Journey to the Heart of Music

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Introduction and Guide to the MOS Examples

MUTABLE NUMBER ANALYSIS

A full description of the MOS (Modulating Oscillatory Systems) model and the Mutable Base Position-Value Number System can be found in the main text of *Journey to the Heart of Music*. This document is an introduction to the book's attached examples, and, although a short summary of MOS ideas and a brief guide are given below referral to the main chapters is recommended.

If one were to draw an analogy between physical theory and music theory, the analysis given by the MOS approach might be likened to a form of quantum mechanics for tonal music: That is, a fundamental approach to tonal music operating at a sufficiently elemental level to be capable of explaining its most basic mechanisms and characteristics. The MOS model, to carry the analogy further, does not enlighten us much regarding the broader human and cultural issues or context, about the purpose of the architecture or the intent of the architect, only the underlying engineering. The model is myopic, essentially seeing no further than the next chord. Other theories concerned with the wider vistas of what might be termed the 'classical mechanics' of music are required to access broader vistas and higher level understandings. In particular, the MOS model might well be integrated into theories which focus on the dynamic aspect of tonal compositions. As in physics, where both the quantum and classical worlds coexist (though not always peacefully), the MOS approach does not overturn the relevance of other tonal theories but rather, for the most part, complements and underpins them; with perhaps mutable numbers playing a similar role in regard to tonal music, as that of the group of integers modulo twelve in atonal music.

At bottom the MOS model of tonal music is a means of construing the physical reality of musical sound as a representation of number. That is, by extrapolation from the frequency relationships of the objective notes – up to common overtones and downward to a unit fundamental – a positional number system can be constructed: Mutable Base Numbers. When applying a mutable number analysis to a composition, while it is almost always possible to derive some combination of nested harmonic series able link together even the most difficult, not to say desperate, chord progressions, it may not be wise or sound analysis so to do in every case. Given the complex resources of the human mind: the ability to skip passed unexpected detail, fill in assumed 'missing' sensory information or leap across gaps of logic to form broad understandings; the MOS model should not endeavour to provide an absolutely compelling analysis for every chord sequence ever written. Rather the model should, hopefully, capture the essence of the structure generally, while also being compelling in its analysis of the detail of 'regular' tonal chord sequences, particularly those of core importance to the tonal structure of the composition.

Equally, the model should not be constrained by the extent of aural cognition, whatever these (as yet unknown) limits turn out to be. It would be remarkable, I think, if the ear and mind were truly able to compute every detail of the extended family of nested harmonic series proposed in the MOS model. More likely our 'ears' grasp the explicit upper level *aggregated* series defined by the relationships of the majority of objective notes, plus more or less the underlying middle level *nested* series which supports it. And probably learning and experience, to a degree, influence the depth of individual perception here. However, for the low level *fundamental* series, the series which encapsulates a sense of key, this would probably be only vaguely computed or implied, in most cases, from the exchanges of the two higher level nested and aggregated series.

Though of course, it might well turn out that the MOS approach proves to have no connection at all with the processes of aural cognition: In which case the theory simply falls back to being yet another mathematical model of tonal music, and hopefully, still a useful analytical tool.

GUIDE

In the following analyses a parallel range of information is delivered in a variety of different formats, going from top to bottom of the page; 1) Mutable Base Numbers in subscript format, 2) traditional Roman numeral harmonic analysis, 3) Mutable Base Numbers in 'stacked' factor format, 4) explicit harmonic series and 5) additions, subtractions and ratio exchanges of the modulation algorithm of symmetrical exchange.

Above the system the current *mutable base number* may be shown in subscript format, for example,

MBN $9_2\ 0_{32}\ 0_1$

which in decimal is 576 or in a generalised positional notation, Dec $5_{10}\ 7_{10}\ 6_1$. The mutable numbers appear once, continuing in force until replaced by another mutable digit sequence – which might represent the same or a different conjunction value – as unlike fixed base number systems (i.e. Decimal, Binary, etc.) mutable base numbers will, most often, have a *range of different digit sequences* for a given value. It is this richness and variety of number representation which enables mutable base numbers to capture tonal chord progressions within the digit sequences of one value. Thus rendering the progression logical and commensurable,... or not logical and incommensurable where no corresponding digit sequence exists! However, it should not be thought that mutable numbers command tonal music, rather the other way about, the chords of tonal music are (parts of) mutable numbers written in sound and the 'rules' or characteristics of tonal harmony, are thus also those of the mutable number system. Tonal harmonic progressions which make sense aurally, will have corresponding mutable number digit sequences which ‘add up’, or in math-speak ‘mathematically exist’. For convenience and ease of reading, both the column digits and column base subscripts of mutable numbers are written in the form of plain decimal numbers. (It would be possible to use letters for extra symbols, as in the hexadecimal system and indeed any other symbols but as there is no limit to the range of either column digits or column bases in the mutable system this strategy would make the written notation extremely cumbersome.)

Between the staves a traditional Roman numeral analysis of the harmony is given, for example,

Key G: I ii V^7 I.

Below the system the current mutable base number is shown in ‘stacked’ *factor format*, with the equivalent decimal value, separated by a line of dashes, placed above it. This decimal number is the value of the conjunction which links adjacent chords together and by means of this commonality, renders the harmonic progression commensurable. The conjunction values used in factor format (and subscript format) are usually fixed at the frequency of the ‘key’ series, this is done so that the mutable numbers remain anchored around the unit H1, which enhances the presentational logic of being in a key/tonal center. (It is equally possible to track the precise course taken by the MOS model from start to finish, by using real values for the unit throughout, e.g. H1.016 or H0.990.) The stacked factor format numbers are updated with each new chord, whether they change digit sequence or not. Also, as in the example below, the note letter equivalent of the conjunction’s frequency ratio is appended to the decimal value.

576-A

9
2
32
1

Sometimes the conjunction value will run across a number of different chords linking them all by a common value; but as often as not, a new conjunction value, either higher or lower, will be required to connect to the next chord in the harmonic progression. A new conjunction is found by counting upward or downward in the ratios of the uppermost harmonic series. When this occurs the gray band running across the page is broken and a new band commenced at the appropriate level. The harmonic motion of a composition will at times lead to a ‘flexing’ of strict whole number relationships within a multi-column mutable number. This occurs where a modulation exchange produces a non-integer result (within the fixed value grid applied to the tonal center) and it is accommodated within the system by means of a fractional unit. That is the unitary period of the absolute fundamental frequency (H1) is allowed a little leeway or tolerance to expand or contract as required to maintain a common conjunction value between chords, for example:

576-A

9
3
21
1.016

Although this might be viewed as something of a fudge or blemish upon the purity of a system essentially rooted in whole numbers, it mirrors the tolerance of small deviations of pitch exhibited by the human ear. Indeed, the equal-tempered scale upon which tonal music depends for access to all keys on the piano and many other instruments, would be impossible without such a tolerance. And, in the next format below this feature disappears into a continuum of smoothly variable values.

The conjunction band mentioned above actually runs through the uppermost ratios of a *nested system of harmonic series*, which are written out below the stacked mutable numbers. These series could be taken to represent the music as a ‘pseudo-physical’ system of relationships evolving by means of addition, subtraction and modulation exchanges (harmonic progression) in a material context. In the ‘classical’ material world, which is of course the environment in which music actually exists, relationships are generally taken to be smoothly variable. And so the ‘flexing’ of relationships here presents no obstacle, for example F-h21 is twenty-one whole steps of the harmonic series above h1 and if those steps happen to take the value of 1.016 arbitrarily defined units of frequency (rounded up from 1.015873...), then the twenty-first step will consist of F-21.333... units. In this format of extended harmonic series, the MOS model (Modulating Oscillatory System) takes the changes in the musical relationships, the modulation exchanges, as they come and computes the results in real numbers, that is, decimals with unlimited fractional expansions. Though here in these analyses, for convenience, only one decimal place is shown. (The reader may calculate to whatever accuracy they require.) Beside these real numbers the ‘logical’ number or relationship to the unit is given in brackets: h1, h2, h3, etc. these are, of course, the ratios of the harmonic series – and the digits of mutable numbers. As for the most part three nested harmonic series are present: a fundamental/nesting series (H1, H2, H3, etc.), a nested middle level series (h1, h2, h3, etc.) and a top level aggregated series marked with asterisks (h3*, h6*, h9*, etc.) – they are distinguished by the symbols given here in brackets. These three nested harmonic series correspond to the columns in mutable base numbers. However, there may be any number of columns in a mutable number or levels of nesting in a MOS analysis. Three columns is usual and convenient, two not uncommon, but if the relatively large fundamental series (e.g. typically H1 through H32, or H1 through H48) were to be broken down into their factors, then double or more columns/nestings would emerge. Such additional detail in the ‘murky depths’ are implied but not shown in most MOS analyses.

Finally, below the written out harmonic series are the additions, subtractions and modulations of the MOS model’s algorithm of symmetrical exchange – the modulation algorithm. Here the operation (addition, subtraction or modulation exchange) is shown within the arrow linking two bracketed descriptions of a nested pair of harmonic series. In black oblique script the movement of the upper aggregated series within the middle level nested series is shown. And in a gray upright script the changes of the middle level nested series paired with the fundamental nesting series is given. Both pairs, aggregated/nested series and nested/fundamental series share the same conjunction. Indeed, there is no theoretical limit to the number of levels of nesting that could be employed, provided there are sufficient elements in the system to support the structure – yet still there would be only one, shared, conjunction. These upper levels of nesting can usefully be thought of in terms of the subdivision(s) of a meter created by the nested/fundamental series, the foundational pair, with the system is driven, from the top, by the motion of the objective notes. As the notes change from beat to beat and measure to measure, the underlying structure of nested harmonic series must respond to accommodate the harmonic motion. Sometimes only change in the upper level is required, at other times the low level nested/fundamental series must also change along with the aggregated series to accommodate the harmonic motion of the objective notes; and where a piece changes key, the whole system of nested series must change to new ground.

CONSTRUCTING A MOS ANALYSIS

Heinrich Schenker famously stated that his analytical approach to music was fundamentally an art, rather than a science. Similarly, in some degree, creating modulating oscillatory system analyses also requires the intuitions of an artist – a musician's insight, knowledge and experience. A MOS analysis cannot be made without reference to how the analyst hears the music and interprets the harmonic progressions. Therefore at points within a MOS analysis where choices and judgements have to be made, different analysts may come to different conclusions, arguing for one or another route through a number of different, but equally viable exchanges. However, sooner or later the two (or more) variant paths will rejoin and the analysis proceed again in undisputed exchanges. The music for this example analysis is the chorale *Ach Gott und Herr,* set by J.S. Bach.

Some useful steps toward making a MOS analysis are:

1) Make a roman numeral analysis of the harmony,
2) From the numeral analysis produce the most likely *configuration of partials* for each chord,
3) Construct a sequence of fundamental bass frequencies which support these configurations,
4) Chose the conjunctions which best connect the configurations together (with reference to the harmonics, and perhaps summation tones, generated by the objective written notes).

The crucial component to find in this method, is the sequence fundamental frequencies underlying the harmony – a concept very much akin to Rameau's 'basse fondamentale'. However, whereas the roots of major chords are reflected much as one would expect in this sequence of tones. That is, as the fundamental tones of harmonic series whose higher harmonics largely encompass the objective chords as *configurations of partials*, and, that these higher harmonics will mostly possess a common factor of 2, or some higher power of 2, (e.g. the chord: C-h4, G-h6, Eh10 and C-h16). In contrast to this, for minor chords, the tones in this sequence of fundamental frequencies are less intuitive, as the common factor (predominantly) bundling-up the underlying harmonic series will be 5, or on occasions 3 or 7 (e.g. chord: E-h10, B-h15, E-h20, G-h24); which leads to the fundamental tone in the underlying sequence being different from the minor chord's perceived root. In the example E-minor chord above, the fundamental tone of the series is 'C', C-h1.

Once arrived at, this sequence of fundamental tones may then be linked together by a chain of ratios of exchange which transform each fundamental tone into the one that follows it; and, this succession of fundamental tone are, effectively, themselves harmonics of a posited absolute fundamental tone 'H1' – which is taken to encapsulate the unifying sense of tonal center or key. This chain of transformations is the backbone of a modulating oscillatory system, it is a framework articulated by the 'modulations' of the algorithm of symmetrical exchange – the *modulation algorithm*.

After making the initial roman numeral analysis by hand, each of the remaining steps may be speeded up with the aid of a computer. The procedure described here gradually homes in upon a final interpretation. For steps two and three the Perl script *txt2mos* may be helpful, and for step four *makecon* and *makehs* are provided. The scripts, which I hope in time to further refine, are on the attached compact disc (CHPT19 folder/directory) and available over the internet from *http://www.pjperry.freeuk.com/scripts/perl.htm*. Some familiarity with using the command terminal of a computer is assumed, as well as access to a Perl interpreter with Sean Burke's MIDI modules installed (*http://www.cpan.org/modules/by-module/MIDI/*). However, it is perfectly possible to produce a complete analysis by hand, indeed, all of the examples G to N were done this way.

Step 1

Figure 1. A Roman numeral analysis of the chorale *Ach Gott und Herr*.

Step 2

Having made the Roman numeral analysis, this can be use to reduce the score to a harmonic skeleton. To use the *txt2mos* script involves first producing a simplified MIDI file that encompasses the harmonic gist of the Roman numeral analysis shorn of any superfluous detail like non-harmonic passing notes, grace-notes, etc. This can most easily be done in a score editing program which is capable of generating MIDI files. The music will need to be exported as a single MIDI track. (The *txt2txt* script will collect multiple tracks into one if need be.) The simplified MIDI score is then given to the *mid2txt* utility (MIDI-to-text) with the '-e' switch set, so as to generate an event list (rather than the default score list). Finally, the event list text file is given to the *txt2mos* utility (text-to-MOS), which generates an outline MOS analysis set out in columns: bar-beat position, MIDI-tick, two blank-fields ("?" and "?:?"), fundamental, objective notes and conjunctions.

The *txt2mos* script is far from perfect or complete in the analysis it produces, in particular, the conjunctions it finds will only be a rough stab at what will be needed and a good number of the fundamental frequencies will require adjustment. However, it does produce a general framework of the whole section or score ready for further development – a first approximation. At this stage it is best to ignore the conjunctions and concentrate on finding the complete sequence of fundamental frequencies. Looking through the list of fundamentals for each chord found by *txt2mos*, in comparison with the roman numeral analysis, it is apparent that the script has 'misunderstood' some of the chords; these can be corrected from the numeral analysis. The fifth column in Figure 2, directly after the blank fields, is the utility's rather unreliable guess; though, overall, the list does give a 'feel' for the likely fundamental frequencies of the crucial middle-level nested series. The best-fit sequence of note-letter fundamentals may be entered in the third column, under the heading 'fund'. For example, the first eight are: C, G, D (not C), G, C, F (not C), G and C.

```
txt2mos    file:            frequencies in hertz : mid C=256Hz
measure    ticks root ratio fundamental - notes - conjunctions
000.750        0  ?    ?:?  64C - 128C 320E 384G 512C - 576->D
001.000       96  ?    ?:?  96G - 192G 288D 384G 480B - 576<-D 576->D
001.250      192  ?    ?:?  16C - 144D 256C 352F# 416A - 576<-D 576->D
001.500      288  ?    ?:?  48G - 96G 240B 288D 384G - 576<-D 576->D
001.750      384  ?    ?:?  32C - 128C 224A# 320E 384G - 576<-D 576->D
002.000      480  ?    ?:?   8C - 168F 208A 256C 416A - 576<-D 576->D
002.250      576  ?    ?:?  24G - 144D 168F 288D 480B - 576<-D 576->D
002.500      672  ?    ?:?  64C - 128C 192G 320E 512C - 576<-D 576->D
002.750      768  ?    ?:?  48G - 96G 240B 384G 576D - 576<-D 576->D
003.000      864  ?    ?:?   8C - 104A 256C 352F# 512C - 576<-D 576->D
003.250      960  ?    ?:?  24G - 120B 288D 384G 480B - 576<-D 576->D
003.500     1056  ?    ?:?  16C - 144D 288D 352F# 416A - 576<-D 576->D
003.750     1152  ?    ?:?  96G - 192G 288D 384G 480B - 576<-D 576->D
004.000     1248  ?    ?:?  32C - 128C 320E 384G 416A - 576<-D 576->D
004.250     1344  ?    ?:?  16C - 144D 256C 352F# 416A - 576<-D 576->D
004.500     1440  ?    ?:?  48G - 96G 240B 288D 384G - 576<-D 576->D
004.750     1536  ?    ?:?  32C - 160E 256C 384G 512C - 576<-D 576->D
005.000     1632  ?    ?:?  48G - 144D 288D 336F 480B - 576<-D 576->D
005.250     1728  ?    ?:?  64C - 128C 320E 384G 512C - 576<-D 576->D
005.500     1824  ?    ?:?  48G - 192G 240B 384G 576D - 576<-D 576->D
006.000     2016  ?    ?:? 128C - 256C 256C 384G 640E - 768<-G 768->G
006.250     2112  ?    ?:?  16C - 208A 256C 352F# 512C - 768<-G 768->G
006.500     2208  ?    ?:?  48G - 192G 240B 384G 576D - 768<-G 768->G
006.750     2304  ?    ?:?  32C - 160E 256C 320E 384G - 768<-G 768->G
007.000     2400  ?    ?:?  16C - 176F# 256C 288D 416A - 768<-G 768->G
007.250     2496  ?    ?:?   8C - 200G# 240B 288D 480B - 768<-G 768->G
007.500     2592  ?    ?:?  16C - 208A 256C 320E 512C - 768<-G 768->G
007.625     2640  ?    ?:?  16C - 208A 256C 336F 576D - 768<-G 768->G
007.750     2688  ?    ?:?  64C - 192G 256C 384G 640E - 768<-G 768->G
007.875     2736  ?    ?:?   8C - 168F 256C 416A 672F - 768<-G 768->G
008.000     2784  ?    ?:?  64C - 192G 256C 384G 576D - 768<-G 768->G
008.250     2880  ?    ?:?  48G - 96G 240B 336F 576D - 768<-G 768->G
008.500     2976  ?    ?:?  64C - 128C 192G 320E 512C - 768<-G
```

Figure 2. The output from *txt2mos* for the chorale *Ach Gott und Herr*. (Notice that the measure positions in the first column are given as decimal fractions and are irregular as they chart the harmonic, rather than metrical, changes.)

Step 3

The reason for filling in the blank fields (with more accurate data) is that the resulting text file can then be passed on to the *makecon* (make-conjunctions) utility, which, from this improved approximation, is able to generate a list of all possible conjunctions between the chords given by the roman numeral analysis. (That is, all possible conjunctions up to a user defined limit.)

Deciding the note-letter fundamental for the major chords is fairly straightforward, it is almost always the root of the chord. The script will often guess right for these. However, for the minor chords, which require the middle level nested series to be bundled-up in groups of five ratios, or less often groups of three ratios, the note-letter of the fundamental will be a major-third or perfect fifth below the root of the chord, and the *txt2mos* utility will get these wrong. In this example, measures 4.000, 7.500 and 7.625 for the chords of A-minor7th, A-minor and D-minor7th respectively.

Quite often a low value fundamental will indicate a position where the script has struggled, for one reason or another, and adjustments are required, for example measure 2.0 where the script posited ‘8C’. In Figure 3 the fundamental note-letters have been filled in, and the rather jagged profile of the fundamental frequencies have be smoothed out. Editing the fundamental frequencies in column five is optional, as later scripts don’t read this data; however, by smoothing the awkward profile of the frequencies into a Rameau-like ‘fundamental bass’, a better idea of the best-fit frequency level for the nested series is obtained.

```
txt2mos    file:            frequencies in hertz : mid C=256Hz
measure    ticks fund ratio fundamental - notes - conjunctions
000.750        0  C    ?:?  64C - 128C 320E 384G 512C - 576->D
001.000       96  G    ?:?  48G - 192G 288D 384G 480B - 576<-D 576->D
001.250      192  D    ?:?  36D - 144D 256C 352F# 416A - 576<-D 576->D
001.500      288  G    ?:?  24G - 96G 240B 288D 384G - 576<-D 576->D
001.750      384  C    ?:?  32C - 128C 224A# 320E 384G - 576<-D 576->D
002.000      480  F    ?:?  21F - 168F 208A 256C 416A - 576<-D 576->D
002.250      576  G    ?:?  24G - 144D 168F 288D 480B - 576<-D 576->D
002.500      672  C    ?:?  32C - 128C 192G 320E 512C - 576<-D 576->D
002.750      768  G    ?:?  24G - 96G 240B 384G 576D - 576<-D 576->D
003.000      864  D    ?:?  18D - 104A 256C 352F# 512C - 576<-D 576->D
003.250      960  G    ?:?  24G - 120B 288D 384G 480B - 576<-D 576->D
003.500     1056  D    ?:?  36D - 144D 288D 352F# 416A - 576<-D 576->D
003.750     1152  G    ?:?  24G - 192G 288D 384G 480B - 576<-D 576->D
004.000     1248  F    ?:?  21F - 128C 320E 384G 416A - 576<-D 576->D
004.250     1344  D    ?:?  18D - 144D 256C 352F# 416A - 576<-D 576->D
004.500     1440  G    ?:?  24G - 96G 240B 288D 384G - 576<-D 576->D
004.750     1536  C    ?:?  16C - 160E 256C 384G 512C - 576<-D 576->D
005.000     1632  G    ?:?  24G - 144D 288D 336F 480B - 576<-D 576->D
005.250     1728  C    ?:?  32C - 128C 320E 384G 512C - 576<-D 576->D
005.500     1824  G    ?:?  24G - 192G 240B 384G 576D - 576<-D 768->G
006.000     2016  C    ?:?  32C - 256C 256C 384G 640E - 768<-G 768->G
006.250     2112  D    ?:?  36D - 208A 256C 352F# 512C - 768<-G 768->G
006.500     2208  G    ?:?  24G - 192G 240B 384G 576D - 768<-G 768->G
006.750     2304  C    ?:?  16C - 160E 256C 320E 384G - 768<-G 768->G
007.000     2400  D    ?:?  18D - 176F# 256C 288D 416A - 768<-G 768->G
007.250     2496  E    ?:?  20E - 200G# 240B 288D 480B - 768<-G 768->G
007.500     2592  F    ?:?  21F - 208A 256C 320E 512C - 768<-G 768->G
007.625     2640  A#   ?:?  14A#- 208A 256C 336F 576D - 768<-G 768->G
007.750     2688  C    ?:?  16C - 192G 256C 384G 640E - 768<-G 768->G
007.875     2736  F    ?:?  21F - 168F 256C 416A 672F - 768<-G 768->G
008.000     2784  G    ?:?  24G - 192G 256C 384G 576D - 768<-G 768->G
008.250     2880  G    ?:?  24G - 96G 240B 336F 576D - 768<-G 768->G
008.500     2976  C    ?:?  32C - 128C 192G 320E 512C - 768<-G
```

Figure 3. The note-letters of the middle level nested series fundamentals have been entered in column three and the column five fundamental frequency values smoothed out and/or corrected. (All the utility scripts require the input fields to be separated by at least one space.)

The next step is to fill in column four with the ratios of exchange which carry the harmony from the first nested fundamental frequency in column five, to each of the succeeding values in turn. At present the smoothed out succession of nested fundamental values are an approximation, expressed as the partial frequencies of the harmonic series of the absolute fundamental tone ‘H1’. This foundational harmonic series embodies the key or tonal center.

From the succession nested fundamentals in column five it is possible to calculate the ratios of column four. Beginning with frequency C-h64 the ratio which will transform 64C into 48G, the next nested fundamental, is 64:48 or by division 4:3. This value is entered in the first row of column four. Next 48G is transformed into 36D, 48:36 or 4:3 again, which is entered in the second row, column four. Third comes 36D to 24G, 36:24 which reduces to 3:2. Then comes 24G to 32C, 24:32 or 3:4. The fifth exchange between C-h32 and F-h21 is more interesting, the perfect fifth relationship of 3:2 doesn’t quite match 32:21. To transform 32C by a perfect fifth down to F produces the value F-h21.333... This is the value we require as a step toward a closer approximation to a purely relational modulating oscillatory system. We enter 3:2 into row five column four. Figure 3 shows all the ratios entered in column four.

```
txt2mos    file:            frequencies in hertz : mid C=256Hz
measure    ticks fund ratio fundamental - notes - conjunctions
000.750        0  C    4:3  64C - 128C 320E 384G 512C - 576->D
001.000       96  G    4:3  48G - 192G 288D 384G 480B - 576<-D 576->D
001.250      192  D    3:2  36D - 144D 256C 352F# 416A - 576<-D 576->D
001.500      288  G    3:4  24G - 96G 240B 288D 384G - 576<-D 576->D
001.750      384  C    3:2  32C - 128C 224A# 320E 384G - 576<-D 576->D
002.000      480  F    9:10 21F - 168F 208A 256C 416A - 576<-D 576->D
002.250      576  G    3:4  24G - 144D 168F 288D 480B - 576<-D 576->D
002.500      672  C    4:3  32C - 128C 192G 320E 512C - 576<-D 576->D
002.750      768  G    4:3  24G - 96G 240B 384G 576D - 576<-D 576->D
003.000      864  D    3:4  18D - 104A 256C 352F# 512C - 576<-D 576->D
003.250      960  G    2:3  24G - 120B 288D 384G 480B - 576<-D 576->D
003.500     1056  D    3:2  36D - 144D 288D 352F# 416A - 576<-D 576->D
003.750     1152  G   10:9  24G - 192G 288D 384G 480B - 576<-D 576->D
004.000     1248  F    6:5  21F - 128C 320E 384G 416A - 576<-D 576->D
004.250     1344  D    3:4  18D - 144D 256C 352F# 416A - 576<-D 576->D
004.500     1440  G    3:2  24G - 96G 240B 288D 384G - 576<-D 576->D
004.750     1536  C    2:3  16C - 160E 256C 384G 512C - 576<-D 576->D
005.000     1632  G    3:4  24G - 144D 288D 336F 480B - 576<-D 576->D
005.250     1728  C    4:3  32C - 128C 320E 384G 512C - 576<-D 576->D
005.500     1824  G    3:4  24G - 192G 240B 384G 576D - 576<-D 768->G
006.000     2016  C    7:8  32C - 256C 256C 384G 640E - 768<-G 768->G
006.250     2112  D    3:2  36D - 208A 256C 352F# 512C - 768<-G 768->G
006.500     2208  G    3:2  24G - 192G 240B 384G 576D - 768<-G 768->G
006.750     2304  C    7:8  16C - 160E 256C 320E 384G - 768<-G 768->G
007.000     2400  D    7:8  18D - 176F# 256C 288D 416A - 768<-G 768->G
007.250     2496  E   15:16 20E - 200G# 240B 288D 480B - 768<-G 768->G
007.500     2592  F    3:2  21F - 208A 256C 320E 512C - 768<-G 768->G
007.625     2640  A#   8:9 14A#- 208A 256C 336F 576D - 768<-G 768->G
007.750     2688  C    3:4  16C - 192G 256C 384G 640E - 768<-G 768->G
007.875     2736  F    8:9  21F - 168F 256C 416A 672F - 768<-G 768->G
008.000     2784  G    1:1  24G - 192G 256C 384G 576D - 768<-G 768->G
008.250     2880  G    3:4  24G - 96G 240B 336F 576D - 768<-G 768->G
008.500     2976  C    1:1  32C - 128C 192G 320E 512C - 768<-G
```

Figure 4. The ratios of exchange placed in column 4 are calculated so as to produce the succession of notes/ frequencies as shown in column5.

The sixth exchange in Figure 4 also raises some interesting points. Here F-h21.3 is transformed by a whole-tone step into G-h24, and the ratio 21.333... is to 24 reduces through division by 2.666... to 8:9 the ratio of the 'just' whole-tone. However, there are other factors to be taken into account. The neat sesquioctava exchange of 8:9 implies a conjunction between the F-major and rootless G-major chord at 768G, top G on the treble staff. But although the third harmonic of middle C in the F-major chord obliges, none of the notes in the rootless G-major chord generate a corresponding harmonic at 768 hertz. Yet all is not lost.

Firstly, one might look to the frequency 768 hertz being generated in the ear by the summation tone created by the two upper notes in the rootless G-major chord (288D + 480B = 768G) but not everyone is convinced of summation tones' existence.

Secondly, one might take some guidance from what the configuration of the notes in the chord is indicating. That is, that the note F occurs in both chords, and therefore, that F or one of its harmonics would provide a good thread of stability to guide the ear and aural cognitive processes through the chord progression. To find this conjunction the whole-tone step must be enlarged from 8:9 to 7:8, this keeps the common notes F in the two chords steady at 170.666...hertz. Alternatively, a narrow whole-step of 9:10 would align the second harmonic of the top note of the F-major chord (853.333...A) with the third harmonic of the alto D in the G-major chord. Although the written note A is not common to both chords, it is important to bear in mind that the notes appearing in a score are far from the whole acoustic story of the piece in performance. Each written note will, to greater or lesser degree, be the source its own harmonic series – as the fundamental tone of a vibrating string or column of air. Indeed, as often as not, the higher harmonics rather than the fundamental frequency will possess the majority of acoustic energy, and in the light of this, my choice of conjunction is A-h853.333... and 9:10 exchange. (Though I might change my mind tomorrow.)

A rather downbeat third choice exists; that is to treat the analysis as a solely formal matter of attaching mutable numbers to chord progressions, irrespective of the dynamical aspects of the music, apprehended by human ears, in performance. In this situation all exchanges are equally viable.

Arising from this examination of the possible whole-tone exchanges (7:8, 8:9, 9:10) is the point that there is a degree of manoeuvre within a MOS analysis. The ratios of the wide intervals of exchange – the octave, fifth, fourth and major-third – generate many closely spaced conjunction frequencies. This is perhaps the reason for tonal music's preference for motion by wide bass intervals, which Rameau noticed and incorporated within his theory of the fundamental bass; and, the reason behind the frequent interleaving of first inversion and root position chords where the bass moves by step. Harmonic motion by fifth and fourth etc. (or implied by inversion) firmly anchors the chord progressions of tonal music within a web of connecting conjunctions. In contrast, the ratios of the narrow intervals of exchange – the minor-third, the whole-tone and the semitone – produce relatively scarce conjunctions. For example, the 'just' semitone of ratio 15:16 has to stretch up to the fourteenth partial before finding a conjunction to connect the chord progression. Sustained chord progressions in semitone steps are usually reserved for special effects, and the ear switches from 'crunching the discrete tonal numbers' to an appreciation of an implied glissando-like sound effect. However, as illustrated above, where narrow steps do occur, within the context of tonal music's normal mode of harmonic computation, the sparseness of conjunctions created by such narrow exchanges is compensated for, by the wealth of choice in precise ratios. For the semitone exchange this can range from a wide 14:15, through the ‘just’ 15:16 and out to beyond the equal-tempered near-equivalent of 17:18. Yet also one should not be carried away by so many choices. Take the advice of the chords themselves for likely conjunctions and be aware of the effect on the overall pitch stability in the choice of ratio. Inevitably, MOS analyses demand a freedom for the pitch to roam but there is no reason to go to extremes. And finally, to this veritable cornucopia of ratios must be added the accepting tolerance of the ear, ever ready to accommodate small irregularities in objective experience, in order to grasp a broader intelligibility.

```
  makecon  file:        frequencies in hertz : mid C approx. 256Hz
  measure               conjunctions ---->
  000.750  64.000C (h01) 192.000G-(h03) 384.000G-(h06) 576.000D-(h09) 768.000G-(h12)*
   4:3 ->  48.000--(h01) 192.000--(h04) 384.000--(h08) 576.000--(h12) 768.000--(h16)*

  001.000  48.000G (h01) 144.000D-(h03) 288.000D-(h06) 432.000A-(h09) 576.000D-(h12)*
   4:3 ->  36.000--(h01) 144.000--(h04) 288.000--(h08) 432.000--(h12) 576.000--(h16)*

  001.250  36.000D (h01)  72.000D-(h02) 144.000D-(h04) 216.000A[snip] 576.000D-(h16)*
   3:2 ->  24.000--(h01)  72.000--(h03) 144.000--(h06) 216.000-[snip] 576.000--(h24)*

  001.500  24.000G (h01)  96.000G-(h04) 192.000G-(h08) 288.000D-(h12) 384.000G-(h16)*
   3:4 ->  32.000--(h01)  96.000--(h03) 192.000--(h06) 288.000--(h09) 384.000--(h12)*

  001.750  32.000C (h01)  64.000C-(h02) 128.000C-(h04) 192.000G[snip] 512.000C-(h16)*
   3:2 ->  21.333--(h01)  64.000--(h03) 128.000--(h06) 192.000-[snip] 512.000--(h24)*

  002.000  21.333F (h01) 213.333A-(h10) 426.667A-(h20) 640.000E-(h30) 853.333A-(h40)*
   9:10->  23.704--(h01) 213.333--(h09) 426.667--(h18) 640.000--(h27) 853.333--(h36)*
>>--------------------->
  007.000  18.347D (h01) 146.773D-(h08) 293.546D-(h16) 440.319A-(h24) 587.092D-(h32)*
   7:8 ->  20.968--(h01) 146.773--(h07) 293.546--(h14) 440.319--(h21) 587.092--(h28)*

  007.250  20.968E (h01) 335.481E-(h16) 670.962E-(h32) 1006.444B-(h48)* 1341.952E-(h64)
  15:16->  22.365--(h01) 335.481--(h15) 670.962--(h30) 1006.444--(h45)* 1341.952--(h60)

  007.500  22.365F (h01)  44.731F-(h02)  89.462F-(h04) 134.192C[snip] 894.617A-(h40)*
   3:2 ->  14.910--(h01)  44.731--(h03)  89.462--(h06) 134.192-[snip] 894.617--(h60)*

  007.625  14.910A#(h01) 134.192C-(h09) 268.385C-(h18) 402.577G[snip] 670.962E-(h45)*
   8:9 ->  16.774--(h01) 134.192--(h08) 268.385--(h16) 402.577-[snip] 670.962--(h40)*

  007.750  16.774C (h01)  67.096C-(h04) 134.192C-(h08) 201.289G[snip] 1073.536C-(h64)*
   3:4 ->  22.365--(h01)  67.096--(h03) 134.192--(h06) 201.289-[snip] 1073.536--(h48)*

  007.875  22.365F (h01) 201.289G-(h09) 402.577G-(h18) 603.866D-(h27) 805.155G-(h36)*
   8:9 ->  25.161--(h01) 201.289--(h08) 402.577--(h16) 603.866--(h24) 805.155--(h32)*

  008.000  25.161G (h01)  50.322G-(h02)  75.483D-(h03) 100.644G[snip] 805.155G-(h32)*
   1:1 ->  25.161--(h01)  50.322--(h02)  75.483--(h03) 100.644-[snip] 805.155--(h32)*

  008.250  25.161G (h01) 100.644G-(h04) 201.289G-(h08) 301.933D[snip] 805.155G-(h32)*
   3:4 ->  33.548--(h01) 100.644--(h03) 201.289--(h06) 301.933-[snip] 805.155--(h24)*
  008.500  33.548C (h01)  67.096C-(h02) 100.644G-(h03) 134.192C-(h04) 805.155G-(h24)*
```

Figure 5. Two examples from the output of the *makecon* script – measures 0-1 and 7-8. The asterisks on the right-hand side have been entered to mark the chosen conjunctions and some pairs of lines ‘snipped’ to page width.

Step 4

Once the text file Figure 4 has been completed with the relevant entries in columns three and four, and remembering that all the entered fields must have spaces around them so that the script can see where one ends and the next begins, the file can be sent to the *makecon* utility for further processing. However, before proceeding to step 4 it is worth reviewing the sequence of ratios of exchange, as this chain of fundamental tones and ratios forms the kernel of the analysis. Upon this information all subsequent procedures depend, and although there remains much working-out to be done and some choices to be made regarding which of a range of possible conjunctions to use, from this point onward in the analysis the die is pretty much cast.

The entry of data into these text files is best handled by a good text editor program, with a monospaced font and a long (user determined) line length selected. A monospaced font (e.g. Courier) makes the columns line up and the long line length (e.g. 1024 characters) allows the calculated conjunctions from each chord exchange (Figure 5) to spread out along a pair of lines, with each conjunction matching up. Each chord appears twice, once for the incoming conjunction and again for the outgoing conjunction; the two entries are adjacent, separated by a blank line. Only a small portion of the output from the *makecon* script is shown in Figure 5, where necessary many of the conjunctions calculated by the script have been snipped out, so as the chosen conjunction can be seen on the right-hand side. The conjunctions have been marked by entering an asterisk into the text file.

Admittedly, the long pairs of lines generated by makecon can be rather awkward to work with, and the files perhaps require a little time and practice to become familiar. (If you are not seeing pairs of lines separated by a blank line, then your text editor's line length is too short, and if you are seeing pairs of lines but the space separated fields aren't align, you need to switch your text editor to a monospaced font.) Each line of *makecon's* output represents a harmonic series that enfolds a chord. First a chord's enfolding harmonic series is paired with its precursor chord's harmonic series, and all the possible conjunctions between these two series are displayed (up to the limit set by the -l switch, default 1000 hertz). To the left is given the chord/harmonic series position in the composition and below this the ratio of exchange.

```
000.750  64.000C (h01) 192.000G-(h03) 384.000G-(h06) 576.000D-(h09) 768.000G-(h12)*
  4:3 ->  48.000--(h01) 192.000--(h04) 384.000--(h08) 576.000--(h12) 768.000--(h16)*
```

From the displayed conjunctions a suitable one may be selected by entering an asterisk, as shown above and in Figure 5. The conjunction to chose will need to equal, or more likely exceed, the pitch of the highest objective note; and, overall, match the bundles or aggregations in the underlying series provoked by the particular configuration of the objective chord. In addition it is also desirable that the conjunction is a harmonic of one or more of the objective notes; that is to say, the conjunction also exists objectively (as well having the formal status of a mutable number value). So, quite a few considerations; but do not be daunted, surprisingly, there is almost always one or more candidate conjunctions which fit all the criteria. In the example above, the first and second chords of the chorale – C-major to G-major – it can be seen that the conjunction chosen, 768G, 1) is higher in pitch than the top objective notes – 512C and 480B; 2) matches the aggregated bundles of the two major chords (groups of 1 and 2) at the conjunction (G-h12/h16); and 3) 768G is the second harmonic of the alto's note 384G in both chords, as well as being the sixth and fourth harmonics, respectively, of the chords' bass notes. At first sight, no doubt, it all seems complex and convoluted, but quite quickly one gains a feel for the numbers and where the conjunctions will lie. The trick is to 'read' the objective chords, the common note G in the alto part suggests where the most likely conjunction may be found to join the chords together – as a single mutable number value expressed in two different digit sequences.

Of the pairs of lines shown in Figure 5 the pair for measure 7.25 to 7.5, with the narrow exchange of 15:16 and therefore comparatively large aggregation in bundles or groups of fifteen and sixteen, most clearly illustrates step four – marking the conjunction. The large steps result in the chosen conjunction falling before the lines reach the right-hand side of the page. This is most like the raw text file output of *makecon*, with the possible conjunctions stretching far off to the right and the asterisk being entered between the paired-up fields of conjoined harmonics.

Once all the asterisks have been entered the text file is ready to be further processed by the utility script *makehs*. The script reads where the chosen conjunctions fall (i.e. at the asterisks) and generates complete harmonic series from fundamental tone up to the asterisk, as lists suitable to be placed within a MOS analysis as illustrated below. The processed data as with earlier scripts is a plain text file, and the output from these scripts contains only one instance of each harmonic series, rather than the paired output of *makecon*. Each harmonic series is complete (all harmonics listed) from fundamental to conjunction; and where the incoming conjunction from the previous chord is higher in pitch than the outgoing conjunction to the succeeding chord, the harmonic series is extended passed the asterisk (conjunction marker) up to this higher frequency. There are two features in this output which the user needs to be aware of: 1) in extending the harmonic series up to the level of the previous conjunction, one extra entry may creep onto the list, which is caused by, 2) the rounding up or down of values to one decimal place. This latter feature means that on occasion a conjunction may be shown, for example, as 'nnn.5' in one harmonic series and 'nnn.4' in another. My apologies for this small infelicity.

From this point on, the the final assembly of the MOS analysis will depend to a considerable extent upon the analyst's particular mode of working, whether by hand or by computer, and if by computer what score editor and graphic program(s) are being used and the facilities they offer. For the example analysis below, the score and MIDI file were produced by the music editor *RISC OS Sibelius* and the final assembly was done in *RISC OS Draw.* One very useful feature offered by this drawing package is that of a 'textarea', which accepts a text file as input and displays the text as a list – a most convenient way of creating sequences of harmonic series. The text file input used (below) with the drawing package, is generated by *makehs* with each line representing one harmonic series. Further formatting commands understood by the drawing package can be added into the individual lines of text, e.g. text color, background color, underlining, line spacing, etc.; generated either from within the script or added by hand afterwards in a text editor. However, before adding any refinements to the lists (harmonic series) generated by *makehs,* it is a good idea to enter all the lists into the drawing package and roughly work through the analysis, marking aggregate groups, conjunctions and exchanges. In this way a 'sketch' analysis may be quickly produced. Almost inevitably many shortcomings will become apparent: positions where the fundamental tone needs to fall instead of rise or vice versa, aggregations which don't match conjunction, narrow exchanges which need fine tuning, etc. After this first work through, a re-run from Step 3 is usually required, and often several more!

MBN $12_1 0_{64} 0_1 = 8_2 0_{48} 0_1$ $6_2 0_{48} 0_1 = 8_2 0_{36} 0_1 = 12_2 0_{24} 0_1$ $8_2 0_{24} 0_1 = 6_2 0_{32} 0_1$ $8_2 0_{32} 0_1 =$

Value:	768:G (decimal) →	768:G	576:D →	576:D →	576:D	384:G →	384:G	512:C →
Mutable Number Digit Sequences	12	8	6	8	12	8	6	8
	1	2	2	2	2	2	2	2
	64	48	48	36	24	24	32	32
	1	1	1	1	1	1	1	1

```
*768.0:G-(h12)>
*704.0:F#(h11)
*640.0:E-(h10)
*576.0:D-(h09)
*512.0:C-(h08)
*448.0:A#(h07)
*384.0:G-(h06)
*320.0:E-(h05)
*256.0:C-(h04)
*192.0:G-(h03)
*128.0:C-(h02)
*64.0:C-(h1/H64)R
Bar:000.750
```

```
*768.0:G-(h16)
720.0:F#(h15)
*672.0:F-(h14)
624.0:E-(h13)
*576.0:D-(h12)>
528.0:C#(h11)
*480.0:B-(h10)
432.0:A-(h09)
*384.0:G-(h08)
336.0:F-(h07)
*288.0:D-(h06)
240.0:B-(h05)
*192.0:G-(h04)
144.0:D-(h03)
*96.0:G-(h02)R
48.0:G-(h1/H48)
Bar:001.000
```

```
*576.0:D-(h16)>
540.0:C#(h15)
*504.0:C-(h14)
468.0:B-(h13)
*432.0:A-(h12)
396.0:G#(h11)
*360.0:F#(h10)
324.0:E-(h09)
*288.0:D-(h08)
252.0:C-(h07)
*216.0:A-(h06)
180.0:F#(h05)
*144.0:D-(h04)
108.0:A-(h03)
*72.0:D-(h02)R
36.0:D-(h1/H36)
Bar:001.250
```

```
*576.0:D-(h24)
552.0:?-(h23)
*528.0:C#(h22)
504.0:C-(h21)
*480.0:B-(h20)
456.0:A#(h19)
*432.0:A-(h18)
408.0:G#(h17)
*384.0:G-(h16)>
360.0:F#(h15)
*336.0:F-(h14)
312.0:E-(h13)
*288.0:D-(h12)
264.0:C#(h11)
*240.0:B-(h10)
216.0:A-(h09)
*192.0:G-(h08)
168.0:F-(h07)
*144.0:D-(h06)
120.0:B-(h05)
*96.0:G-(h04)
72.0:D-(h03)
*48.0:G-(h02)R
24.0:G-(h01/H24)
Bar:001.500
```

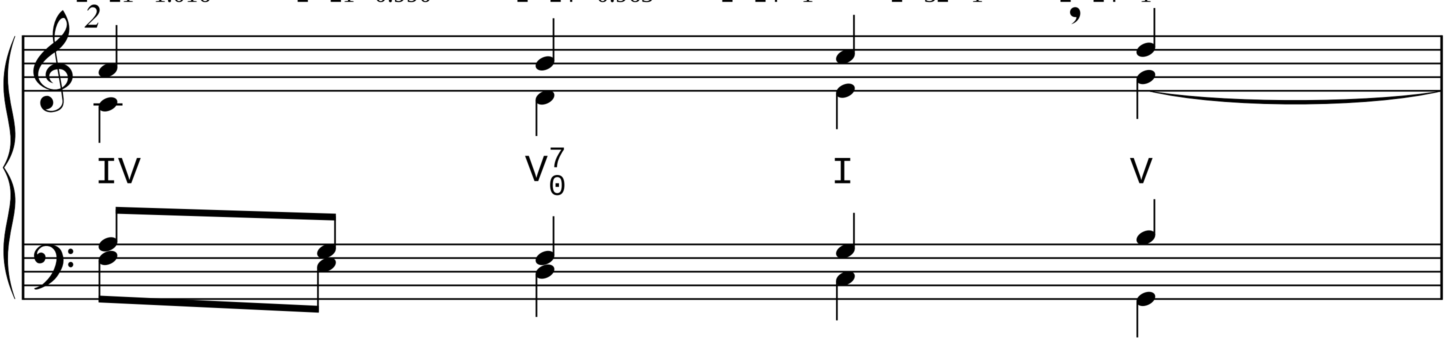
```
*512.0:C-(h16)>→
480.0:B-(h15)
*448.0:A#(h14)
416.0:A-(h13)
*384.0:G-(h12)
352.0:F#(h11)
*320.0:E-(h10)
288.0:D-(h09)
*256.0:C-(h08)
224.0:A#(h07)
*192.0:G-(h06)
160.0:E-(h05)
*128.0:C-(h04)
96.0:G-(h03)
*64.0:C-(h02)R
32.0:C-(h1/H32)
Bar:001.750
```

Aggregated Series:
(12 groups of 1)-2:3->(8 groups of 2)
(-2 grps of 2)
6 groups of 2)-4:3->(8 grps of 2)-3:2->(12 groups of 2)
(-2 grps of 2)
(8 groups of 2)-3:4->(6 groups of 2)
(+2 grps of 2)
(8 grps of 2)-3:2->

Nested Series:
(4 x three)-----4:3---->(4 x four)
(-4 x one)
(4 x three)-----4:3---->(4 x four)
-1:2->(8 x two)--3:2-->(8 x three)
(-8 x one)
(4 x four)----3:4---->(4 x three)
(+4 x one)
(8 x two)-3:2->

Fundamental Series: C-H1 to Hn, not shown,
except for joining harmonic, e.g. H64 in first chord.

$= 12_2 0_{21} 0_{1.016} \quad 20_2 0_{21} 0_{0.990} = 18_2 0_{24} 0_{0.963} \quad 16_2 0_{24} 0_1 = 12_2 0_{32} 0_1 = 16_2 0_{24} 0_1$ *etc.*



→ 512:C	832:A →	832:A	768:G →	768:G →	768:G	832:A →
-----	-----	-----	-----	-----	-----	-----
12	20	18	16	12	16	18
2	2	2	2	2	2	2
21	21	24	24	32	24	24
1.016	0.990	0.963	1	1	1	0.963

*853.3:A-(h40)>
832.0:A-(h39)
*810.7:G#(h38)
789.3:G-(h37)
*768.0:G-(h36)
746.7:G-(h35)
*725.3:F#(h34)
704.0:F-(h33)
*682.7:F-(h32)
661.3:?-(h31)
*640.0:E-(h30)
618.7:?-(h29)
*597.3:D#(h28)
576.0:D-(h27)
*554.7:D-(h26)
533.3:C#(h25)
→ *512.0:C-(h24)
490.7:?-(h23)
*469.3:B-(h22)
448.0:A#(h21)
*426.7:A-(h20)
405.3:G#(h19)
*384.0:G-(h18)
362.7:F#(h17)
*341.3:F-(h16)
320.0:E-(h15)
*298.7:D#(h14)
277.3:D-(h13)
*256.0:C-(h12)
234.7:B-(h11)
*213.3:A-(h10)
192.0:G-(h09)
*170.7:F-(h08)
149.3:D#(h07)
*128.0:C-(h06)
106.7:A-(h05)
*85.3:F-(h04)
64.0:C-(h03)
*42.7:F-(h02)R
21.3:F-(H21)
Bar:002.000

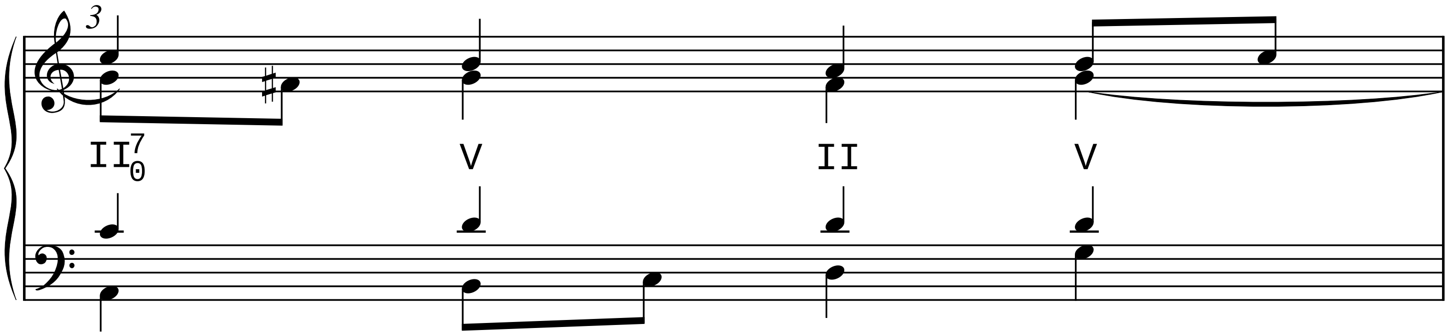
*853.3:A-(h36)
829.6:A-(h35)
*805.9:G#(h34)
782.2:G-(h33)
*758.5:G-(h32)>
734.8:?-(h31)
*711.1:F#(h30)
687.4:?-(h29)
*663.7:F-(h28)
640.0:E-(h27)
*616.3:E-(h26)
592.6:D#(h25)
*568.9:D-(h24)
545.2:?-(h23)
*521.5:C#(h22)
497.8:C-(h21)
*474.1:B-(h20)
450.4:A#(h19)
*426.7:A-(h18)
403.0:G#(h17)
*379.3:G-(h16)
355.6:F#(h15)
*331.9:F-(h14)
308.2:E-(h13)
*284.4:D-(h12)
260.7:C#(h11)
*237.0:B-(h10)
213.3:A-(h09)
*189.6:G-(h08)
165.9:F-(h07)
*142.2:D-(h06)
118.5:B-(h05)
*94.8:G-(h04)
71.1:D-(h03)
*47.4:G-(h02)R
23.7:G-(H24)
Bar:002.250

*758.5:G-(h24)>
726.9:?-(h23)
*695.3:F#(h22)
663.7:F-(h21)
*632.1:E-(h20)
600.5:D#(h19)
*568.9:D-(h18)
537.3:C#(h17)
*505.7:C-(h16)
474.1:B-(h15)
*442.5:A#(h14)
410.9:A-(h13)
*379.3:G-(h12)
347.7:F#(h11)
*316.1:E-(h10)
284.4:D-(h09)
*252.8:C-(h08)
221.2:A#(h07)
*189.6:G-(h06)
158.0:E-(h05)
*126.4:C-(h04)
94.8:G-(h03)
*63.2:C-(h02)R
31.6:C-(H32)
Bar:002.500

*853.3:A-(h36)> →
829.6:A-(h35)
*805.9:G#(h34)
782.2:G-(h33)
*758.5:G-(h32)
734.8:?-(h31)
*711.1:F#(h30)
687.4:?-(h29)
*663.7:F-(h28)
640.0:E-(h27)
*616.3:E-(h26)
592.6:D#(h25)
*568.9:D-(h24)
545.2:?-(h23)
*521.5:C#(h22)
497.8:C-(h21)
*474.1:B-(h20)
450.4:A#(h19)
*426.7:A-(h18)
403.0:G#(h17)
*379.3:G-(h16)
355.6:F#(h15)
*331.9:F-(h14)
308.2:E-(h13)
*284.4:D-(h12)
260.7:C#(h11)
*237.0:B-(h10)
213.3:A-(h09)
*189.6:G-(h08)
165.9:F-(h07)
*142.2:D-(h06)
118.5:B-(h05)
*94.8:G-(h04)
71.1:D-(h03)
*47.4:G-(h02)R
23.7:G-(H24)
Bar:002.750

-3:2->(12 groups of 2)
(+8 groups of 2)
(20 groups of 2) ------9:10-----> (18 groups of 2)
(16 groups of 2) ----3:4----> (12 groups of 2) ----4:3-----> (16 groups of 2)
(-2 groups of 2)
(+2 groups of 2)
(18 groups of 2) -4:3->

--3:2->(8 x three)
(+16 x one)
(4 x ten) -----------9:10----------> (4 x nine)
(8 x four) ---------3:4---------> (8 x three) -------4:3------> (8 x four)
(-4 x one)
(+4 x one)
(12 x three) --4:3-->



832:A	832:A	832:A	832:A
-----	-----	-----	-----
24	18	12	9
2	2	2	4
18	24	36	24
0.963	0.963	0.963	0.963

*853.3:A-(h48)>
835.6:A-(h47)
*817.8:?-(h46)
800.0:G#(h45)
*782.2:G#(h44)
764.5:G#(h43)
*746.7:G-(h42)
728.9:F#(h41)
*711.1:F#(h40)
693.3:F#(h39)
*675.6:F-(h38)
657.8:E-(h37)
*640.0:E-(h36)
622.2:E-(h35)
*604.5:D#(h34)
586.7:D-(h33)
*568.9:D-(h32)
551.1:?-(h31)
*533.3:C#(h30)
515.6:?-(h29)
*497.8:C-(h28)
480.0:B-(h27)
*462.2:B-(h26)
444.4:A#(h25)
*426.7:A-(h24)
408.9:?-(h23)
*391.1:G#(h22)
373.3:G-(h21)
*355.6:F#(h20)
337.8:F-(h19)
*320.0:E-(h18)
302.2:D#(h17)
*284.4:D-(h16)
266.7:C#(h15)
*248.9:C-(h14)
231.1:B-(h13)
*213.3:A-(h12)
195.6:G#(h11)
*177.8:F#(h10)
160.0:E-(h09)
*142.2:D-(h08)
124.4:C-(h07)
*106.7:A-(h06)
88.9:F#(h05)
*71.1:D-(h04)
53.3:A-(h03)
*35.6:D-(h02)R
17.8:D-(H18)
Bar:003.000

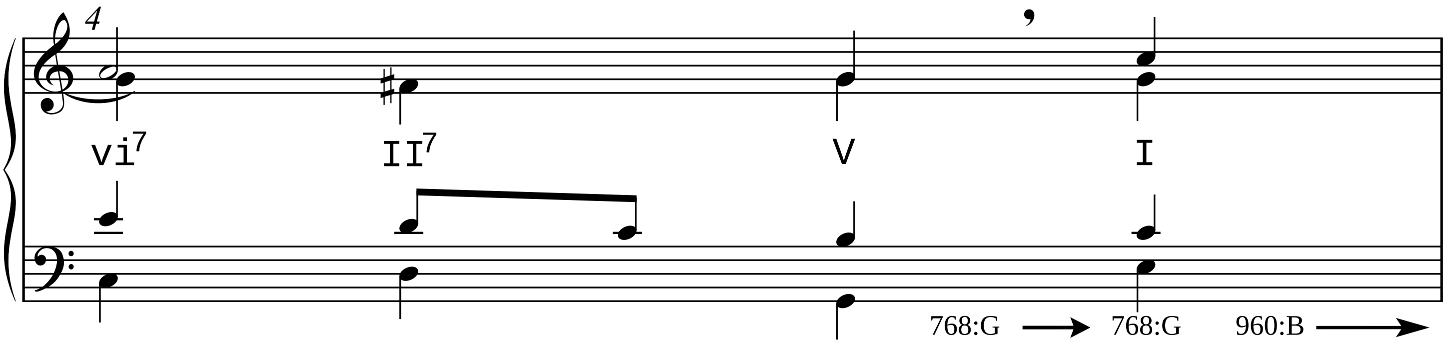
*853.3:A-(h36)>
829.6:A-(h35)
*805.9:G#(h34)
782.2:G-(h33)
*758.5:G-(h32)
734.8:?-(h31)
*711.1:F#(h30)
687.4:?-(h29)
*663.7:F-(h28)
640.0:E-(h27)
*616.3:E-(h26)
592.6:D#(h25)
*568.9:D-(h24)
545.2:?-(h23)
*521.5:C#(h22)
497.8:C-(h21)
*474.1:B-(h20)
450.4:A#(h19)
*426.7:A-(h18)
403.0:G#(h17)
*379.3:G-(h16)
355.6:F#(h15)
*331.9:F-(h14)
308.2:E-(h13)
*284.4:D-(h12)
260.7:C#(h11)
*237.0:B-(h10)
213.3:A-(h09)
*189.6:G-(h08)
165.9:F-(h07)
*142.2:D-(h06)
118.5:B-(h05)
*94.8:G-(h04)
71.1:D-(h03)
*47.4:G-(h02)R
23.7:G-(H24)
Bar:003.250

*853.3:A-(h24)>
817.8:?-(h23)
*782.2:G#(h22)
746.7:G-(h21)
*711.1:F#(h20)
675.6:F-(h19)
*640.0:E-(h18)
604.5:D#(h17)
*568.9:D-(h16)
533.3:C#(h15)
*497.8:C-(h14)
462.2:B-(h13)
*426.7:A-(h12)
391.1:G#(h11)
*355.6:F#(h10)
320.0:E-(h09)
*284.4:D-(h08)
248.9:C-(h07)
*213.3:A-(h06)
177.8:F#(h05)
*142.2:D-(h04)
106.7:A-(h03)
*71.1:D-(h02)R
35.6:D-(H36)
Bar:003.500

*853.3:A-(h36)>
829.6:A-(h35)
805.9:G#(h34)
782.2:G-(h33)
*758.5:G-(h32)
734.8:?-(h31)
711.1:F#(h30)
687.4:?-(h29)
*663.7:F-(h28)
640.0:E-(h27)
616.3:E-(h26)
592.6:D#(h25)
*568.9:D-(h24)
545.2:?-(h23)
521.5:C#(h22)
497.8:C-(h21)
*474.1:B-(h20)
450.4:A#(h19)
426.7:A-(h18)
403.0:G#(h17)
*379.3:G-(h16)
355.6:F#(h15)
331.9:F-(h14)
308.2:E-(h13)
*284.4:D-(h12)
260.7:C#(h11)
237.0:B-(h10)
213.3:A-(h09)
*189.6:G-(h08)
165.9:F-(h07)
142.2:D-(h06)
118.5:B-(h05)
*94.8:G-(h04)R
71.1:D-(h03)
47.4:G-(h02)
23.7:G-(H24)
Bar:003.750

-4:3->(24 groups of 2) -----3:4-----> (18 groups of 2) ------2:3----->(12 groups of 2) ----3:4------> (9 groups of 4) ---8:9-->

-4:3->(12 x four) -----------3:4--------> (12 x three) -----------2:3-------> (12 x two) --------3:2-------> (12 x three) (4 x nine) --10:9-->



→ 832:A → 832:A 576:D → 576:D 768:G → 768:G 960:B →

832:A	832:A	576:D	576:D	768:G	768:G	960:B
-----	-----	-----	-----	-----	-----	-----
8	12	8	12	16	24	30
5	4	4	2	2	2	2
21	18	18	24	24	16	16
0.990	0.963	1	1	1	1	1

→ *853.3:A-(h40)>
832.0:A-(h39)
810.7:G#(h38)
789.3:G-(h37)
768.0:G-(h36)
*746.7:G-(h35)
725.3:F#(h34)
704.0:F-(h33)
682.7:F-(h32)
661.3:?-(h31)
*640.0:E-(h30)
618.7:?-(h29)
597.3:D#(h28)
576.0:D-(h27)
554.7:D-(h26)
*533.3:C#(h25)
512.0:C-(h24)
490.7:?-(h23)
469.3:B-(h22)
448.0:A#(h21)
*426.7:A-(h20)
405.3:G#(h19)
384.0:G-(h18)
362.7:F#(h17)
341.3:F-(h16)
*320.0:E-(h15)
298.7:D#(h14)
277.3:D-(h13)
256.0:C-(h12)
234.7:B-(h11)
*213.3:A-(h10)
192.0:G-(h09)
170.7:F-(h08)
149.3:D#(h07)
128.0:C-(h06)
*106.7:A-(h05)R
85.3:F-(h04)
64.0:C-(h03)
42.7:F-(h02)
21.3:F-(H21)
Bar:004.000

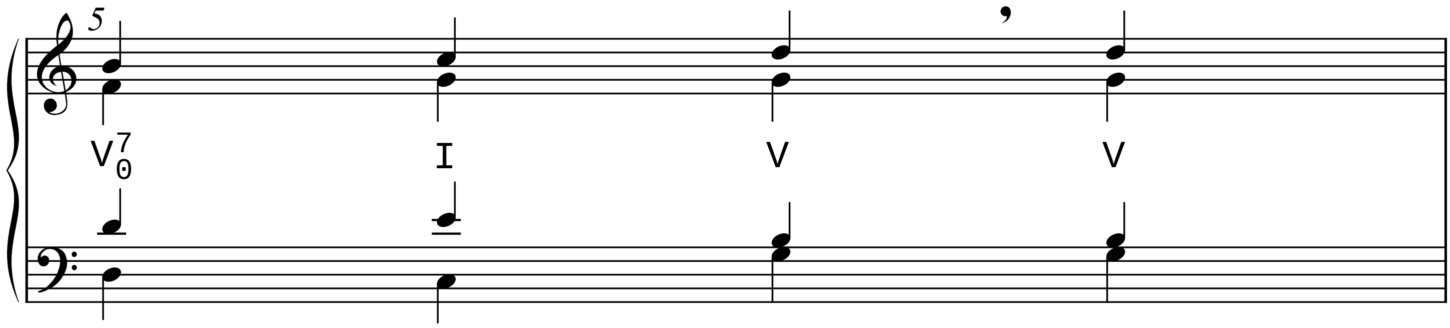
*853.3:A-(h48)
835.6:A-(h47)
817.8:?-(h46)
800.0:G#(h45)
*782.2:G#(h44)
764.5:G#(h43)
746.7:G-(h42)
728.9:F#(h41)
*711.1:F#(h40)
693.3:F#(h39)
675.6:F-(h38)
657.8:E-(h37)
*640.0:E-(h36)
622.2:E-(h35)
604.5:D#(h34)
586.7:D-(h33)
*568.9:D-(h32)>
551.1:?-(h31)
533.3:C#(h30)
515.6:?-(h29)
*497.8:C-(h28)
480.0:B-(h27)
462.2:B-(h26)
444.4:A#(h25)
*426.7:A-(h24)
408.9:?-(h23)
391.1:G#(h22)
373.3:G-(h21)
*355.6:F#(h20)
337.8:F-(h19)
320.0:E-(h18)
302.2:D#(h17)
*284.4:D-(h16)
266.7:C#(h15)
248.9:C-(h14)
231.1:B-(h13)
*213.3:A-(h12)
195.6:G#(h11)
177.8:F#(h10)
160.0:E-(h09)
*142.2:D-(h08)
124.4:C-(h07)
106.7:A-(h06)
88.9:F#(h05)
*71.1:D-(h04)R
53.3:A-(h03)
35.6:D-(h02)
17.8:D-(H18)
Bar:004.250

*758.5:G-(h32)>
734.8:?-(h31)
*711.1:F#(h30)R
687.4:?-(h29)
*663.7:F-(h28)
640.0:E-(h27)
*616.3:E-(h26)
592.6:D#(h25)
*568.9:D-(h24)
545.2:?-(h23)
*521.5:C#(h22)
497.8:C-(h21)
*474.1:B-(h20)
450.4:A#(h19)
*426.7:A-(h18)
403.0:G#(h17)
*379.3:G-(h16)
355.6:F#(h15)
*331.9:F-(h14)
308.2:E-(h13)
*284.4:D-(h12)
260.7:C#(h11)
*237.0:B-(h10)
213.3:A-(h09)
*189.6:G-(h08)
165.9:F-(h07)
*142.2:D-(h06)
118.5:B-(h05)
*94.8:G-(h04)
71.1:D-(h03)
*47.4:G-(h02)R
23.7:G-(H24)
Bar:004.500

*948.1:B-(h60)> →
932.3:B-(h59)
*916.5:?-(h58)
900.7:A#(h57)
*884.9:A#(h56)
869.1:A#(h55)
*853.3:A-(h54)
837.5:A-(h53)
*821.7:A-(h52)
805.9:A-(h51)
*790.1:G#(h50)
774.3:G-(h49)
*758.5:G-(h48)
742.7:G-(h47)
*726.9:?-(h46)
711.1:F#(h45)
*695.3:F#(h44)
679.5:F#(h43)
*663.7:F-(h42)
647.9:E-(h41)
*632.1:E-(h40)
616.3:E-(h39)
*600.5:D#(h38)
584.7:D-(h37)
*568.9:D-(h36)
553.1:D-(h35)
*537.3:C#(h34)
521.5:C-(h33)
*505.7:C-(h32)
489.9:?-(h31)
*474.1:B-(h30)
458.3:?-(h29)
*442.5:A#(h28)
426.7:A-(h27)
*410.9:A-(h26)
395.1:G#(h25)
*379.2:G-(h24)
363.4:?-(h23)
*347.6:F#(h22)
331.8:F-(h21)
*316.0:E-(h20)
300.2:D#(h19)
*284.4:D-(h18)
268.6:C#(h17)
*252.8:C-(h16)
237.0:B-(h15)
*221.2:A#(h14)
205.4:A-(h13)
*189.6:G-(h12)
173.8:F#(h11)
*158.0:E-(h10)
142.2:D-(h09)
*126.4:C-(h08)
110.6:A#(h07)
*94.8:G-(h06)
79.0:E-(h05)
*63.2:C-(h04)
47.4:G-(h03)
*31.6:C-(h02)R
15.8:C-(H16)
Bar:004.750

-8:9->(8 groups of 5) ---3:2---> (12 groups of 4)
(-4 groups of 4)
(8 groups of 4) -----3:2-----> (12 groups of 2)
(16 groups of 2) ------3:2------> (24 groups of 2)
(+4 groups of 2)
(+6 groups of 2)
(30 groups of 2)-2:3->

-10:9-> (4 x ten)
-2:1->(8 x five) ------6:5-----> (8 x six)
(-16 x one)
(8 x four) -----------3:4----------> (8 x three)
(16 x two) -----------3:2----------> (16 x three)
(+8 x one)
(+12 x one)
(20 x three)--2:3-->



→ 960:B → 960:B → 960:B 768:G →

960:B	960:B	960:B	768:G
-----	-----	-----	-----
20	15	20	16
2	2	2	2
24	32	24	24
1	1	1	1

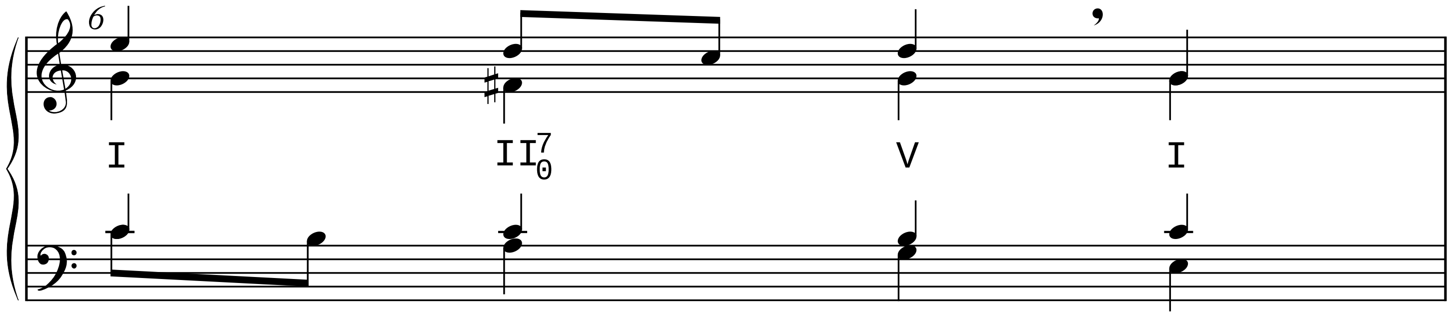
→ *948.2:B-(h40)>
924.5:B-(h39)
*900.8:A#(h38)
877.0:A-(h37)
*853.3:A-(h36)
829.6:A-(h35)
*805.9:G#(h34)
782.2:G-(h33)
*758.5:G-(h32)
734.8:?-(h31)
*711.1:F#(h30)
687.4:?-(h29)
*663.7:F-(h28)
640.0:E-(h27)
*616.3:E-(h26)
592.6:D#(h25)
*568.9:D-(h24)
545.2:?-(h23)
*521.5:C#(h22)
497.8:C-(h21)
*474.1:B-(h20)
450.4:A#(h19)
*426.7:A-(h18)
403.0:G#(h17)
*379.3:G-(h16)
355.6:F#(h15)
*331.9:F-(h14)
308.2:E-(h13)
*284.4:D-(h12)
260.7:C#(h11)
*237.0:B-(h10)
213.3:A-(h09)
*189.6:G-(h08)
165.9:F-(h07)
*142.2:D-(h06)
118.5:B-(h05)
*94.8:G-(h04)
71.1:D-(h03)
*47.4:G-(h02)R
23.7:G-(H24)
Bar:005.000

*948.1:B-(h30)>
916.5:?-(h29)
*884.9:A#(h28)
853.3:A-(h27)
*821.7:A-(h26)
790.1:G#(h25)
*758.5:G-(h24)
726.9:?-(h23)
*695.3:F#(h22)
663.7:F-(h21)
*632.1:E-(h20)
600.5:D#(h19)
*568.9:D-(h18)
537.3:C#(h17)
*505.7:C-(h16)
474.1:B-(h15)
*442.5:A#(h14)
410.9:A-(h13)
*379.3:G-(h12)
347.7:F#(h11)
*316.1:E-(h10)
284.4:D-(h09)
*252.8:C-(h08)
221.2:A#(h07)
*189.6:G-(h06)
158.0:E-(h05)
*126.4:C-(h04)
94.8:G-(h03)
*63.2:C-(h02)R
31.6:C-(H32)
Bar:005.250

*948.2:B-(h40)
924.5:B-(h39)
*900.8:A#(h38)
877.0:A-(h37)
*853.3:A-(h36)
829.6:A-(h35)
*805.9:G#(h34)
782.2:G-(h33)
*758.5:G-(h32)> →
734.8:?-(h31)
*711.1:F#(h30)
687.4:?-(h29)
*663.7:F-(h28)
640.0:E-(h27)
*616.3:E-(h26)
592.6:D#(h25)
*568.9:D-(h24)
545.2:?-(h23)
*521.5:C#(h22)
497.8:C-(h21)
*474.1:B-(h20)
450.4:A#(h19)
*426.7:A-(h18)
403.0:G#(h17)
*379.3:G-(h16)
355.6:F#(h15)
*331.9:F-(h14)
308.2:E-(h13)
*284.4:D-(h12)
260.7:C#(h11)
*237.0:B-(h10)
213.3:A-(h09)
*189.6:G-(h08)
165.9:F-(h07)
*142.2:D-(h06)
118.5:B-(h05)
*94.8:G-(h04)
71.1:D-(h03)
*47.4:G-(h02)R
23.7:G-(H24)
Bar:005.500

-2:3->(20 groups of 2) -------3:4-------> (15 groups of 2) ---------4:3--------> (20 groups of 2)
(-2 groups of 2)
(16 groups of 2) ---------------------3:4---->

-2:3->(20 x two)
-1:2->(10 x four) ----------3:4---------> (10 x three) ------------4:3------------> (10 x four)
(-4 x one)
(8 x four) ---------------------3:4------>



→ 768:G	1024:C →	1024:C	832:A →	832:A	768:G →	768:G	512:C →
-----	-----	-----	-----	-----	-----	-----	-----
12	16	14	12	18	16	24	16
2	2	2	2	2	2	2	2
32	32	36	36	24	24	16	16
1	1	1.016	0.963	0.963	1	1	1

```
*1011.4:C-(h32)>
979.8:?-(h31)
*948.1:B-(h30)
916.5:?-(h29)
*884.9:A#(h28)
853.3:A-(h27)
*821.7:A-(h26)
790.1:G#(h25)
→ *758.5:G-(h24)
726.9:?-(h23)
*695.3:F#(h22)
663.7:F-(h21)
*632.1:E-(h20)
600.5:D#(h19)
*568.9:D-(h18)
537.3:C#(h17)
*505.7:C-(h16)
474.1:B-(h15)
*442.5:A#(h14)
410.9:A-(h13)
*379.3:G-(h12)
347.7:F#(h11)
*316.1:E-(h10)
284.4:D-(h09)
*252.8:C-(h08)
221.2:A#(h07)
*189.6:G-(h06)
158.0:E-(h05)
*126.4:C-(h04)
94.8:G-(h03)
*63.2:C-(h02)R
31.6:C-(H32)
Bar:006.000
```

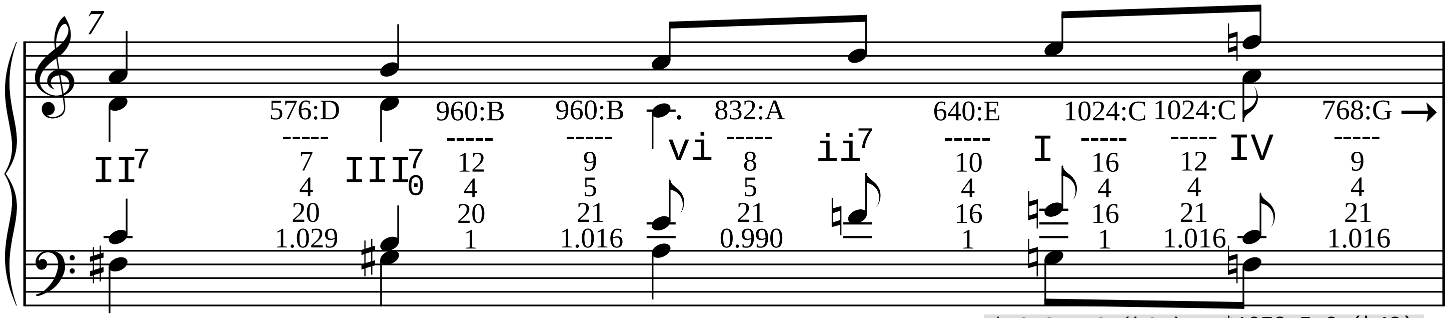
```
*1011.4:C-(h28)
975.2:B-(h27)
*939.1:B-(h26)
903.0:A#(h25)
*866.9:A-(h24)>
830.8:?-(h23)
*794.6:G#(h22)
758.5:G-(h21)
*722.4:F#(h20)
686.3:F-(h19)
*650.2:E-(h18)
614.0:D#(h17)
*577.9:D-(h16)
541.8:C#(h15)
*505.7:C-(h14)
469.6:B-(h13)
*433.4:A-(h12)
397.3:G#(h11)
*361.2:F#(h10)
325.1:E-(h09)
*289.0:D-(h08)
252.8:C-(h07)
*216.7:A-(h06)
180.6:F#(h05)
*144.5:D-(h04)
108.4:A-(h03)
*72.2:D-(h02)R
36.1:D-(H36)
Bar:006.250
```

```
*866.9:A-(h36)
842.8:A-(h35)
*818.7:G#(h34)
794.6:G-(h33)
*770.6:G-(h32)>
746.5:?-(h31)
*722.4:F#(h30)
698.3:?-(h29)
*674.2:F-(h28)
650.2:E-(h27)
*626.1:E-(h26)
602.0:D#(h25)
*577.9:D-(h24)
553.8:?-(h23)
*529.8:C#(h22)
505.7:C-(h21)
*481.6:B-(h20)
457.5:A#(h19)
*433.4:A-(h18)
409.4:G#(h17)
*385.3:G-(h16)
361.2:F#(h15)
*337.1:F-(h14)
313.0:E-(h13)
*289.0:D-(h12)
264.9:C#(h11)
*240.8:B-(h10)
216.7:A-(h09)
*192.6:G-(h08)
168.6:F-(h07)
*144.5:D-(h06)
120.4:B-(h05)
*96.3:G-(h04)
72.2:D-(h03)
*48.2:G-(h02)R
24.1:G-(H24)
Bar:006.500
```

```
*770.5:G-(h48)
754.5:G-(h47)
*738.4:?-(h46)
722.4:F#(h45)
*706.3:F#(h44)
690.3:F#(h43)
*674.2:F-(h42)
658.2:E-(h41)
*642.1:E-(h40)
626.1:E-(h39)
*610.0:D#(h38)
594.0:D-(h37)
*577.9:D-(h36)
561.9:D-(h35)
*545.8:C#(h34)
529.7:C-(h33)
*513.7:C-(h32)> →
497.6:?-(h31)
*481.6:B-(h30)
465.5:?-(h29)
*449.5:A#(h28)
433.4:A-(h27)
*417.4:A-(h26)
401.3:G#(h25)
*385.3:G-(h24)
369.2:?-(h23)
*353.2:F#(h22)
337.1:F-(h21)
*321.1:E-(h20)
305.0:D#(h19)
*289.0:D-(h18)
272.9:C#(h17)
*256.8:C-(h16)
240.8:B-(h15)
*224.7:A#(h14)
208.7:A-(h13)
*192.6:G-(h12)
176.6:F#(h11)
*160.5:E-(h10)
144.5:D-(h09)
*128.4:C-(h08)
112.4:A#(h07)
*96.3:G-(h06)
80.3:E-(h05)
*64.2:C-(h04)
48.2:G-(h03)
*32.1:C-(h02)R
16.1:C-(H16)
Bar:006.750
```

-3:4->(12 groups of 2)
(+4 groups of 2)
(16 groups of 2) ---------7:8--------> (14 groups of 2)
(-2 groups of 2)
(12 groups of 2) ------3:2------> (18 groups of 2)
(-2 groups of 2)
(16 groups of 2) ----3:2---> (24 groups of 2)
(-8 groups of 2)
(16 groups of 2) -7:8->

-3:4->(8 x three)
(+8 x one)
(4 x eight) ------------7:8------------> (4 x seven)
(-4 x one)
(12 x two) ------------3:2----------> (12 x three)
(-4 x one)
(16 x two) --------3:2-------> (16 x three)
(-16 x one)
(4 x eight) --7:8-->



512:C	576:D	576:D
14	16	8
2	2	4
18	18	18
1.016	1	1

*587.1:D-(h32)>
568.8:?-(h31)
*550.4:C#(h30)
532.1:?-(h29)
→ *513.7:C-(h28)
495.4:B-(h27)
*477.0:B-(h26)
458.7:A#(h25)
*440.3:A-(h24)
422.0:?-(h23)
*403.6:G#(h22)
385.3:G-(h21)
*366.9:F#(h20)
348.6:F-(h19)
*330.2:E-(h18)
311.9:D#(h17)
*293.6:D-(h16)
275.2:C#(h15)
*256.9:C-(h14)
238.5:B-(h13)
*220.2:A-(h12)
201.8:G#(h11)
*183.5:F#(h10)
165.1:E-(h09)
*146.8:D-(h08)
128.4:C-(h07)
*110.1:A-(h06)
91.7:F#(h05)
*73.4:D-(h04)R
55.0:A-(h03)
*36.7:D-(h02)R
18.3:D-(H18)
Bar:007.000

*1006.5:B-(h48)>
985.5:B-(h47)
964.5:?-(h46)
943.6:A#(h45)
*922.6:A#(h44)
901.6:A#(h43)
880.7:A-(h42)
859.7:G#(h41)
*838.7:G#(h40)
817.8:G#(h39)
796.8:G-(h38)
775.8:F#(h37)
*754.8:F#(h36)
733.9:F-(h35)
712.9:F-(h34)
691.9:E-(h33)
*671.0:E-(h32)
650.0:?-(h31)
629.0:D#(h30)
608.1:?-(h29)
*587.1:D-(h28)
566.1:C#(h27)
545.2:C#(h26)
524.2:C-(h25)
*503.2:B-(h24)
482.3:?-(h23)
461.3:A#(h22)
440.3:A-(h21)
*419.4:G#(h20)
398.4:G-(h19)
377.4:F#(h18)
356.5:F-(h17)
*335.5:E-(h16)
314.5:D#(h15)
293.6:D-(h14)
272.6:C#(h13)
*251.6:B-(h12)
230.6:A#(h11)
209.7:G#(h10)
188.7:F#(h09)
*167.7:E-(h08)
146.8:D-(h07)
125.8:B-(h06)
104.8:G#(h05)
*83.9:E-(h04)R
62.9:B-(h03)
41.9:E-(h02)
21.0:E-(H20)
Bar:007.250

*1006.4:B-(h45)
984.1:B-(h44)
961.7:B-(h43)
939.3:A#(h42)
917.0:A-(h41)
*894.6:A-(h40)>
872.2:A-(h39)
849.9:G#(h38)
827.5:G-(h37)
805.1:G-(h36)
*782.8:G-(h35)
760.4:F#(h34)
738.0:F-(h33)
715.7:F-(h32)
693.3:?-(h31)
*670.9:E-(h30)
648.6:?-(h29)
626.2:D#(h28)
603.9:D-(h27)
581.5:D-(h26)
*559.1:C#(h25)
536.8:C-(h24)
514.4:?-(h23)
492.0:B-(h22)
469.7:A#(h21)
*447.3:A-(h20)
424.9:G#(h19)
402.6:G-(h18)
380.2:F#(h17)
357.8:F-(h16)
*335.5:E-(h15)
313.1:D#(h14)
290.7:D-(h13)
268.4:C-(h12)
246.0:B-(h11)
*223.6:A-(h10)
201.3:G-(h09)
178.9:F-(h08)
156.6:D#(h07)
134.2:C-(h06)
*111.8:A-(h05)R
89.5:F-(h04)
67.1:C-(h03)
44.7:F-(h02)
22.4:F-(H21)
Bar:007.500

832:A	640:E
12	9
5	5
14	14
0.990	1.016

*894.6:A-(h60)
879.7:A-(h59)
864.8:?-(h58)
849.9:G#(h57)
835.0:G#(h56)
*820.0:G#(h55)
805.1:G-(h54)
790.2:G-(h53)
775.3:G-(h52)
760.4:G-(h51)
*745.5:F#(h50)
730.6:F-(h49)
715.7:F-(h48)
700.8:F-(h47)
685.9:?-(h46)
*671.0:E-(h45)>
656.0:E-(h44)
641.1:E-(h43)
626.2:D#(h42)
611.3:D-(h41)
*596.4:D-(h40)
581.5:D-(h39)
566.6:C#(h38)
551.7:C-(h37)
536.8:C-(h36)
*521.9:C-(h35)
506.9:B-(h34)
492.0:A#(h33)
477.1:A#(h32)
462.2:?-(h31)
*447.3:A-(h30)
432.4:?-(h29)
417.5:G#(h28)
402.6:G-(h27)
387.7:G-(h26)
*372.8:F#(h25)
357.8:F-(h24)
342.9:?-(h23)
328.0:E-(h22)
313.1:D#(h21)
*298.2:D-(h20)
283.3:C#(h19)
268.4:C-(h18)
253.5:B-(h17)
238.6:A#(h16)
*223.7:A-(h15)
208.7:G#(h14)
193.8:G-(h13)
178.9:F-(h12)
164.0:E-(h11)
*149.1:D-(h10)
134.2:C-(h09)
119.3:A#(h08)
104.4:G#(h07)
89.5:F-(h06)
*74.5:D-(h05)R
59.6:A#(h04)
44.7:F-(h03)
29.8:A#(h02)
14.9:A#(H14)
Bar:007.625

*1073.5:C-(h64)>
1056.8:?-(h63)
1040.0:?-(h62)
1023.2:B-(h61)
*1006.4:B-(h60)
989.7:B-(h59)
972.9:?-(h58)
956.1:A#(h57)
*939.3:A#(h56)
922.6:A#(h55)
905.8:A-(h54)
889.0:A-(h53)
*872.2:A-(h52)
855.5:A-(h51)
838.7:G#(h50)
821.9:G-(h49)
*805.2:G-(h48)
788.4:G-(h47)
771.6:?-(h46)
754.8:F#(h45)
*738.1:F#(h44)
721.3:F#(h43)
704.5:F-(h42)
687.7:E-(h41)
*671.0:E-(h40)
654.2:E-(h39)
637.4:D#(h38)
620.6:D-(h37)
*603.9:D-(h36)
587.1:D-(h35)
570.3:C#(h34)
553.5:C-(h33)
*536.8:C-(h32)
520.0:?-(h31)
503.2:B-(h30)
486.4:?-(h29)
*469.7:A#(h28)
452.9:A-(h27)
436.1:A-(h26)
419.4:G#(h25)
*402.6:G-(h24)
385.8:?-(h23)
369.0:F#(h22)
352.3:F-(h21)
*335.5:E-(h20)
318.7:D#(h19)
301.9:D-(h18)
285.2:C#(h17)
*268.4:C-(h16)
251.6:B-(h15)
234.8:A#(h14)
218.1:A-(h13)
*201.3:G-(h12)
184.5:F#(h11)
167.7:E-(h10)
151.0:D-(h09)
*134.2:C-(h08)
117.4:A#(h07)
100.6:G-(h06)
83.9:E-(h05)
*67.1:C-(h04)R
50.3:G-(h03)
33.5:C-(h02)
16.8:C-(H16)
Bar:007.750

*1073.5:C-(h48)
1051.2:C-(h47)
1028.8:?-(h46)
1006.4:B-(h45)
*984.1:B-(h44)
961.7:B-(h43)
939.3:A#(h42)
917.0:A-(h41)
*894.6:A-(h40)
872.2:A-(h39)
849.9:G#(h38)
827.5:G-(h37)
*805.1:G-(h36)>→
782.8:G-(h35)
760.4:F#(h34)
738.0:F-(h33)
*715.7:F-(h32)
693.3:?-(h31)
670.9:E-(h30)
648.6:?-(h29)
*626.2:D#(h28)
603.9:D-(h27)
581.5:D-(h26)
559.1:C#(h25)
*536.8:C-(h24)
514.4:?-(h23)
492.0:B-(h22)
469.7:A#(h21)
*447.3:A-(h20)
424.9:G#(h19)
402.6:G-(h18)
380.2:F#(h17)
*357.8:F-(h16)
335.5:E-(h15)
313.1:D#(h14)
290.7:D-(h13)
*268.4:C-(h12)
246.0:B-(h11)
223.6:A-(h10)
201.3:G-(h09)
*178.9:F-(h08)
156.6:D#(h07)
134.2:C-(h06)
111.8:A-(h05)
*89.5:F-(h04)R
67.1:C-(h03)
44.7:F-(h02)
22.4:F-(H21)
Bar:007.875

7:8->(14 groups of 2)
(+2 groups of 2)
(16 groups of 2)-7:8->(14 groups of 2)
-1:2->(+5 groups of 4)
(12 groups of 4)-3:4->(9 groups of 5)
(-1 groups of 5)
(8 groups of 5)-3:2->(12 groups of 5)
(-3 groups of 5)
(9 groups of 5)10:9(10 groups of 4)
(16 groups of 4)3:4(12 groups of 4)
(+6 groups of 4)
(9 groups of 4)8:9->
(-3 groups of 4)

7:8->(4 x seven)
(+4 x one)
(4 x eight) ----7:8---> (4 x seven)
(+20 x one)
(3 x sixteen) -15:16-> (3 x fifteen)
(-5 x one)
(20 x two) ---3:2--> (20 x three)
(-15 x one)
(5 x nine) ----8:9---> (5 x eight)
(16 x four) --3:4--> (16 x three)
(+24 x one)
(4 x nine) -8:9->
(-12 x one)

MBN $8_4 0_{24} 0_1 = 12_2 0_{32} 0_1$

→ 768:G	→ 768:G	→ 768:G
-----	-----	-----
8	8	12
4	4	2
24	24	32
1	1	1

```
→ *805.2:G-(h32)>
   780.0:?-(h31)
   754.8:F#(h30)
   729.7:?-(h29)
  *704.5:F-(h28)
   679.3:E-(h27)
   654.2:E-(h26)
   629.0:D#(h25)
  *603.9:D-(h24)
   578.7:?-(h23)
   553.5:C#(h22)
   528.4:C-(h21)
  *503.2:B-(h20)
   478.1:A#(h19)
   452.9:A-(h18)
   427.7:G#(h17)
  *402.6:G-(h16)
   377.4:F#(h15)
   352.3:F-(h14)
   327.1:E-(h13)
  *301.9:D-(h12)
   276.8:C#(h11)
⇨  268.4:C-suspn
   251.6:B-(h10)
   226.4:A-(h09)
  *201.3:G-(h08)
   176.1:F-(h07)
   151.0:D-(h06)
   125.8:B-(h05)
 *100.6:G-(h04)R
   75.5:D-(h03)
   50.3:G-(h02)
   25.2:G-(H24)
   Bar:008.000
```

```
 *805.2:G-(h32)>
   780.0:?-(h31)
   754.8:F#(h30)
   729.7:?-(h29)
  *704.5:F-(h28)
   679.3:E-(h27)
   654.2:E-(h26)
   629.0:D#(h25)
  *603.9:D-(h24)
   578.7:?-(h23)
   553.5:C#(h22)
   528.4:C-(h21)
  *503.2:B-(h20)
   478.1:A#(h19)
   452.9:A-(h18)
   427.7:G#(h17)
  *402.6:G-(h16)
   377.4:F#(h15)
   352.3:F-(h14)
   327.1:E-(h13)
  *301.9:D-(h12)
   276.8:C#(h11)
   251.6:B-(h10)
   226.4:A-(h09)
  *201.3:G-(h08)
   176.1:F-(h07)
   151.0:D-(h06)
   125.8:B-(h05)
 *100.6:G-(h04)R
   75.5:D-(h03)
   50.3:G-(h02)
   25.2:G-(H24)
   Bar:008.250
```

```
 *805.2:G-(h24)
  771.6:?-(h23)
 *738.1:F#(h22)
  704.5:F-(h21)
 *671.0:E-(h20)
  637.4:D#(h19)
 *603.9:D-(h18)
  570.3:C#(h17)
 *536.8:C-(h16)
  503.2:B-(h15)
 *469.7:A#(h14)
  436.1:A-(h13)
 *402.6:G-(h12)
  369.0:F#(h11)
 *335.5:E-(h10)
  301.9:D-(h09)
 *268.4:C-(h08)
  234.8:A#(h07)
 *201.3:G-(h06)
  167.7:E-(h05)
 *134.2:C-(h04)
  100.6:G-(h03)
 *67.1:C-(h02)R
  33.5:C-(H32)
  Bar:008.500
```

-8:9-> (8 groups of 4) --------1:1---------> (8 groups of 4) ---------3:2---------> (12 groups of 2)

-8:9-> (4 x eight) -----------------------------> (8 x four) ------------3:4------------> (8 x three)