Journey to the Heart of Music

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Example I
J.S.Bach – Chorale
Christus, der ist mein Leben

MUTABLE NUMBER ANALYSIS

Viewed from the perspective of the MOS model, 'tonal computation' – number processing by means of harmonic progression – did not spring into existence fully formed at the dawn of the tonal era (circa 1400) but rather gradually evolved over many generations, until by the eighteenth century the system of mutable numbers, written in musical sound, finally found its most clearly focused expression in the music of the baroque and classical periods. Musicians and composers of polyphonic music, no matter what their intentions and preconceived notions where, found that in the process of making music in many parts, their instinctive musical compasses would inevitably swing around, to guide their harmony toward a tonal pole. Unrecognised for what it actually was, but intuitively understood, musicians found and developed a language of number in their harmonic logic. During tonality's long zenith from Corelli to Schubert the stylistic course travelled by western music passed across this center of tonal attraction: Then continuing onward, driven by the momentum the system of tonality had attained, western music progressed through a period of romanticism, coasting on to dissipate its last vestige of harmonic energy and developmental vitality in the fractured trajectories of the late and post romantics – nationalism, neo-classicism, impressionism. However, by this stage in the early twentieth century, most of the participants in the great tonal endeavour, both audience and many musicians, discovered that their own internal compasses were again swinging about, reversing direction and pointing back to the music of the past rather than forward to the unfathomable ocean of atonality. This set the scene for the twentieth century's rediscovery of the historic canon of tonal music, begun earlier by Mendelssohn with the popularisation of works by J.S.Bach, the process of rediscovery gathered pace in later years to eventually encompass every corner and byway of tonality's historic landscape.

Bach's settings of the chorales probably have too much of his individual touch to lay claim to being the absolutely clearest expression of tonal computation in musical form, nevertheless they are certainly not far off this, and perhaps all the more interesting examples for the harmonic ingenuity with which he invests these noble melodies.

Though only eight measures of F major, in this short chorale Bach manages both to hint at a change of tonal center (measures 6 and 7) and to invigorate a basically simple harmonic framework with the addition of many suspensions and passing notes plus a touch of drama (measure 8, beat 3) before the final cadence. The harmonic language is avowedly straightforward common practice, with four phrases beginning on the tonic or subdominant and closing on the tonic or dominant. The cadential progressions being IV-V-I, I-V-I, vi^7-II-V (ii-V-I in the dominant key) and ii^7-V^7-I, respectively. All but one of these highly directional chord progressions (i.e. IV-V) involve the powerful sesquitertia (3:4) and sesquialtera (2:3) ratios of exchange. These cadences show mutable number processing at its most explicit, with each harmonic step clearly defined by large changes of a quarter or a third (gain or loss) in the number of elements in the surface level aggregated series. Which translates into relatively marked changes to the notional energy of the system. In contrast, between the opening of a phrase and the cadential progression the harmonic motion can be more stepwise (measures 4, 6 and 8) deploying bouts of sesquioctava (8:9) and sesquinona (9:10) modulation exchanges, in the foreground aggregated series, yielding gentle shades of gain or loss.

Overall the values computed in the chorale are on the low side, ranging from decimal 384 to just below one thousand, which is to be expected in a relatively short piece of prime common practice harmony. The high point of decimal 960 (MBN $10_4\ 0_{24}\ 0_1$) is attained in the harmonically climatic third phrase, where the piece veers toward the key of C major; and from which it descends in the fourth phrase to cadence with the number with which it began, MBN $6_4\ 0_{16}\ 0_1$ (decimal 384).

Perhaps the most interesting interpretive choice is presented by the touch of harmonic drama Bach introduces into the final cadence (measure 8, beat 3). Is this chord a first inversion ii^7 (G-minor) with flattened fifth or a root position iv (Bflat-minor) with added major sixth? This is in part the age old question of the ambiguity of chord ii versus chord IV, discussed at some length by R.F. Goldman in Chapter 5 of *Harmony in Western Music*. In this chorale Bach gives the ambiguity an added twist by flattening the sixth scale degree (Dflat). The perspective the MOS approach brings to this conundrum is that either interpretation, chord ii or chord iv/IV, sit equally comfortably within the same background nested series. It is here merely a question of dividing the underlying nested series (built on D#h1/H14) into aggregates of three or five. The lower energy configuration is provided by groups of five. This yields chord ii, probably the predominant way most listeners would hear the chord progression. Alternatively, grouping the nested series into aggregates of three yields chord iv (or IV), a more energetic arrangement and an interpretation perhaps less likely to be placed upon the progression. (The alternative possibility of a nested series built on F#h1/H17, could host an aggregated series in groups of five supporting chord iv^6 but would preclude chord ii, and would be totally alien to a full and final cadential progression in F major.) An interesting historical side light to this is J-P. Rameau use of the concept of 'double emploi' in this situation, which though motivated by a desire to maintain root (fundamental bass) motion by fifths, contains a similar harmonic flexibility to the MOS interpretation.

Typical of the many lesser choices are, for example, the nested series C-h1/H24 (measure 3, beats 1 and 2) and C-h1/H12 (measure 4, beat 1). In the former ‘short series’ the passing notes have been given by implication to the fundamental series built on F-H1, where any non-harmonic tones can ultimately find a place. Ideally, at least, the MOS model will account for all the notes in a composition, with the awkward dissonant and passing notes dropping through to lower level series, where necessary. In contrast, the latter ‘long series’ accommodates the suspended dissonant F, but at the expense of a more complex and extended aggregated series. However, as the conjunction value for this series (shown by the gray band) is equal to the second harmonic of the chorale melody note, the distance between the objective sound and the rather complex mutable digit sequence is perhaps not that great.

In charting a mutable number analysis of a composition, the general ‘rule of thumb’ is to join adjacent chords together by the lowest value conjunction equal to or greater than, the highest objective note. Above this first conjunction there will be a theoretically unlimited number of additional higher frequency conjunctions. The further one goes the more there are within each octave. Thus in providing the first, or perhaps one of the lowest conjunctions in an analysis, one is implicitly demonstrating that there are many more higher frequencies which also conjoin the harmonies together. (The example of two simultaneous conjunctions is shown in full at the final cadence of the chorale below, plus five higher conjunctions in a dashed line box.) Though when listening to music ones attention is naturally drawn to the perceived objective notes, the ear without conscious awareness, processes sound over the entire range of hearing, which includes the many overtones generated by musical instruments and the human voice. (As well as some other non-objective pitches generated within the ear.) The range of greatest interest lies between the two octaves from 1kHz and 4kHz, the top two octaves on the piano keyboard. Amongst the considerable array of frequencies in this range produced in performance, there will most probably be some objective frequencies (and perhaps also some subjective tones) which also match the conjunctions associated with the particular harmonic exchange used in the MOS analysis. To write out in detail all these frequencies would be difficult to fit upon the page and so just a few example frequencies are shown at measures 8 to 9. All the higher frequency conjunctions illustrated in the dashed line box are present in the harmonic series of the objective notes of the two chords to a greater or lesser extent. For example, E-1280Hz appears as an overtone in the harmonic series of three of the four notes in both the C-major and F-major cadential chords.

meter

(Metrical MOS)

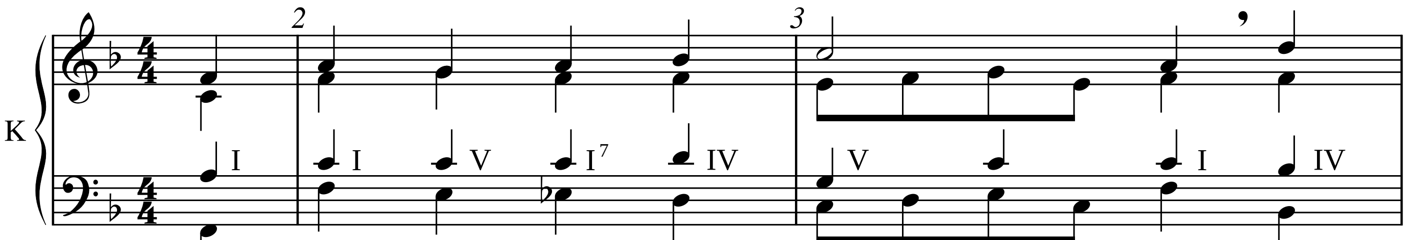
MBN: $4_4 0_1$

```
h4...:sixteenthnote figuration
h3
h2...:eighthnote figuration
h1/H4:quarternote pulse
   H3
   H2
   H1:meter 4/4 time
```

Measures 1 through 9

Figure I.1 In these measures the finest durational subdivision consists of sixteenthnotes, giving the chorale’s common 4/4 time signature the pleasingly similar mutable number representation of MBN: $4_4 0_1$ (which is equal to Decimal 16).

I.4 - CHORALE, J.S.BACH



Value:	C-384 (decimal)	C-384	C-384	C-384	C-384	C-384	C-384	C-384	F-512	F-512
	------	------	------	------	------	------	------	------	------	------
Mutable	6	4	6	12	9	8	4	6	8	6
Digit	4	4	4	2	2	2	4	4	4	4
Sequence	16	24	16	16	21	24	24	16	16	21
	1 } Tonal Center F	1	1	1	1.016	1	1	1	1	1.016

Objective
Notes: ◯

Aggregates: *

Mid.C=256Hz

Conjunction:>

						F-h32*> 682.6Hz	F-h24*>
						- h31	- h23
						E-h30	E-h22
						- h29	D#h21
						D#h28*	D-h20*
						- h27	C#h19
						D-h26	C-h18
						C#h25	B-h17
Conjunction	C-512.0(h24)*>	C-h16*>	C-h24**>	C-h18*> 512.0Hz	C-h16**>	C-h24*	A#h16*
	- 490.6(h23)	B-h15	- h23	B-h17	B-h15	- h23	A-h15
	B-469.3(h22)	A#h14	B-h22*	A#h16*	A#h14*	B-h22	G#h14
	A#448.0(h21)	A-h13	A#h21	A-h15	A-h13	A#h21	G-h13
	A-426.6(h20)*	G-h12*	A-h20**	G#h14*	G-h12**	A-h20*	F-h12*
	G#405.3(h19)	F#h11	G#h19	G-h13	F#h11	G#h19	E-h11
	G-384.0(h18)	E-h10	G-h18*	F-h12*	E-h10*	G-h18	D-h10
	F#362.6(h17)	D- h9	F#h17	E-h11	D- h9	F#h17	C- h9
	F-341.3(h16)*	C- h8*	F-h16**	D-h10*	C- h8**	F-h16*	A# h8*
	E-320.0(h15)	A# h7	E-h15	C- h9	A# h7	E-h15	G# h7
	D#298.6(h14)	G- h6	D#h14*	A# h8*	G- h6*	D#h14	F- h6
	D-277.3(h13)	E- h5	D-h13	G# h7	E- h5	D-h13	D- h5
	C-256.0(h12)*	C- h4*R	C-h12**	F- h6*	C- h4**	C-h12*	A# h4*R
	B-234.6(h11)	G- h3	B-h11	D- h5	G- h3	B-h11	F- h3
	A-213.3(h10)	C- h2	A-h10*	A# h4*	C- h2*R	A-h10	A# h2
	G-192.0 (h9)	C-h1/H24	G- h9	F- h3	C-h1/H24	G- h9	A# h1/H21
	F-170.6 (h8)*	(32.000Hz)	F- h8**	A# h2*R	(32.000Hz)	F- h8*	(28.444Hz)
	D#149.3 (h7)		D# h7	A# h1/H21		D# h7	
	C-128.0 (h6)		C- h6*	(28.444Hz)		C- h6	
	A-106.6 (h5)		A- h5			A- h5	
	F- 85.3 (h4)*R		F- h4**			F- h4*R	
	C- 64.0 (h3)		C- h3			C- h3	
	F- 42.6 (h2)		F- h2*R			F- h2	
	F- 21.3 (h1/H16)		F- h1/H16			F- h1/H16	
			(21.333Hz)			(21.333Hz)	
	....						
	F- 1.333Hz (H1)						

Aggregated
Series: *(6 groups of 4) -2:3-> (4 groups of 4) 3:2 (6 grps of 4)*
[Secondary Sesquialtera Exchange]
(2:1)
(12 grps of 2) 3:4 (9 grps of 2) 8:9 (8 grps of 2)
[Tertiary Sesquitertia Exchange]
(1:2)
(4 grps of 4) 3:2 (6 grps of 4)
(+2 grps of 4)
(8 grps of 4) --3:4-> (6 grps of 4) ->

Nested
Series: (8 x three) -----2:3-----> (8 x two) -3:2-> (8 x three)
[Eightfold Sesquialtera Exchange]
(6 x four) 3:4 (6 x three)
(2 x nine) ----8:9----> (2 x eight)
(8 x two) -3:2-> (8 x three)
(+8 x one)
(8 x four) --3:4--> (8 x three) -->

I.5 - CHORALE, J.S.BACH

F-512	G-576	G-576	G-576	G-576	G-576	G-576	G-576	G-576	E-960	E-960	E-960
------	------	------	------	------	------	------	------	------	------	------	------
8	9	8	9	12	9	9	8	6	10	9	6
4	4	5	4	4	4	4	4	4	4	5	6
16	16	28	16	12	16	16	18	24	24	21	26
1	1	1.029	1	1	1	1	1	1	1	1.016	1.026

```
G-h36*>
F-h32*
D#h28*
- h27
D-h26
C#h25
C-h24*
- h23
B-h22
A#h21
A-h20*
G#h19
G-h18
F#h17
F-h16*
E-h15
D#h14
D-h13
C-h12*
B-h11
A-h10
G- h9
F- h8*
D# h7
C- h6
A- h5
F- h4*R
C- h3
F- h2
F- h1/H16
(21.333Hz)
```

```
768.0Hz
G-h40*>
- h35*
D-h30*
- h29
C#h28
- h27
C-h26
B-h25*
A#h24
- h23
A-h22
G#h21
G-h20*
F#h19
F-h18
E-h17
D#h16
D-h15*
C#h14
C-h13
A#h12
A-h11
G-h10*
F- h9
D# h8
C# h7
A# h6
G- h5*R
D# h4
A# h3
D# h2
D#h1/H14
(19.200Hz)
```

```
G-h36*>
F-h32*
D#h28*
- h27
D-h26
C#h25
C-h24*
- h23
B-h22
A#h21
A-h20*
G#h19
G-h18
F#h17
F-h16*
E-h15
D#h14
D-h13
C-h12*
B-h11
A-h10
G- h9
F- h8*
D# h7
C- h6
A- h5
F- h4*R
C- h3
F- h2
F- h1/H16
(21.333Hz)
```

```
G-h48*>
F#h44*
E-h40*
D-h36*
C-h32*
A#h28*
G#h25
G-h24*
- h23
F#h22
F-h21
E-h20*
D#h19
D-h18
C#h17
C-h16*
B-h15
A#h14
A-h13
G-h12*
F#h11
E-h10
D- h9
C- h8*
A# h7
G- h6
E- h5
C- h4*R
G- h3
C- h2
C-h1/H12
(16.000Hz)
```

```
G-h36*>
F-h32*
D#h28*
- h27
D-h26
C#h25
C-h24*
- h23
B-h22
A#h21
A-h20*
G#h19
G-h18
F#h17
F-h16*
E-h15
D#h14
D-h13
C-h12*
B-h11
A-h10
G- h9
F- h8*
D# h7
C- h6
A- h5
F- h4*R
C- h3
F- h2
F- h1/H16
(21.333Hz)
```

```
G-h36*>
F-h32*
D#h28*
- h27
D-h26
C#h25
C-h24*
- h23
B-h22
A#h21
A-h20*
G#h19
G-h18
F#h17
F-h16*
E-h15
D#h14
D-h13
C-h12*
B-h11
A-h10
G- h9
F- h8*
D# h7
C- h6
A- h5
F- h4*R
C- h3
F- h2
F- h1/H16
(21.333Hz)
```

```
G-h32*>
- h31
F#h30
- h29
F-h28*
- h27
E-h26
D#h25
D-h24*
- h23
C#h22
C-h21
B-h20*
A#h19
A-h18
G#h17
G-h16*
F#h15
F-h14
E-h13
D-h12*
C#h11
B-h10
A- h9
G- h8*
F- h7
D- h6
B- h5
G- h4*R
D- h3
G- h2
G-h1/H18
(24.000Hz)
```

```
1280.0Hz
E-h40*>
D-h36*
C-h32*
A#h28*
G#h25
G-h24*
- h23
F#h22
F-h21
E-h20*
D#h19
D-h18
C#h17
C-h16*
B-h15
A#h14
A-h13
G-h12*
F#h11
E-h10
D- h9
C- h8*
A# h7
G- h6
E- h5
C- h4*R
G- h3
C- h2
C-h1/H24
(32.000Hz)
```

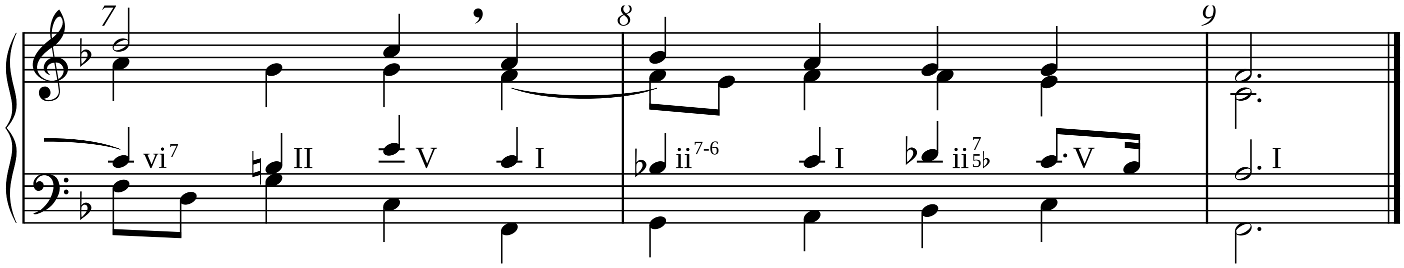
```
E-h45*>
D-h40*
- h35*
A-h30*
- h29
G#h28
- h27
G-h26
F#h25*
F-h24
- h23
E-h22
D#h21
D-h20*
C#h19
C-h18
B-h17
A#h16
A-h15*
G#h14
G-h13
F-h12
E-h11
D-h10*
C- h9
A# h8
G# h7
F- h6
D- h5*R
A# h4
F- h3
A# h2
A# h1/H21
(28.444Hz)
```

```
E-h36**>
- h35
D#h34
- h33*
D-h32
C#h30**
C-h28
- h27*
B-h26
A#h25
A-h24**
- h23
G#h22
G-h21*
F#h20
F-h19
E-h18**
D#h17
D-h16
C#h15*
C-h14
B-h13
A-h12**
G#h11
F#h10
E- h9*
D- h8
C- h7
A- h6**
F# h5
D- h4
A- h3*R
D- h2
D- h1/H26
(35.555Hz)
```

4:3 (8 grps 4)
(+1 grp of 4)
(9 grps 4) 10:9 (10 grps 5) 9:10 (9 grps 4) 4:3 (12 grps 4) 3:4 (9 grps of 4) ---8:9----> (8 grps 4) 3:4 (6 grps 4)
(+4 grps of 4)
(10 grps of 4) 9:10 (9 grps 5) 2:3 (6 grps 6)

4:3 (8 x four)
(+4 x one)
(4 x nine) 10:9 (4 x ten) 9:10 (4 x nine)
(12 x three) 4:3 (12 x four) 3:4 (12 x three)->(4 x nine) 8:9 (4 x eight)
(8 x four) 3:4 (8 x three)
(+16 x one)
(5 x eight) 9:8 (5 x nine)
(9 x five) 4:5 (9 x four)

I.6 - CHORALE, J.S.BACH



E-960	D-832	D-832	D-832	G-576	G-576	G-576	G-576	G-576	G-576	C-384	C-384
9	8	12	9	6	9	8	9	8	6	4	6
5	5	4	4	4	4	5	4	5	4	4	4
21	21	18	24	24	16	14	16	14	24	24	16
1.016	0.990	0.963	0.963	1	1	1.029	1	1.029	1	1	1

2048Hz	C-h64*>	C-h96*
1792Hz	A#h56*>	A#h84*
1536Hz	G-h48*>	G-h72*
1280Hz	E-h40*>	E-h60
1024Hz	C-h32*>	C-h48*

E-h45*								
D-h40*>	D-h48*>	D-h36* 1152.0Hz						
- h35*	C#h44*	C-h32*						
A-h30*	B-h40*	A#h28*						
- h29	A-h36*	G#h25						
G#h28	G-h32*	G-h24*>	G-h36*>	G-h40*>	G-h36*>	G-h40*>	G-h24* 768.0Hz	G-h36*
- h27	F-h28*	- h23	F-h32*	- h35*	F-h32*	- h35*	- h23	F-h32*
G-h26	D-h24*	F#h22	D#h28*	D-h30*	D#h28*	D-h30*	F#h22	D#h28*
F#h25*	- h23	F-h21	- h27	- h29	- h27	- h29	F-h21	
F-h24	C#h22	E-h20*	D-h26	C#h28	D-h26	C#h28	E-h20*	- h27
- h23	C-h21	D#h19	C#h25	- h27	C#h25	- h27	D#h19	D-h26
E-h22	B-h20*	D-h18	C-h24*	C-h26	C-h24*	C-h26	D-h18	C#h25
D#h21	A#h19	C#h17	- h23	B-h25*	- h23	B-h25*	C#h17	
D-h20*	A-h18	C-h16*	B-h22	A#h24	B-h22	A#h24	C-h16*> 512.0Hz	C-h24*
C#h19	G#h17	B-h15	A#h21	- h23	A#h21	- h23	B-h15	- h23
C-h18	G-h16*	A#h14	A-h20*	A-h22	A-h20*	A-h22	A#h14	B-h22
B-h17	F#h15	A-h13	G#h19	G#h21	G#h19	G#h21	A-h13	A#h21
A#h16	F-h14	G-h12*	G-h18	G-h20*	G-h18	G-h20*	G-h12*	A-h20*
A-h15*	E-h13	F#h11	F#h17	F#h19	F#h17	F#h19	F#h11	G#h19
G#h14	D-h12*	E-h10	F-h16*	F-h18	F-h16*	F-h18	E-h10	G-h18
G-h13	C#h11	D- h9	E-h15	E-h17	E-h15	E-h17	D- h9	F#h17
F-h12	B-h10	C- h8*	D#h14	D#h16	D#h14	D#h16	C- h8*	F-h16*
E-h11	A- h9	A# h7	D-h13	D-h15*	D-h13	D-h15*	A# h7	E-h15
D-h10*	G- h8*	G- h6	C-h12*	C#h14	C-h12*	C#h14	G- h6	D#h14
C- h9	F- h7	E- h5	B-h11	C-h13	B-h11	C-h13	E- h5	D-h13
A# h8	D- h6	C- h4*R	A-h10	A#h12	A-h10	A#h12	C- h4*R	C-h12*
G# h7	B- h5	G- h3	G- h9	A-h11	G- h9	A-h11	G- h3	B-h11
F- h6	G- h4*R	C- h2	F- h8*	G-h10*	F- h8*	G-h10*	C- h2	A-h10
D- h5*R	D- h3	C-h1/H24	D# h7	F- h9	D# h7	F- h9	C-h1/H24	G- h9
A# h4	G- h2	(32.000Hz)	C- h6	D# h8	C- h6	D# h8	(32.000Hz)	F- h8*
F- h3	G-h1/H18		A- h5	C# h7	A- h5	C# h7		D# h7
A# h2	(24.000Hz)		F- h4*R	A# h6	F- h4*R	A# h6		C- h6
A# h1/H21			C- h3	G- h5*R	C- h3	G- h5*R		A- h5
(28.444Hz)			F- h2	D# h4	F- h2	D# h4		F- h4*R
			F- h1/H16	A# h3	F- h1/H16	A# h3		C- h3
			(21.333Hz)	D# h2	(21.333Hz)	D# h2		F- h2
				D#h1/H14		D#h1/H14		F- h1/H16
				(19.200Hz)		(19.200Hz)		(21.333Hz)

3:2 (9 grps 5)
(-1 grp 5)
(8 grps 5) 3:2 (12 grps 4) 3:4 (9 grps 4)
(-3 grps 4)
[Tertiary Sesquialtera Exchange]
(6 grps 4) 3:2 (9 grps 4) 9:8 (8 grps 5) 8:9 (9 grps 4) 9:8 (8 grps 5) -3:4-> (6 grps 4) -3:2-> (9 grps of 4)
[Primary Sesquioctava Modulation Exchange]
(-2 grps 4)
(4 grps of 4) -3:2-> (6 grps of 4)
[Secondary Sesquialtera Exchange]

5:4 (9 x five)
(-5 x one)
(8 x five) -6:5-> (8 x six)
(12 x four) -3:4-> (12 x three)
(-12 x one)
(12 x two) 3:2 (12 x three)
(4 x nine) 10:9 (4 x ten) 9:10 (4x nine) 10:9 (4 x ten)
[Quaternary Sesquinona Modulation Exchange]
(8 x five) 3:5 (8 x three)
(-8 x one)
(8 x two) --3:2--> (8 x three)
[Octonary Sesquialtera Exchange]