Journey to the Heart of Music

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Introduction and Guide to the MOS Examples

MUTABLE NUMBER ANALYSIS

A full description of the MOS (Modulating Oscillatory Systems) model and the Mutable Base Position-Value Number System can be found in the main text of *Journey to the Heart of Music*. The first part of this document is an introduction to the book's attached examples, in particular Example G. Following this brief introduction there is a guide to the techniques involved in constructing MOS analyses, referral to the main chapters is also recommended.

If one were to draw an analogy between physical theory and music theory, the analysis given by the MOS approach might be likened to a form of quantum mechanics for tonal music: That is, a fundamental approach to tonal music operating at a sufficiently elemental level so as to be capable of explaining its most basic mechanisms and characteristics. The MOS model, to carry the analogy further, does not enlighten us much regarding the broader human and cultural issues or context, about the purpose of the architecture or the intent of the architect, only the underlying engineering is elucidated. The model is myopic, essentially seeing no further than the next chord. Other theories concerned with the wider perspectives of what might be termed the 'classical mechanics' of music are required to access broader vistas and higher level understandings. In particular, the MOS model might well be integrated into theories which focus on the dynamic aspect of tonal compositions. As in physics, where both the quantum and classical worlds coexist (though not always peacefully), the MOS approach does not overturn the relevance of other tonal theories but rather, for the most part, complements and underpins them; with perhaps mutable numbers playing a similar role in regard to tonal music, as that of the group of integers modulo twelve in atonal music.

At bottom the MOS model of tonal music is a means of construing the physical reality of musical sound as a representation of number. That is, by extrapolation from the frequency relationships of the objective written notes – up to common overtones and downward to a unit fundamental – a positional number system can be constructed: Mutable Base Numbers. When applying a mutable number analysis to a composition, while it is almost always possible to derive some combination of nested harmonic series able link together even the most difficult, not to say desperate, chord progressions, it may not be wise or sound analysis so to do in every case. Given the complex resources of the human mind; the ability to skip passed unexpected detail, fill in assumed ‘missing’ sensory information or leap across gaps of logic to form broad understandings, the MOS model should not endeavour to provide an absolutely compelling analysis for every chord sequence ever written. Rather the model should, hopefully, capture the essence of the structure generally, while also being compelling in its analysis of the detail of ‘regular’ tonal chord sequences, particularly those of core importance to the tonal structure of the composition.

Equally, the model should not be constrained by the extent of aural cognition, whatever these (as yet unknown) limits turn out to be. It would be remarkable, I think, if the ear and mind were truly able to acquire every detail of the extended family of nested harmonic series proposed in the MOS model. More likely our ‘ears’ grasp the explicit upper level *aggregated* series defined by the principal relationships of the objective musical sound and from of this network of tones (notes plus partials) aural cognition computes the essence of the mutable number digit exchanges described by the model without explicitly reaching down to the foundations of the system. Though of course, it might well turn out that the MOS approach proves to have no connection at all with the processes of aural cognition, in which case the theory simply falls back to being yet another mathematical model of tonal music, and hopefully, still a useful analytical tool.

INTRODUCTION TO THE MOS EXAMPLES

In the Example G analysis a parallel range of information is delivered in a variety of different formats, going from top to bottom of the page; 1) Mutable Base Numbers in subscript format, 2) traditional Roman numeral harmonic analysis, 3) Mutable Base Numbers in ‘stacked’ factor format, 4) explicit harmonic series and 5) additions, subtractions and ratio exchanges of the modulation algorithm of symmetrical exchange.

Above the system the current *mutable base number* may be shown in subscript format, for example,

$$\text{MBN } 9_2\, 0_{32}\, 0_1$$

which in decimal is 576 or in a generalised positional notation, Dec $5_{10}\, 7_{10}\, 6_1$. The mutable numbers appear once, continuing in force until replaced by another mutable digit sequence – which might represent the same or a different conjunction value – as unlike fixed base number systems (i.e. Decimal, Binary, etc.) mutable base numbers will, most often, have a *range of different digit sequences* for a given value. It is this richness and variety of number representation which enables mutable base numbers to capture tonal chord progressions within the digit sequences of one value – thus rendering the progression logical and commensurable,... or incommensurable where no corresponding digit sequence exists! However, it should not be thought that mutable numbers command tonal music, rather the other way about, the chords of tonal music are (parts of) mutable numbers written in sound and the ‘rules’ or characteristics of tonal harmony are thus also those of the mutable number system. Tonal harmonic progressions which make sense aurally, will have corresponding mutable number digit sequences which ‘add up’, or in math-speak ‘mathematically exist’. For convenience and ease of reading, both the column digits and column base subscripts of mutable numbers are written in the form of plain decimal numbers. (It would be possible to use letters for extra symbols, as in the hexadecimal system and indeed any other symbols but as there is no limit to the range of either column digits or column bases in the mutable system this strategy would make the written notation extremely cumbersome.)

Between the staves a traditional Roman numeral analysis of the harmony is given, for example,

Key G: I ii V^7 I.

Below the system the current mutable base number is shown in 'stacked' *factor format*, with the equivalent decimal value, separated by a line of dashes, placed above it. This decimal number is the value of the conjunction which links adjacent chords together and by means of this commonality, renders the harmonic progression commensurable. The conjunction values used in factor format (and subscript format) are usually fixed at the frequency of the 'key' series, this is done so that the mutable numbers remain anchored around the unit H1, which enhances the presentational logic of being in a key/tonal center. (It is equally possible to track the precise course taken by the MOS model from start to finish, by using freely varying real values, which is what the full MOS analysis, below, does.) The stacked factor format numbers are updated with each new chord, whether they change digit sequence or not. Also, as in the example below, the note letter equivalent of the conjunction's frequency ratio is often appended to the decimal value.

576-A

9
2
32
1

Sometimes the conjunction value will run across a number of different chords linking them all by a common value; but as often as not, a new conjunction value, either higher or lower, will be required to connect to the next chord in a harmonic progression. A new conjunction is found by counting upward or downward in the ratios of the uppermost harmonic series. When this occurs the gray band running across the page is broken and a new band commenced at the appropriate level. The harmonic motion of a composition will at times lead to a 'flexing' of strict whole number relationships within a multi-column mutable number. This occurs where a modulation exchange produces a non-integer result (within the fixed value grid applied to the tonal center) and it is accommodated within the system by means of a fractional unit. That is, the unitary period of the absolute fundamental frequency (H1) is allowed a little leeway or tolerance to expand or contract as required to maintain a common conjunction value between chords, for example:

576-A

9
3
21
1.016

Although this might be viewed as something of a fudge or blemish upon the purity of a system essentially rooted in whole numbers, it mirrors the tolerance of small deviations of pitch exhibited by the human ear. Indeed, the equal-tempered scale upon which tonal music depends for access to all keys on the piano and many other instruments, would be impossible without such a tolerance. And, in the next format below this feature disappears into a continuum of smoothly variable values.

The conjunction band mentioned above actually runs through the uppermost ratios of a *nested system of harmonic series,* which are written out below the stacked mutable numbers. These series could be taken to represent the music as a ‘pseudo-physical’ system of relationships evolving by means of addition, subtraction and modulation exchanges (harmonic progression) in a material context. In the ‘classical’ material world, which is of course the environment in which music actually exists, relationships are generally taken to be smoothly variable. And so the ‘flexing’ of relationships here presents no obstacle, for example F-h21 is twenty-one whole steps of the harmonic series above h1 and if those steps happen to take the value of 1.016 arbitrarily defined units of frequency (rounded up from 1.015873...), then the twenty-first step will consist of F-21.333... units. In this format of extended harmonic series, the MOS model (Modulating Oscillatory System) takes the changes in the musical relationships, the modulation exchanges, as they come and computes the results in real numbers, that is, decimals with unlimited fractional expansions. Though here in these analyses, for convenience, only one decimal place is shown. (The reader may calculate to whatever accuracy they require.) Beside these real numbers the ‘logical’ number or relationship to the unit is given in brackets: h1, h2, h3, etc. these are, of course, the ratios of the harmonic series – and the digits of mutable numbers. As for the most part three nested harmonic series are present: a fundamental/nesting series (H1, H2, H3, etc.), a nested middle level series (h1, h2, h3, etc.) and a top level aggregated series marked with asterisks (h3*, h6*, h9*, etc.) – they are distinguished by the symbols given here in brackets. These three nested harmonic series correspond to the columns in mutable base numbers. However, there may be any number of columns in a mutable number or levels of nesting in a MOS analysis. Three columns is usual and convenient, two not uncommon, but if the relatively large fundamental series (e.g. typically H1 through H32, or H1 through H48) were to be broken down into their factors, then double or more columns/nestings would emerge. Such additional detail in the ‘murky depths’ are implied but not shown in most MOS analyses, beyond the connection to the middle level nested series, for example: C-h1/H32.

Finally, below the written out harmonic series is the ‘algebra’ of additions, subtractions and modulations of the MOS model’s algorithm of symmetrical exchange. Here the operations (addition, subtraction or modulation exchange) are shown with arrows linking two bracketed descriptions of a nested pair of harmonic series. In black oblique script the movement of the upper aggregated series within the middle level nested series is shown. And in a gray upright script the changes of the middle level nested series paired with the fundamental nesting series is given. Both pairs, aggregated/nested series and nested/ fundamental series share the same conjunction. Indeed, there is no theoretical limit to the number of levels of nesting that could be employed, provided there are sufficient elements in the system to support the structure – yet still there would be only one, shared, conjunction. A MOS system is driven from the top by the motion of the objective notes. As the notes change from beat to beat and measure to measure, the underlying structure of nested harmonic series must respond to accommodate the harmonic motion. Sometimes only change in the upper level is required, at other times the low level nested/fundamental series must also change along with the aggregated series to accommodate the harmonic motion of the objective notes; and where a piece changes key, the whole system of nested series must change to new ground – a new absolute fundamental tone. Finally, though in these examples the model is employed for harmonic analysis, the MOS approach is equally amenable to metrical analysis.