7

G#-h7

G-h6

F-h6

E-h5

D-h5

C-h4*R

A#-h4*R

G-h3

F-h3

C-h2

A#-h2

C-h1/H16

A#-h1/H14

(8.229Hz)

(7.200Hz)

--> (24 groups of 4) -----5:3-----> (40 groups of 4)

(-32 groups of 4)

(8 groups of 4) -----7:8-----> (7 groups of 4)

--> (32 x three) -----5:3-----> (32 x five)

(-128 x one)

(4 x eight) -----7:8-----> (4 x seven)

(+1 group of 4)

(8 groups of 4) ->

(+4 x one)

(8 x four) ->

K.9 - WALDSTEIN SONATA, L.v.BEETHOVEN

CM: IV iv V⁷ iv

512-C 2048-C 2048-C 2688-F 2688-F 2688-F

6 24 24 32 28 32
 4 4 5 5 5 5
 21 21 17 17 19 17
 1.016 1.016 1.004 0.988 1.011 0.988

		F-h160*>	1404.3Hz	F-h140*>	F-h160*>
		- h150*		E-h136	- h150*
		- h140*		D-h120*	D#h144
		- h130*		- h110*	- h140*
C-h96*>	1053.2Hz	C-h120*		B-h100*	D-h136
B-h88*		B-h112		- h90*	- h130*
A-h80*		A#h104		G-h80*	C-h120*
G-h72*		A-h100*		- h70*	- h100*
F-h64*		G#h96		D-h60*	F-h80*
D-h52*		G-h88		-----	-----
C-h48*		F-h80*		C#h28	C-h30*
-----		-----		- h27	A-h25*
C#h25		C-h30*		C-h26	G#h24
263.3Hz C-h24*		A-h25*		B-h25*	- h23
- h23		G#h24		A#h24	G-h22
B-h22		- h23		- h23	F#h21
A#h21		G-h22		A-h22	F-h20*
A-h20*		F#h21		G#h21	E-h19
G#h19		F-h20*		G-h20*	D#h18
G-h18		E-h19		F#h19	D-h17
F#h17		D#h18		F-h18	C#h16
F-h16*		D-h17		E-h17	C-h15*
E-h15		C#h16		D#h16	B-h14
D#h14		C-h15*		D-h15*	A#h13
D-h13		B-h14		C#h14	G#h12
C-h12*		A#h13		C-h13	G-h11
B-h11		G#h12		A#h12	F-h10*
A-h10		G-h11		A-h11	D# h9
G- h9		F-h10*		G-h10*	C# h8
F-h8*		D# h9		F-h9	B- h7
D# h7		C# h8		D# h8	G# h6
C- h6		B- h7		C# h7	F- h5*R
A- h5		G# h6		A# h6	C# h4
F- h4*R		F- h5*R		G- h5*R	G# h3
C- h3		C# h4		D# h4	C# h2
F- h2		G# h3		A# h3	C# h1/H17
F- h1/H21		C# h2		D# h2	(8.777Hz)
(10.971Hz)		C# h1/H17		D#h1/H19	
		(8.777Hz)		(10.031Hz)	

-3:4-> (6 groups of 4)
 (+18 groups of 4)
 (24 groups of 4) -----1:1-----> (24 groups of 5)
 (+8 groups of 5)

-3:4-> (8 x three)
 (+72 x one)
 (24 x four) -----5:4-----> (24 x five)
 (+40 x one)
 (20 x eight) -----7:8-----> (20 x seven) -----8:7-----> (20 x eight)
 (16 x ten) 7:10

K.10 - WALDSTEIN SONATA, L.v.BEETHOVEN

CM: V⁷ iv V⁷ iv V⁷ V

2688-F 2688-F 2688-F 2688-F 2688-F 1536-G 1536-G 1536-G 768-G

28 32 28 32 28 16 24 16 8
4 5 4 5 4 4 3 4 8
24 17 24 17 24 21 24 24
1 0.988 1 0.988 1 1 1.016 1 1

F-h112*>	F-h160*>	F-h112*>	F-h160*>	F-h112*>	F-h112*	1404.4Hz
E-h104*	- h150*	E-h104*	- h150*	E-h104*	E-h104*	
D#h100*	D#h144	D#h100*	D#h144	D#h100*	D#h100*	
D-h96*	- h140*	D-h96*	- h140*	D-h96*	D-h96*	
C#h88*	D-h136	C#h88*	D-h136	C#h88*	C#h88*	
C-h84*	- h130*	C-h84*	- h130*	C-h84*	C-h84*	
B-h80*	C-h120*	B-h80*	C-h120*	B-h80*	B-h80*	
G-h32*	F-h40*	G-h32*	F-h40*	G-h32*	G-h64*>	802.5Hz
F-h28*	- h35*	F-h28*	- h35*	F-h28*	F-h56*	G-h72*>
D-h24*	C-h30*	D-h24*	C-h30*	D-h24*	D#h50	F-h63*
- h23	A-h25*	- h23	A-h25*	- h23	D-h48*	E-h60*
C#h22	G#h24	C#h22	G#h24	C#h22	C#h44*	- h54*
C-h21	- h23	C-h21	- h23	C-h21	C-h42	C-h48*
B-h20*	G-h22	B-h20*	G-h22	B-h20*	B-h40*	A#h42*
A#h19	F#h21	A#h19	F#h21	A#h19	-----	G-h36*
A-h18	F-h20*	A-h18	F-h20*	A-h18	B-h20*	- h33*
G#h17	E-h19	G#h17	E-h19	G#h17	A-h18	E-h30*
G-h16*	D#h18	G-h16*	D#h18	G-h16*	G-h16*	- h29
F#h15	D-h17	F#h15	D-h17	F#h15	F#h15	D#h28
F-h14	C#h16	F-h14	C#h16	F-h14	F-h14	- h27*
E-h13	C-h15*	E-h13	C-h15*	E-h13	E-h13	C-h24*
D-h12*	B-h14	D-h12*	B-h14	D-h12*	D-h12*	A#h21*
C#h11	A#h13	C#h11	A#h13	C#h11	C#h11	G-h18*
B-h10	G#h12	B-h10	G#h12	B-h10	B-h10	F#h17
A- h9	G-h11	A- h9	G-h11	A- h9	A- h9	F-h16
G- h8*	F-h10*	G- h8*	F-h10*	G- h8*	G- h8*	E-h15*
F- h7	D# h9	F- h7	D# h9	F- h7	F- h7	D#h14
D- h6	C# h8	D- h6	C# h8	D- h6	D- h6	D-h13
B- h5	B- h7	B- h5	B- h7	B- h5	B- h5	C-h12*
G- h4*R	G# h6	G- h4*R	G# h6	G- h4*R	G- h4*R	B-h11
D- h3	F- h5*R	D- h3	F- h5*R	D- h3	D- h3	A-h10
G- h2	C# h4	G- h2	C# h4	G- h2	G- h2	G- h9*
G-h1/H24	G# h3	G-h1/H24	G# h3	G-h1/H24	G-h1/H24	F- h8
(12.539Hz)	C# h2	(12.539Hz)	C# h2	(12.539Hz)	(12.539Hz)	D# h7
	C# h1/H17		C# h1/H17			C- h6*
	(8.777Hz)		(8.777Hz)			A- h5
						F- h4
						C- h3*R
						F- h2
						F- h1/H21
						(11.146Hz)

(28 grps 4)8:7(32 grps 5)7:8(28 grps 4)8:7(32 grps 5) --7:8--> (28 grps 4)
(-12 grps 4)
(16 groups of 4) ----3:2----> (24 groups of 3) ---2:3--> (16 groups of 4)
(16 x seven)10:7(16 x ten)7:10(16 x seven)10:7(16 x ten)7:10(16 x seven)
(-48 x one)
(8 x eight) -----9:8-----> (8 x nine) -----8:9-----> (8 x eight)
(-32 x one)
(8 x four) ->

K.11 - WALDSTEIN SONATA, L.v.BEETHOVEN

D - h48*> 1222.8Hz

611.4Hz

D - h32*>

D - h24*

- h31
C#h30
- h29

- h23
C#h22
C - h21

C - h32*>

535.0Hz

C - h28*

B - h20*

- h31
B - h30
- h29
A#h28*
- h27
A - h26
G#h25

- h27
B - h26
A#h25
A - h24*
- h23
G#h22
G - h21

A#h19
A - h18
G#h17
G - h16*
F#h15
F - h14
E - h13

G - h24*>

401.2Hz

G - h24*

- h23
F#h22
F - h21
E - h20*
D#h19
D - h18
C#h17
C - h16*
B - h15
A#h14
A - h13
G - h12*
F#h11
E - h10
D - h9
C - h8*
A# h7
G - h6
E - h5
C - h4*R
G - h3
C - h2
C - h1/H32
(16.718Hz)

- h23
F#h22
F - h21
E - h20*
D#h19
D - h18
C#h17
C - h16*
B - h15
A#h14
A - h13
G - h12*
F#h11
E - h10
D - h9
C - h8*
A# h7
G - h6
E - h5
C - h4*R
G - h3
C - h2
C - h1/H32
(16.718Hz)

F#h20*
F - h19
E - h18
D#h17
D - h16*
C#h15
C - h14
B - h13
A - h12*
G#h11
F#h10
E - h9
D - h8*
C - h7
A - h6
F# h5
D - h4*R
A - h3
D - h2
D - h1/H36
(19.107Hz)

D - h12*
C#h11
B - h10
A - h9
G - h8*
F - h7
E - h6
D - h5
C - h4*R
D - h3
G - h2
G - h1/H48
(25.476Hz)

-3:4-> (6 groups of 4) -----> (6 groups of 4)

(+2 groups of 4)

(8 groups of 4) -----7:8-----> (7 groups of 4)

(+1 group of 4)

-3:4-> (6 x four) -----> (6 x four)

(+8 x one)

(8 x four)

(8 groups of 4) -3:4-> (6 groups of 4)

(+6 group of 4)

(12 group of 4) ----->

-----7:8-----> (7 x four)

(+4 x one)

(8 x four)

--3:4--> (8 x three)

(+24 x one)

(12 x four) ----->

K.12 - WALDSTEIN SONATA, L.v.BEETHOVEN

17 CM: V 18 ii 19 III⁷

2304-D 2304-D 2304-D 2304-D 2560-E

12 16 32 28 32
4 3 3 4 4
48 48 24 20 20
1 1 1 1.029 1

1397.5Hz E-h128*>

D-h48*>	1222.8Hz	D-h48*>	D-h96*>	D-h112*>
C#h44*		C#h44	-----	B-h96*
C-h42*		C-h42*	D-h48*	-----
B-h40*		B-h40	C#h44*	E-h64*
A-h36*		A-h36*	C-h42*	D-h56*
G-h32*		G-h32	B-h40*	B-h48*
D-h24*		D-h24*	A-h36*	A#h44*
- h23		- h23	G-h32*	G#h40*
C#h22		C#h22	F-h28*	G-h38
C-h21		C-h21*	D#h25	F#h36*
B-h20*		B-h20	D-h24*	F-h34
A#h19		A#h19	- h23	E-h32*
A-h18		A-h18*	C#h22	D#h30
G#h17		G#h17	C-h21*	D-h28*
G-h16*		G-h16	B-h20	C#h26
F#h15		F#h15*	A#h19	C-h25
F-h14		F-h14	A-h18*	B-h24*
E-h13		E-h13	G#h17	- h23
D-h12*		D-h12*	G-h16	A#h22
C#h11		C#h11	F#h15*	A-h21
B-h10		B-h10	F-h14	G#h20*
A- h9		A- h9*	E-h13	G-h19
G- h8*		G- h8	D-h12*	F#h18
F- h7		F- h7	C#h11	F-h17
D- h6		D- h6*	B-h10	E-h16*
B- h5		B- h5	A- h9*	D#h15
G- h4*R		G- h4	G- h8	D-h14
D- h3		D- h3*R	F- h7	C#h13
G- h2		G- h2	D- h6*	B-h12*
G-h1/H48		G-h1/H48	B- h5	A#h11
(25.476Hz)		(25.476Hz)	G- h4	G#h10
			D- h3*R	F# h9
			G- h2	E- h8*
			G-h1/H24	D- h7
			(12.738Hz)	B- h6
				G# h5
				E- h4*R
				B- h3
				E- h2
				E- h1/H20
				(10.918Hz)

----> (12 groups of 4) -----4:3-----> (16 groups of 3) -----2:1-----> (32 groups of 3) ----7:8----> (28 groups of 4)
 (+4 groups of 4)
 (32 groups of 4)

-----> (12 x four) -----> (48 x one) -----2:1-----> (48 x two)
 (16 x six) -----7:6-----> (16 x seven)
 (+16 x one)
 (8 x sixteen)

K.13 - WALDSTEIN SONATA, L.v.BEETHOVEN

CM: vi

I⁷ (Em:VI⁷)

2560-E

2560-E-640

2560-E 1920-B

24

5

21

1.016

24 -- 6

5

21

1.016

20

15

8

16

1

1

E-h120*> 1397.5Hz

- h100*

A-h80*

F-h64

E-h60*

- h50*

C-h48

B-h44

A-h40*

- h35*

E-h30*

C#h25*

C-h24

- h23

B-h22

A#h21

A-h20*

G#h19

G-h18

F#h17

F-h16

E-h15*

D#h14

D-h13

C-h12

B-h11

A-h10*

G-h9

F-h8

D#h7

C-h6

A-h5*R

F-h4

C-h3

F-h2

F-h1/H21

(11.646Hz)

(E)-h120*>

D#h112

D-h104

C-h96

B-h88

A#h84

A-h80*

C-h48

B-h44

A-h40*

- h35*

E-h30*

C#h25*

C-h24

- h23

B-h22

A#h21

A-h20*

G#h19

G-h18

F#h17

F-h16

E-h15*

D#h14

D-h13

C-h12

B-h11

A-h10*

G-h9

F-h8

D#h7

C-h6

A-h5*R

F-h4

C-h3

F-h2

F-h1/H21

(11.646Hz)

(E)-h160**

D#h152**

D-h144**

C-h128**

B-h120**> 1048.1Hz

A#h112**

E-h80**

C-h64**

B-h60*

G-h48**

E-h40**

C-h32**

A#h28*

- h27

A-h26

G#h25

G-h24**

- h23

F#h22

F-h21

E-h20*

D#h19

D-h18

C#h17

C-h16**

B-h15

A#h14

A-h13

G-h12*

F#h11

E-h10

D-h9

C-h8**

A#h7

G-h6

E-h5

C-h4*R

G-h3

C-h2

C-h1/H16

(8.734Hz)

-3:4-> (24 groups of 5) -----> (24 groups of 5) -----5:6-----> (20 groups of 8)

(-5 groups of 8)

(15 groups of 8) -----16:15----->

-15:16-> (8 x fifteen) -----> (40 x three) -----4:3-----> (40 x four)

(-40 x one)

(8 x fifteen) -----16:15----->

K.14 - WALDSTEIN SONATA, L.v.BEETHOVEN

1920-B

16
8
15
1

1920-B

12
10
16
1

1920-B-480

16 -- 4
8
15
1

1920-B-480

12 -- 3
10
16
1

(B)-h128**> 1048.1Hz

A#h120**
A-h112**
G#h104**
G-h100**
F#h96**
E-h84**
D#h80**
C#h72**
B-h64**
A#h60**
A-h56**

B-h120**>

A#h112
A-h104
G#h100**
G-h96
F#h88
E-h80**
D#h76
C#h68
C-h64
B-h60**
A#h56

B-h128**>*

A#h120**
A-h112**
G#h104**
G-h100**
F#h96**
D#h80**
D-h76**
C#h72**
B-h64**
A#h60**
A-h56**

B-h120**>*

A#h112
A-h104
G#h100**
G-h96
F#h88
E-h80**
D#h76
C#h68
C-h64
B-h60**
A#h56

(B)-h32**> 262.0Hz

- h31
A#h30
- h29
A-h28*
- h27
G#h26
G-h25
F#h24**
- h23
F-h22
E-h21
D#h20*
D-h19
C#h18
C-h17
B-h16**
A#h15
A-h14
G#h13
F#h12*
F-h11
D#h10
C# h9
B- h8**R
A- h7
F# h6
D# h5
B- h4*
F# h3
B- h2
B- h1/H15
(8.189Hz)

(B)-h30**>

A#h28
- h27
A-h26
G#h25*
G-h24
- h23
F#h22
F-h21
E-h20**
D#h19
D-h18
C#h17
C-h16
B-h15*
A#h14
A-h13
G-h12
F#h11
E-h10**R
D- h9
C- h8
A# h7
G- h6
E- h5*
C- h4
G- h3
C- h2
C-h1/H16
(8.734Hz)

(B)-h32**>*

- h31
A#h30
- h29
A-h28*
- h27
G#h26
G-h25
F#h24**
- h23
F-h22
E-h21
D#h20*
D-h19
C#h18
C-h17
B-h16**
A#h15
A-h14
G#h13
F#h12*
F-h11
D#h10
C# h9
B- h8**R
A- h7
F# h6
D# h5
B- h4*
F# h3
B- h2
B- h1/H15
(8.189Hz)

(B)-h30**>*

A#h28
- h27
A-h26
G#h25*
G-h24
- h23
F#h22
F-h21
E-h20**
D#h19
D-h18
C#h17
C-h16
B-h15*
A#h14
A-h13
G-h12
F#h11
E-h10**R
D- h9
C- h8
A# h7
G- h6
E- h5*
C- h4
G- h3
C- h2
C-h1/H16
(8.734Hz)

(16 groups of 8) -----3:4-----> (12 groups of 10) -----4:3-----> (16 groups of 8) -----3:4-----> (12 groups of 10) -----4:3----->

(8 groups of 4) -----3:4-----> (6 groups of 5) -----4:3-----> (8 groups of 4) -----3:4-----> (6 groups of 5) -----4:3----->

--> (8 x sixteen) -----15:16-----> (8 x fifteen) -----16:15-----> (8 x sixteen) -----15:16-----> (8 x fifteen) -----16:15----->

K.15 - WALDSTEIN SONATA, L.v.BEETHOVEN

25

Em: V

i

26

V7

i

1920-B

16

8

15

1

1920-B

12

10

16

1

1920-B

16

8

15

1

1920-B

12

10

16

1

27. *Em: V*

1920-B 12 8 10 15 16 1

28. *V i V i*

1920-B 12 8 10 15 16 1

29 30

Em: V V

1920-B 1408F# 1216D# 960-B 704F# 608D# 480-B 240-B 1920-B

16 12 10 8 6 5 4 2 16

8 8 8 8 8 8 8 8 8

15 15 15 15 15 15 15 15 15

1 1 1 1 1 1 1 1 1

K.16 - WALDSTEIN SONATA, L.v.BEETHOVEN

31 32 33 34

Em: V

1920-B 2176C#

16 18
8 8
15 15
1 1.007

2176C#

24 18
5 30
1.007 1.007

2176C# 1920-B

18 16
4 4
30 30
1.007 1

C#h144*>

C#h144*>

1179.2Hz

C#h120*>

C#h72*

B-h128* 1048.1Hz

G#h104*

A#h100*

B-h64*> 1048.1Hz

F#h96*

F#h96*

A-h96

A-h56*

E-h84*

E-h84*

G#h88

G#h52*

D#h80*

D#h80*

F#h80*

F#h48*

C#h72*

C#h72*

- h70*

E-h42

B-h64*

B-h64*

D-h64

D#h40*

A-h56*

A-h56*

C#h60*

C#h36*

G#h52*

G#h52*

B-h52

B-h32*

F#h48*

F#h48*

A#h50*

- h31

E-h42

E-h42

A-h48

A#h30

D#h40*

D#h40*

- h29

C#h36*

C#h36*

A#h25*

A-h28*

B-h32*

B-h32*

A-h24

- h27

- h31

- h31

- h23

G#h26

A#h30

A#h30

G#h22

G-h25

- h29

- h29

G-h21

F#h24*

A-h28*

A-h28*

F#h20*

- h23

- h27

- h27

F-h19

F-h22

G#h26

G#h26

E-h18

E-h21

G-h25

G-h25

D#h17

D#h20*

F#h24*

F#h24*

D-h16

D-h19

- h23

- h23

C#h15*

C#h18

F-h22

F-h22

C-h14

C-h17

E-h21

E-h21

B-h13

B-h16*

D#h20*

D#h20*

A-h12

A#h15

D-h19

D-h19

G#h11

A-h14

C#h18

C#h18

F#h10*

G#h13

C-h17

C-h17

E-h9

F#h12*

B-h16*

B-h16*

D-h8

F-h11

A#h15

A#h15

C-h7

D#h10

A-h14

A-h14

A-h6

C#h9

G#h13

G#h13

F#h5*R

B-h8*

F#h12*

F#h12*

D-h4

A-h7

F-h11

F-h11

A-h3

F#h6

D#h10

D#h10

D-h2

D#h5

C#h9

C#h9

D-h1/H18

B-h4*R

B-h8*

B-h8*

(9.826Hz)

F#h3

A-h7

A-h7

B-h2

F#h6

F#h6

B-h1/H30

D#h5

D#h5

(16.377Hz)

B-h4*R

B-h4*R

F#h3

F#h3

B-h2

B-h2

B-h1/H15

B-h1/H15

(8.189Hz)

(8.189Hz)

--> (16 groups of 8)
(+2 groups of 8)
(18 groups of 8)

(16 groups of 4) --3:4-->
(-2 groups of 4)
(18 groups of 4)

--> (16 x eight)
(+16 x one)
(24 x six)

(16 x four) -----3:4----->
(-8 x one)
(24 x three)

4:3----->

(24 groups of 5) --3:4-->

5:6----->

(24 x five) -----3:5----->

K.17 - WALDSTEIN SONATA, L.v.BEETHOVEN

[illegible]

K.18 - WALDSTEIN SONATA, L.v.BEETHOVEN

[illegible]

K.19 - WALDSTEIN SONATA, L.v.BEETHOVEN

960-B	1920-B		1920-B	1920-B	1920-B	1600G#	1600G#	1600G#	1600G#	1664-A
6	12		12	8	7	6	8	16	15	8
4	4		4	6	5	5	4	2	2	4
40	40		40	40	52	52	50	50	52	52
1	1		1	1	1.055	1.026	1	1	1.026	1

B-h48*>	1078.1Hz	B-h48*>	B-h48*>	B-h35*	A-h32*>	958.3Hz
A#h44*		A-h42	- h45*	G#h30*>	G#h32*>	G#h30*
G#h40*		G#h40*	A-h42**	F#h26	- h31	G-h28*
G#h38		G#h38	G#h40	F-h25*	G-h30*	F#h26*
F#h36*		F#h36*	F#h36**	E-h24	- h29	F-h25
F-h34		F-h34	F-h34	- h23	F#h28**	E-h24*
E-h32*		E-h32*	E-h32	D#h22	- h27	- h23
D#h30		D#h30	D#h30**	D-h21	F-h26*	D#h22*
D-h28*		D-h28*	D-h28	C#h20*	E-h25	D-h21
- h27		- h27	- h27*	C-h19	D#h24**	C#h20*
C#h26		C#h26	C-h25	B-h18	- h23	C-h19
C-h25		C-h25	B-h24**	A#h17	D-h22*	B-h18*
B-h24*	539.0Hz	B-h24*	A#h22	A-h16	C#h21	A#h17
- h23		- h23	A-h21*	G#h15*	C-h20**	A-h16*
A#h22		A#h22	G#h20	G-h14	B-h19	G#h15
A-h21		A-h21	G-h19	F#h13	A#h18*	G-h14*
G#h20*		G#h20*	F#h18**	E-h12	A-h17	F#h13
G-h19		G-h19	F-h17	D#h11	G#h16**	E-h12*
F#h18		F#h18	E-h16	C#h10*	G-h15	D#h11
F-h17		F-h17	D#h15*	B-h9	F#h14*	C#h10*
E-h16*		E-h16*	D-h14	A-h8	F-h13	B-h9
D#h15		D#h15	C#h13	G-h7	D#h12**	A-h8*
D-h14		D-h14	B-h12**	E-h6	D-h11	G-h7
C#h13		C#h13	A#h11	C#h5*R	C-h10*	E-h6*
B-h12*		B-h12*	G#h10	A-h4	A#h9	C#h5
A#h11		A#h11	F#h9*	E-h3	G#h8**	A-h4*
G#h10		G#h10	E-h8	A-h2	F#h7	E-h3
F#h9		F#h9	D-h7	A-h1/H52	D#h6*	A-h2*R
E-h8*		E-h8*	B-h6**	(29.947Hz)	C-h5	A-h1/H52
D-h7		D-h7	G#h5		G#h4**	(29.947Hz)
B-h6		B-h6	E-h4		D#h3	
G#h5		G#h5	B-h3*R		G#h2*R	
E-h4*R		E-h4*R	E-h2		G#h1/H50	
B-h3		B-h3	E-h1/H40		(28.075Hz)	
E-h2		E-h2	(22.460Hz)			
E-h1/H40		E-h1/H40	(22.460Hz)			

3:4-> (6 groups of 4) --> (+6 grps 4) -----> (12 grps 4)-2:3->(8 grps 6) -7:8-> (7 grps 5)
 (-1 grps 5)
 (6 grps 5) -4:3-> (8 grps 4)
 -2:1-> (16 grps 2) -15:16-> (15 grps 2)
 (+1 grp 2)
 -----> (6 x four) -----> (+24 x one) -----> (12 x four)-3:4->(12 ~ three)
 (-6 x one)
 (2 x fifteen) 16:15 (2 x sixteen)-15:16->(2 x fifteen)
 (+2 x one)
 (4 x eight) -7:8->

1664-A 1920-B	1920-B	2816F#	2816F#	1920-B	1920-B	1920-B	1920-B	1920-B	1088C#	1088C#				
7 4 60 0.990	8 4 60 1	12 4 40 1	18 4 40 0.978	16 5 36 0.978	18 4 40 0.978	12 4 40 1	16 3 40 1	16 4 30 1	12 4 40 1	960-B ----- 6 4 40 1	960-B ----- 8 4 30 1	9 4 30 1.007	8 5 26 1.046	800G# ----- 6 5 26 1.026
		F#h72*>	F#h80*>	F#h72*	1642.8Hz									
		E-h64*	- h75*	E-h64*										
		D-h56*	- h70*	D-h56*										
	1095.2Hz	B-h32*>	B-h48*	B-h48*		B-h48*>	B-h48*>	B-h64*>	B-h48*	1095.2Hz				
		A#h30*	A#h44*	C#h60*	A-h42	A-h42*	A-h42*	A-h56*	A#h44*					
958.3Hz	A-h28*	G#h40*	A#h50*	G#h40*	G#h40	G#h40	G#h40	F#h48*	G#h40*					
	G#h26	G-h38	G#h44	G-h38	G-h38	G-h38	G-h38	D#h40*	F#h36*					
	G-h25	F#h36*	F#h40*	F#h36*	F#h36*	F#h36*	F#h36*	C#h36*	E-h32*					
	F#h24*	F-h34	E-h36	F-h34	F-h34	F-h34	F-h34	B-h32*	D-h28*					
	- h23	E-h32*	- h35*	E-h32*	E-h32	E-h32	E-h32	A#h30	C-h25			C#h36*>	C#h40*	
	F-h22	D#h30	C#h30*	D#h30	D#h30	D#h30	D#h30	A-h28*	B-h24*>			B-h32*	- h35*	
	E-h21	D-h28*	A#h25*	D-h28*	D-h28	D-h28	D-h28	G#h26	- h23			A-h28*	G#h30*>	
	D#h20*	C#h26	A-h24	C#h26	C#h26	C#h26	C#h26	G-h25	A#h22			G#h26	F#h26	
	D-h19	C-h25	- h23	C-h25	C-h25	C-h25	C-h25	F#h24*	A-h21			G-h25	F-h25*	
	C#h18	B-h24*	G#h22	B-h24*	B-h24*	B-h24*	B-h24*	- h23	G#h20*			F#h24*	E-h24	
	C-h17	- h23	G-h21	- h23	- h23	- h23	- h23	F-h22	G-h19			F-h22	D#h22	
	B-h16*	A#h22	F#h20*	A#h22	A#h22	A#h22	A#h22	E-h21	F#h18			E-h21	D-h21	
	A#h15	A-h21	F-h19	A-h21	A-h21	A-h21	A-h21	D#h20*	F-h17			D#h20*	C#h20*	
	A-h14	G#h20*	E-h18	G#h20*	G#h20	G#h20	G#h20	D-h19	E-h16*			D-h19	C-h19	
	G#h13	G-h19	D#h17	G-h19	G-h19	G-h19	G-h19	D#h18	D#h15			C#h18	B-h18	
	F#h12*	F#h18	D-h16	F#h18	F#h18	F#h18	F#h18	C-h17	D-h14			C-h17	A#h17	
	F-h11	F-h17	C#h15*	F-h17	F-h17	F-h17	F-h17	B-h16*	C#h13			B-h16*	A-h16	
	D#h10	E-h16*	C-h14	E-h16*	E-h16	E-h16	E-h16	A#h15	B-h12*			A#h15	G#h15*	
	C# h9	D#h15	B-h13	D#h15	D#h15	D#h15	D#h15	A-h14	A#h11			A-h14	G-h14	
	B-h8*	D-h14	A-h12	D-h14	D-h14	D-h14	D-h14	G#h13	G#h10			G#h13	F#h13	
	A-h7	C#h13	G#h11	C#h13	C#h13	C#h13	C#h13	F#h12*	F# h9			F#h12*	E-h12	
	F-h6	B-h12*	F#h10*	B-h12*	B-h12*	B-h12*	B-h12*	F-h11	E-h8*			F-h11	D#h11	
	D# h5	A#h11	E-h9	A#h11	A#h11	A#h11	A#h11	D#h10	D-h7			D#h10	C#h10*	
	B-h4*R	G#h10	D-h8	G#h10	G#h10	G#h10	G#h10	C# h9	B-h6			C# h9	B-h9	
	F# h3	F# h9	C-h7	F# h9	F# h9	F# h9	F# h9	B-h8*	G# h5			B-h8*	A-h8	
	B-h2	E-h8*	A-h6	E-h8*	E-h8	E-h8	E-h8	A-h7	E-h4*R			A-h7	G-h7	
	B-h1/H60	D-h7	F# h5*R	D-h7	D-h7	D-h7	D-h7	F# h6	B-h3			F# h6	E-h6	
	(34.225Hz)	B-h6	D-h4	B-h6	B-h6	B-h6	B-h6	D# h5	E-h2			D# h5	C# h5*R	
		G# h5	A-h3	G# h5	G# h5	G# h5	G# h5	B-h4*R	E-h1/H40			B-h4*R	A-h4	
		E-h4*R	D-h2	E-h4*R	E-h4	E-h4	E-h4	F# h3	(22.817Hz)			F# h3	E-h3	
		B-h3	D-h1/H36	B-h3	B-h3	B-h3	B-h3	B-h2				B-h2	A-h2	
		E-h2	(20.535Hz)	E-h2	E-h2	E-h2	E-h2	B-h1/H30				B-h1/H30	A-h1/H26	
		E-h1/H40		E-h1/H40	E-h1/H40	E-h1/H40	E-h1/H40	(17.112Hz)				(17.112Hz)	(15.401Hz)	
-7:8-> (7 grps of 4)	(+1 grp of 4)													(6 grps 5)->
														(-2 grps 5)
														(9 grps 4)8:9(8 grps 5)
														(+1 grps 4)
														(6 grps 4)4:3(8 grps 4)
														(-6 grps 4)
-7:8-> (4 x seven)	(+4 x one)													(2 x fifteen)->
														(-10 x one)
														(4 x nine) 10:9 (4 x ten)
														(+4 x one)
														(8 x three) 4:3 (8 x four)
														(-24 x one)
														(12 x four)----->(16 x three) 4:3 (16 x four) 3:4 (16 x three)

K.21 - WALDSTEIN SONATA, L.v.BEETHOVEN

800G#	800G# 640-E	640-E 704F#	704-F#	704F# 960-B	960-B	960-B	1920-B
8 --16	15 12	8 9	12	6 8	6	4	8
4 -- 2	2 2	4 4	3	4 4	40	4	4
25	26 26	20 20	20	30 30	1	60	60
1	1.026 1.026	1 0.978	0.978	0.978 1		1	1

B-h32*> 1108.9Hz

F#h24*

B-h32*> B-h24*> 554.4Hz B-h16*

462.0Hz G#h32*>

G-h30*

F#h28*

F-h26*

E-h25

D#h24**

- h23

D-h22*

C#h21

C-h20**

B-h19

A#h18*

A-h17

G#h16**

G-h15

F#h14*

F-h13

D#h12**

D-h11

C-h10*

A# h9

G# h8**

F# h7

D# h6*

C- h5

G# h4**

D# h3

G# h2*R

G# h1/H25

(14.439Hz)

E-h24*> 369.6Hz

- h23

D#h22*

D-h21

C#h20*

C-h19

B-h18*

A#h17

A-h16*

G#h15

G-h14*

F#h13

E-h12*

D#h11

C#h10*

B- h9

A- h8*

G- h7

E- h6*

C# h5

A- h4*

E- h3

A- h2*R

A- h1/H27

(15.401Hz)

F#h36*> 415.8Hz

E-h32*

D-h28*

C-h25

B-h24*

- h23

A#h22

A-h21

G#h20*

G-h19

F#h18

F-h17

E-h16*

D#h15

D-h14

C#h13

B-h12*

A#h11

G#h10

F# h9

E- h8*

D- h7

B- h6

G# h5

E- h4*R

B- h3

E- h2

E- h1/H20

(11.551Hz)

F#h36*>

- h33*

D#h30**

- h27*

C-h25

B-h24**

A#h22

A-h21

G#h20

G-h19

F#h18**

F-h17

E-h16

D#h15*

D-h14

C#h13

B-h12**

A#h11

G#h10

F# h9*

E- h8

D- h7

B- h6**

G# h5

E- h4

B- h3*R

E- h2

E- h1/H20

(11.551Hz)

F#h24*

- h23

F-h22

E-h21

D#h20*

D-h19

C#h18

C-h17

B-h16*

A#h15

A-h14

G#h13

F#h12*

F-h11

D#h10

C# h9

B- h8*

A- h7

F# h6

D# h5

B- h4*R

F# h3

B- h2

B- h1/H30

(17.326Hz)

G#h20*

G-h19

F#h18

F-h17

E-h16*

D#h15

D-h14

C#h13

B-h12*

A#h11

G#h10

F# h9

E- h8*

D- h7

B- h6

G# h5

E- h4*R

B- h3

E- h2

E- h1/H40

(23.102Hz)

B-h16*

A#h15

A-h14

G#h13

F#h12*

F-h11

D#h10

C# h9

B- h8*

A- h7

F# h6

D# h5

B- h4*R

F# h3

B- h2

B- h1/H60

(34.653Hz)

-4:3-> (8 grps of 4)

2:1(16 grps of 2)15:16(15 grps of 2)

(-3 grps of 2)

(12 grps of 2) -2:3-> (8 grps of 4)

(+1 grp of 4)

(9 grps of 4) 4:3 (12 grps of 3) 1:2 (6 grps of 4)

(+2 grps of 4)

(8 grps of 4)-3:4->(6 grps of 4) -2:3-> (4 grps of 4)

(+4 grps of 4)

(8 grps of 4) -3:2->

16:15 (2 x sixteen) 15:16 (2 x fifteen)

(-6 x one)

(8 x three) ---4:3---> (8 x four)

(+4 x one)

(12 x three) -----2:3-> (12 x two)

(+8 x one)

(8 x four) -3:4-> (8 x three) ---2:3---> (8 x two)

(+16 x one)

(16 x two) -3:2->

K.22 - WALDSTEIN SONATA, L.v.BEETHOVEN

51 52 53

EM:I V7 simile...

1920-B 1920-B 1920-B 1920-B 1920-B 1920-B

12	8	12	8	12	8
4	4	4	4	4	4
40	60	40	60	40	60
1	1	1	1	1	1

54 55 56

1920-B 1920-B 1920-B 1920-B 3840-B 3840-B 3840-B

12	8	12	8	16	24	16
4	4	4	4	4	4	4
40	60	40	60	60	40	60
1	1	1	1	1	1	1

57 58

3840-B 3840-B 3840-B 3840-B 3840-B 3840-B

24	16	24	16	24	16
4	4	4	4	4	4
40	60	40	60	40	60
1	1	1	1	1	1

59 60

3840-B-3840 simile..... 3840-B

24 -- 16	24
4 -- 4	4
40 - 60	40
1	1

K.23 - WALDSTEIN SONATA, L.v.BEETHOVEN

3840-B

24
4
40
1

2560-E

12
4
52
1.026

B-h96*>

2217.8Hz

E-h64*

D-h56*

B-h48*

A#h44*

G#h40*

G-h38

F#h36*

F-h34

E-h32*

D#h30

D-h28*

C#h26

C-h25

B-h24*

- h23

A#h22

A-h21

G#h20*

G-h19

F#h18

F-h17

E-h16*

D#h15

D-h14

C#h13

B-h12*

A#h11

G#h10

F# h9

E-h8*

D-h7

B-h6

G# h5

E-h4*R

B-h3

E-h2

E-h1/H40

(23.102Hz)

(24 groups of 4) -----3:4-----> (18 groups of 4)

(-6 groups of 4)

(12 groups of 4) ----->

(24 x four) -----3:4-----> (24 x three)

(-24 x one)

(12 x four) ----->

B-h72*

A-h64*

G-h56*

E-h48*>

1478.5Hz

C#h40*

B-h36*

A-h32*

G-h28*

F-h25

E-h24*

- h23

D#h22

D-h21

C#h20*

C-h19

B-h18

A#h17

A-h16*

G#h15

G-h14

F#h13

E-h12*

D#h11

C#h10

B-h9

A-h8*

G-h7

E-h6

C# h5

A-h4*R

E-h3

A-h2

A-h1/H52

(30.802Hz)

K.24 - WALDSTEIN SONATA, L.v.BEETHOVEN

2560-E 4352C#

 12 20
 4 4
 52 52
 1.026 1.046

4352C# 2560-E

 24 14
 4 4
 44 44
 1.030 1.039

C#h80*>	2464.2Hz	C#h96*
B-h72*		C-h88*
A-h64*		A#h80*
G-h56*		G#h72*
1478.5Hz E-h48*		F#h64*
D#h44*		(E)-h56*>
(C#)h40*		1437.4Hz
(C)-h38		D#h52*
(A)-h32*		(C#)h48*
(G#)h30		(C)-h44*
G-h28*		A#h40*
F-h25		F#h32*
(E)-h24*		E-h28*
- h23		D-h25
D#h22		C#h24*
D-h21		- h23
C#h20*		C-h22
C-h19		B-h21
B-h18		A#h20*
A#h17		A-h19
A-h16*		G#h18
G#h15		G-h17
G-h14		F#h16*
F#h13		F-h15
E-h12*		E-h14
D#h11		D#h13
C#h10		C#h12*
B-h9		C-h11
A-h8*		A#h10
G-h7		G# h9
(E)-h6		(F#)h8*
(C#)h5		(E)-h7
(A)-h4*R		(C#)h6
E-h3		(A#)h5
A-h2		F# h4*R
A-h1/H52		C# h3
(30.802Hz)		F# h2
		F# h1/H44
		(25.669Hz)

--> (12 groups of 4)
 (+8 groups of 4)
 (20 groups of 4) -----6:5-----> (24 groups of 4)
 (-10 groups of 4)
 (14 groups of 4) ----->
 (+32 x one)
 --> (16 x five) -----6:5-----> (16 x six)
 (-40 x one)
 (8 x seven) ----->

K.25 - WALDSTEIN SONATA, L.v.BEETHOVEN

65 EM: II^7 66 I^6_4

2560-E

14
4
44
1.039

2560-E

16
4
40
1

2560-E

16
4
40
1

(E)-h56*>
(D#)-h52*
(C#)-h48*
(C)-h44*
A#-h40*
F#-h32*
E-h28*
D-h25
C#-h24*
- h23
C-h22
B-h21
A#-h20*
A-h19
G#-h18
G-h17
F#-h16*
F-h15
E-h14
D#-h13
C#-h12*
C-h11
A#-h10
G# h9
(F#) h8*
(E) h7
(C#) h6
(A#) h5
F# h4*R
C# h3
F# h2
F# h1/H44
(25.669Hz)

1437.4Hz

E-h64*>
D-h56*
(B)-h48*
A#-h44*
(G#)-h40*
G-h38
F#-h36*
F-h34
(E)-h32*
D#-h30
D-h28*
C#-h26
C-h25
(B)-h24*
- h23
A#-h22
A-h21
(G#)-h20*
G-h19
F#-h18
F-h17
(E)-h16*
D#-h15
D-h14
C#-h13
(B)-h12*
A#-h11
(G#)-h10
F# h9
(E)-h8*
D-h7
(B)-h6
G# h5
E-h4*R
B-h3
E-h2
E-h1/H40
(22.460Hz)

(E)-h64*>
D-h56*
(B)-h48*
A#-h44*
(G#)-h40*
G-h38
F#-h36*
F-h34
(E)-h32*
D#-h30
D-h28*
C#-h26
C-h25
(B)-h24*
- h23
A#-h22
A-h21
(G#)-h20*
G-h19
F#-h18
F-h17
E-h16*
D#-h15
D-h14
C#-h13
(B)-h12*
A#-h11
(G#)-h10
F# h9
(E)-h8*
D-h7
(B)-h6
G# h5
E-h4*R
B-h3
E-h2
E-h1/H40
(22.460Hz)

--> (14 groups of 4) -----8:7-----> (16 groups of 4) -----> (16 groups of 4) ----->

--> (8 x seven) -----8:7-----> (8 x eight) ----->

K.26 - WALDSTEIN SONATA, L.v.BEETHOVEN

2560-E

16
4
40
1

2560-E

16
4
40
1

2560-E

8
4
80
1

2560-E

8
4
80
1

E-h64*> 1437.4Hz
D-h56*
B-h48*
A#h44*
G#h40*
G-h38
F#h36*
F-h34
E-h32*
D#h30
D-h28*
C#h26
C-h25
B-h24*
- h23
A#h22
A-h21
G#h20*
G-h19
F#h18
F-h17
E-h16*
D#h15
D-h14
C#h13
B-h12*
A#h11
G#h10
F# h9
E- h8*
D- h7
B- h6
G# h5
E- h4*R
B- h3
E- h2
E- h1/H40
(22.460Hz)

E-h64*>
D-h56*
B-h48*
A#h44*
G#h40*
G-h38
F#h36*
F-h34
E-h32*
D#h30
D-h28*
C#h26
C-h25
B-h24*
- h23
A#h22
A-h21
G#h20*
G-h19
F#h18
F-h17
E-h16*
D#h15
D-h14
C#h13
B-h12*
A#h11
G#h10
F# h9
E- h8*
D- h7
B- h6
G# h5
E- h4*R
B- h3
E- h2
E- h1/H40
(22.460Hz)

E-h32*>
D#h30
D-h28*
C#h26
C-h25
B-h24*
- h23
A#h22
A-h21
G#h20*
G-h19
F#h18
F-h17
E-h16*
D#h15
D-h14
C#h13
B-h12*
A#h11
G#h10
F# h9
E- h8*
D- h7
B- h6
G# h5
E- h4*R
B- h3
E- h2
E- h1/H80
(44.920Hz)

E-h32*>
D#h30
D-h28*
C#h26
C-h25
B-h24*
- h23
A#h22
A-h21
G#h20*
G-h19
F#h18
F-h17
E-h16*
D#h15
D-h14
C#h13
B-h12*
A#h11
G#h10
F# h9
E- h8*
D- h7
B- h6
G# h5
E- h4*R
B- h3
E- h2
E- h1/H80
(44.920Hz)

--> (16 groups of 4) -----1:2--> (8 groups of 4) ----->

---> (8 x eight) -----1:2--> (8 x four) ----->

K.27 - WALDSTEIN SONATA, L.v.BEETHOVEN

2560-E 2816F#

8 9
4 4
80 80
1 0.978

F#h36*> 1617.1Hz

F-h34

1437.4Hz E-h32*

D#h30

D-h28*

C#h26

C-h25

(B)-h24*

- h23

A#h22

A-h21

(G#)h20*

G-h19

F#h18

F-h17

(E)-h16*

D#h15

D-h14

C#h13

(B)-h12*

A#h11

G#h10

F# h9

E- h8*

D- h7

B- h6

G# h5

E- h4*R

B- h3

E- h2

E- h1/H80

(44.920Hz)

2816F#

12
3
80
0.978

F#h36*>

- h33*

E-h32

D#h30*

- h27*

C#h26

(C)-h25

(B)-h24*

- h23

A#h22

(A)-h21*

G#h20

G-h19

F#h18*

F-h17

E-h16

(D#)h15*

D-h14

C#h13

(B)-h12*

A#h11

G#h10

F# h9*

E- h8

D- h7

B- h6*

G# h5

E- h4

B- h3*R

E- h2

E- h1/H80

(44.920Hz)

--> (8 groups of 4)

(+1 group of 4)

(9 groups of 4) -----4:3-----> (12 groups of 3) ----->

---> (4 x eight) -----> (+4 x one) -----> (9 x four) ----->

K.28 - WALDSTEIN SONATA, L.v.BEETHOVEN

2816F#

12
3
80
0.978

2816F#

12
3
80
0.978

F#h36**>

1617.1Hz

- h33*

E-h32

D#h30**

- h27*

C#h26

Ⓢh25

Ⓢh24**

- h23

A#h22

Ⓢh21*

G#h20

G-h19

F#h18**

F-h17

E-h16

Ⓢh15*

D-h14

C#h13

Ⓢh12**

A#h11

G#h10

F# h9*

E- h8

D- h7

B- h6**

G# h5

E- h4

B- h3*R

E- h2

E- h1/H80

(44.920Hz)

F#h36**>

- h33*

E-h32

D#h30**

- h27*

Ⓢh26

Ⓢh25

Ⓢh24**

- h23

A#h22

Ⓢh21*

G#h20

G-h19

F#h18**

F-h17

Ⓢh16

Ⓢh15*

D-h14

C#h13

B-h12**

A#h11

G#h10

F# h9*

E- h8

D- h7

B- h6**

G# h5

E- h4

B- h3*R

E- h2

E- h1/H80

(44.920Hz)

-1:2-> (6 groups of 6) -----> (6 groups of 6) -----1:1---->

-----> (6 x six) -----> (12 x three) -----2:3---->

K.29 - WALDSTEIN SONATA, L.v.BEETHOVEN

2816F#

6
4
120
0.978

2816F# 2560-E 2560-E

9 8 16
4 4 4
80 80 40
0.978 1 1

(F#h24*>
- h23
F-h22
E-h21
D#h20*
D-h19
C#h18
C-h17
(B)-h16*
A#h15
(A)-h14
G#h13
F#h12*
F-h11
(D#)h10
C# h9
B- h8*
A- h7
F# h6
D# h5
B- h4*R
F# h3
B- h2
B- h1/H120
(67.380Hz)

1617.1Hz

F#h36*

F-h34

(E)-h32*>

D#h30

D-h28*

C#h26

C-h25

(B)-h24*

- h23

A#h22

A-h21

(G#)h20*

G-h19

F#h18

F-h17

(E)-h16*

D#h15

D-h14

C#h13

B-h12*

A#h11

G#h10

F# h9

(E)-h8*

D- h7

B- h6

G# h5

E- h4*R

B- h3

E- h2

E- h1/H80

(44.920Hz)

(E)-h16*

D#h15

D-h14

C#h13

B-h12*

A#h11

G#h10

F# h9

E- h8*

D- h7

B- h6

G# h5

E- h4*R

B- h3

E- h2

E- h1/H40

(22.460Hz)

(E)-h64*> 1437.4Hz

D#h60*

D-h56*

C#h50

(B)-h48*

A#h44*

G#h40*

F#h36*

F-h34

E-h32*

D#h30

(D)-h28*

C#h26

C-h25

(B)-h24*

- h23

A#h22

A-h21

(G#)h20*

G-h19

F#h18

F-h17

(E)-h16*

D#h15

D-h14

C#h13

B-h12*

A#h11

G#h10

F# h9

E- h8*

D- h7

B- h6

G# h5

E- h4*R

B- h3

E- h2

E- h1/H40

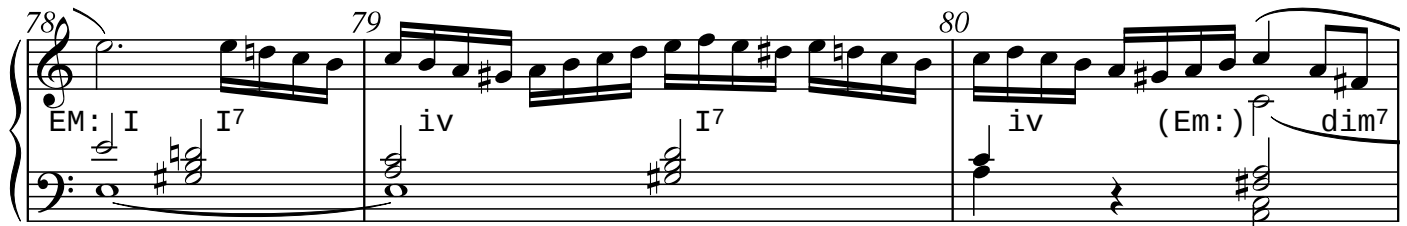
(22.460Hz)

--> (6 groups of 4) -----3:2-----> (9 grps of 4)
(-1 grp of 4)
(8 grps of 4) -2:1-> (16 grps of 4) --3:4-->
---> (12 x two) -----3:2-----> (12 x three)
(-4 x one)
(2 x sixteen) -2:1-> (4 x sixteen) -15:16->

K.30 - WALDSTEIN SONATA, L.v.BEETHOVEN

E-h60*>	(E)-h64*>	1437.4Hz	E-h60*>	E-h60*>	E-h80*
(D)-h52	(D#)h60*		(D)-h52	D-h52	- h70*
- h50*	(D+)h56*		- h50*	- h50	- h65*
(C)-h48	(C+)h50		(C)-h48	(C)-h48	(B)-h60*>
(B)-h44	(B)h48*		(B)-h44	B-h44	- h55*
(A)-h40*	A#h44*		(A)-h40*	(A)-h40	G#h50*
(G#)h38	G#h40*		(G#)h38	- h35	(G)-h48
- h35*	F#h36*		(F#)h34	(F#)h34	- h45*
E-h30*	F-h34		E-h30*	E-h30	(E)-h40*
C#h25*	E-h32*		C#h25*	C#h25	- h35*
(C)-h24	D#h30		(C)-h24	(C)-h24	(B)-h30*
- h23	(D)-h28*		- h23	- h23	A#h28
B-h22	C#h26		B-h22	B-h22	- h27
A#h21	C-h25		A#h21	A#h21	A-h26
(A)-h20*	(B)-h24*		(A)-h20*	(A)-h20	G#h25*
G#h19	- h23		G#h19	G#h19	(G)-h24
G-h18	A#h22		G-h18	G-h18	- h23
F#h17	A-h21		(F#)h17	(F#)h17	F-h22
F-h16	(G#)h20*		F-h16	F-h16	F-h21
(E)-h15*	G-h19		E-h15*	E-h15	(E)-h20*
D#h14	F#h18		D#h14	D#h14	D#h19
D-h13	F-h17		D-h13	D-h13	D-h18
C-h12	(E)-h16*		C-h12	(C)-h12	C#h17
B-h11	D#h15		B-h11	B-h11	C-h16
A-h10*	D-h14		A-h10*	(A)-h10	(B)-h15*
G- h9	C#h13		G- h9	G- h9	A#h14
F- h8	B-h12*		F- h8	F- h8	A-h13
D# h7	A#h11		D# h7	D# h7	G-h12
C- h6	G#h10		C- h6	C- h6	F#h11
A- h5*R	F# h9		A- h5	A- h5	E-h10*
F- h4	E- h8*		F- h4	F- h4	D- h9
C- h3	D- h7		C- h3	C- h3	C- h8
F- h2	B- h6		F- h2	F- h2	A# h7
F- h1/H42	G# h5		F- h1/H42	F- h1/H42	G- h6
(23.957Hz)	E- h4*R		(23.957Hz)	(23.957Hz)	E- h5*R
	B- h3				C- h4
	E- h2				G- h3
	E- h1/H40				C- h2
	(22.460Hz)				C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.968Hz)
					E- h5*R
					C- h4
					G- h3
					C- h2
					C-h1/H32
					(17.9

K.31 - WALDSTEIN SONATA, L.v.BEETHOVEN



1920-B	1280-E	1280-E	1280-E	1280-E	1280-E	1280-E
12	8	16	12	16	12	**
4	4	4	5	4	5	60
40	40	20	21	20	21	21
1	1	1	1.016	1	1.016	1.016

B-h48* 1078.1Hz

A#h44*

G#h40*

F#h36*

F-h34

F-h68

718.7Hz

(E)-h32*>	(E)-h64*>	E-h60*>	(E)-h64*>	E-h60*>	E-h60*>
D#h30	D#h60*	(D)-h52	(D)-h60*	(D)-h52	D-h52
D-h28*	(D)-h56*	- h50*	(D)-h56*	- h50*	- h50
C#h26	(C)-h50	(C)-h48	(C)-h50	(C)-h48	(C)-h48
C-h25	(B)-h48*	(B)-h44	(B)-h48*	(B)-h44	B-h44
B-h24*	A#h44*	(A)-h40*	A#h44*	(A)-h40*	(A)-h40
- h23	G#h40*	(G)-h38	G#h40*	(G)-h38	- h35
A#h22	F#h36*	- h35*	F#h36*	- h35*	(F)-h34
A-h21	F-h34	E-h30*	F-h34	E-h30*	E-h30
G#h20*	E-h32*	C#h25*	E-h32*	C#h25*	C#h25
G-h19	D#h30	(C)-h24	D#h30	(C)-h24	(C)-h24
F#h18	(D)-h28*	- h23	(D)-h28*	- h23	- h23
F-h17	C#h26	B-h22	C#h26	B-h22	B-h22
(E)-h16*	C-h25	A#h21	C-h25	A#h21	A#h21
D#h15	(B)-h24*	(A)-h20*	(B)-h24*	(A)-h20*	(A)-h20
D-h14	- h23	G#h19	- h23	G#h19	G#h19
C#h13	A#h22	G-h18	A#h22	G-h18	G-h18
B-h12*	A-h21	F#h17	A-h21	F#h17	(F)-h17
A#h11	(G)-h20*	F-h16	(G)-h20*	F-h16	F-h16
G#h10	G-h19	(E)-h15*	G-h19	E-h15*	E-h15
F# h9	F#h18	D#h14	F#h18	D#h14	D#h14
(E)- h8*	F-h17	D-h13	F-h17	D-h13	D-h13
D- h7	(E)-h16*	C-h12	(E)-h16*	C-h12	(C)-h12
B- h6	D#h15	B-h11	D#h15	B-h11	B-h11
G# h5	D-h14	A-h10*	D-h14	A-h10*	(A)-h10
E- h4*R	C#h13	G- h9	C#h13	G- h9	G- h9
B- h3	B-h12*	F- h8	B-h12*	F- h8	F- h8
E- h2	A#h11	D# h7	A#h11	D# h7	D# h7
E- h1/H40	G#h10	C- h6	G#h10	C- h6	C- h6
(22.460Hz)	F# h9	A- h5*R	F# h9	A- h5*R	A- h5
	E- h8*	F- h4	E- h8*	F- h4	F- h4
	D- h7	C- h3	D- h7	C- h3	C- h3
	B- h6	F- h2	B- h6	F- h2	F- h2
	G# h5	F- h1/H21	G# h5	F- h1/H21	F- h1/H21
	E- h4*R	(11.979Hz)	E- h4*R	(11.979Hz)	(11.979Hz)
	B- h3		B- h3		
	E- h2		E- h2		
	E- h1/H20		E- h1/H20		
	(11.230Hz)		(11.230Hz)		

->(12 grps of 4)

(-4 grps of 4)

(8 grps of 4) 2:1 (16 grps of 4) 3:4 (12 groups of 5) ---4:3---> (16 groups of 4) ----3:4----> (12 groups of 5) --3:4---/ /--->

->(16 x three)

(-16 x one)

(32 x one) 2:1 (32 x two)

(4 x sixteen) 15:16 (4 x fifteen) -----16:15-----> (4 x sixteen) -----15:16----->(4 x fifteen) -----> (20 x three) -4:3->

K.32 - WALDSTEIN SONATA, L.v.BEETHOVEN

1280-E	960-B	960-B	960-B	1280-E	1280-E	960-B	960-B	960-B	1280-E	1280-E
16	12	16	12	16	12	16	15	29	15	
5	5	4	5	5	5	5	4	4	4	
16	16	15	16	16	21	16	15	16	16	
1	1	1	1	1	1.016	1	1	1	1	1.016

E-h80* 718.7Hz	E-h80*>	E-h60*>	E-h80* 718.7Hz	E-h80*>	E-h60*>
- h70*	- h72	- h52	- h72	- h72*	D#h56*
- h65*	- h65*	- h50*	- h65*	- h64*	D-h52*
(B)h60*>	(B)h64*>	(B)h60*	(B)h60*>	(B)h60*	C-h48*
- h55*	(A)h56*	- h55*	- h55*	(A)h56*	B-h44
G#h50*	(F#)h48*	G#h50*	G#h50*	(F#)h48*	(A)h40*
(G)h48	(D#)h40*	G-h48	(G)h48	(D#)h40*	G-h36*
- h45*	C#h36*	- h45*	- h45*	C#h36*	(F)h32*
(E)h40*	B-h32*	(E)h40*	(E)h40*	B-h32*	- h31
- h35*	- h31	- h35*	- h35*	- h31	E-h30
(B)h30*	A#h30	B-h30*	(B)h30*	A#h30	- h29
A#h28	- h29	A#h28	A#h28	- h29	D#h28*
- h27	(A)h28*	- h27	- h27	(A)h28*	- h27
A-h26	- h27	A-h26	A-h26	- h27	(D)h26
G#h25*	G#h26	G#h25*	G#h25*	G#h26	C#h25
(G)h24	G-h25	(G)h24	(G)h24	G-h25	C-h24*
- h23	(F#)h24*	- h23	- h23	(F#)h24*	- h23
F#h22	- h23	F#h22	F#h22	- h23	B-h22
F-h21	F-h22	F-h21	F-h21	F-h22	A#h21
(E)h20*	E-h21	(E)h20*	(E)h20*	E-h21	A-h20*
D#h19	(D#)h20*	D#h19	D#h19	(D#)h20*	G#h19
D-h18	D-h19	D-h18	D-h18	D-h19	G-h18
C#h17	C#h18	C#h17	C#h17	C#h18	F#h17
C-h16	C-h17	C-h16	C-h16	C-h17	(F)h16*
(B)h15*	(B)h16*	B-h15*	(B)h15*	(B)h16*	(E)h15
A#h14	A#h15	A#h14	A#h14	A#h15	D#h14
A-h13	A-h14	A-h13	A-h13	A-h14	(D)h13
G-h12	G#h13	(G)h12	G-h12	G#h13	(C)h12*
F#h11	F#h12*	F#h11	F#h11	F#h12*	B-h11
E-h10*	F-h11	E-h10*	E-h10*	F-h11	A-h10
D- h9	D#h10	D- h9	D- h9	D#h10	G- h9
C- h8	C# h9	C- h8	C- h8	C# h9	(F)h8*
A# h7	B- h8*	A# h7	A# h7	B- h8*	D# h7
G- h6	A- h7	G- h6	G- h6	A- h7	C- h6
E- h5*R	F# h6	E- h5*R	E- h5*R	F# h6	A- h5
C- h4	D# h5	C- h4	C- h4	D# h5	F- h4*R
G- h3	B- h4*R	G- h3	G- h3	B- h4*R	C- h3
C- h2	F# h3	C- h2	C- h2	F# h3	F- h2
C-h1/H16	B- h2	C-h1/H16	C-h1/H16	B- h2	F- h1/H21
(8.984Hz)	B- h1/H15	(8.984Hz)	(8.984Hz)	B- h1/H15	(11.979Hz)
	(8.423Hz)			(8.423Hz)	

--> (16 grps 5)

(-4 grps 5)

(12 grps 5) 4:3 (16 grps 4) -3:4-> (12 grps 5)

(+4 grps 5)

(16 grps 5)-3:4->(12 grps 5)-4:3->(16 grps 5)

(-4 grps 5)

(12 grps 5) 4:3 (16 grps 4) 15:16 (15 grps 4)

--> (20 x four)

(-20 x one)

(4 x fifteen)16:15(4 x sixteen)15:16(4 x fifteen)

(+20 x one)

(20 x four) -3:4-> (20 x three) -4:3-> (20 x four)

(-20 x one)

(4 x fifteen)16:15(4 x sixteen)15:16(4 x fifteen)

(+20 x one)

(20 x four) 3:4 (20 x three) ->

K.33 - WALDSTEIN SONATA, L.v.BEETHOVEN

1280-E	1280-E	960-B	960-B														
16	20	15	20	960-B	512-C					512-C					512-C	576-D	
5	4	4	3														
16	16	16	16	15	8					8					7	8	
1	1	1	1	4	4					4					4	4	
				16	16					16					18	18	
E-h80*>	E-h80*	718.7Hz		1	1					1					1.016	1	

E-h80*>	E-h80*	718.7Hz					
- h70*	D-h72*						
- h65*	C-h64*						
B-h60*>	B-h60*>	539.0Hz	B-h60*				
- h55*	A#h56*		A-h54*	A#h56*			
G#h50*	G#h50*		G-h48*	G#h50*			
Ⓞh48	Ⓞh48*		- h45*	G-h48*			
- h45*	F#h44*		Ⓞh42*	F#h44*			
Ⓞh40*	Ⓞh40*		Ⓞh36*	E-h40*			
- h35*	D-h36*		Ⓞh30*	D-h36*			
B-h30*	Ⓞh32*		A#h28*	Ⓞh32*>	287.5Hz	C-h32*>	C-h28*
A#h28	A#h28*		- h27*	A#h28*		- h31	- h27
- h27	- h27		A-h26	- h27		B-h30	B-h26
A-h26	A-h26		G#h25	A-h26		- h29	A#h25
G#h25*	G#h25		G-h24*	G#h25		A#h28*	A-h24*
Ⓞh24	Ⓞh24*		- h23	G-h24*		- h27	- h23
- h23	- h23		F#h22	- h23		A-h26	G#h22
F#h22	F#h22		Ⓞh21*	F#h22		G#h25	G-h21
F-h21	F-h21		E-h20	F-h21		G-h24*	Ⓞh20*
Ⓞh20*	Ⓞh20*		D#h19	Ⓞh20*		- h23	F-h19
D#h19	D#h19		Ⓞh18*	D#h19		F#h22	E-h18
D-h18	D-h18		C#h17	D-h18		F-h21	D#h17
C#h17	C#h17		C-h16	C#h17		Ⓞh20*	Ⓞh16*
C-h16	C-h16*		B-h15*	Ⓞh16*		D#h19	C#h15
B-h15*	B-h15		A#h14	B-h15		D-h18	C-h14
A#h14	A#h14		A-h13	A#h14		C#h17	B-h13
A-h13	A-h13		Ⓞh12*	A-h13		Ⓞh16*	Ⓞh12*
Ⓞh12	Ⓞh12*		F#h11	G-h12*		B-h15	G#h11
F#h11	F#h11		E-h10	F#h11		A#h14	F#h10
E-h10*	E-h10		D-h9*	E-h10		A-h13	E-h9
D-h9	D-h9		C-h8	D-h9		Ⓞh12*	D-h8*
C-h8	C-h8*		A#h7	Ⓞh8*		F#h11	Ⓞh7
A#h7	A#h7		G-h6*	A#h7		E-h10	A-h6
G-h6	G-h6		E-h5	G-h6		D-h9	F#h5
E-h5*R	E-h5		C-h4	E-h5		Ⓞh8*	D-h4*R
C-h4	C-h4*R		G-h3*R	C-h4*R		A#h7	A-h3
G-h3	G-h3		C-h2	G-h3		G-h6	D-h2
C-h2	C-h2		C-h1/H16	C-h2		E-h5	D-h1/H18
C-h1/H16	C-h1/H16	(8.984Hz)		C-h1/H16		C-h4*R	(10.267Hz)
(8.984Hz)	(8.984Hz)			(8.984Hz)		G-h3	
						C-h2	
						C-h1/H16	
						(8.984Hz)	

(16 grps 5) 5:4 (20 grps 4)

(-5 grps 4)

(15 grps 4) 4:3 (20 grps 3) -3:4-> (15 grps 4)

(-7 grps 4)

(16 groups of 4)

-> (8 groups of 4) -----7:8-----> (7 groups of 4)

(+1 group of 4)

(8 groups of 4) ->

---4:3---> (20 x four)

(-20 x one)

(15 x four)-----> (-28 x one) -----> (8 x four) -----> (4 x eight) -----> 7:8-----> (4 x seven)

(+4 x one)

(8 x four)-3:4->