Journey to the Heart of Music
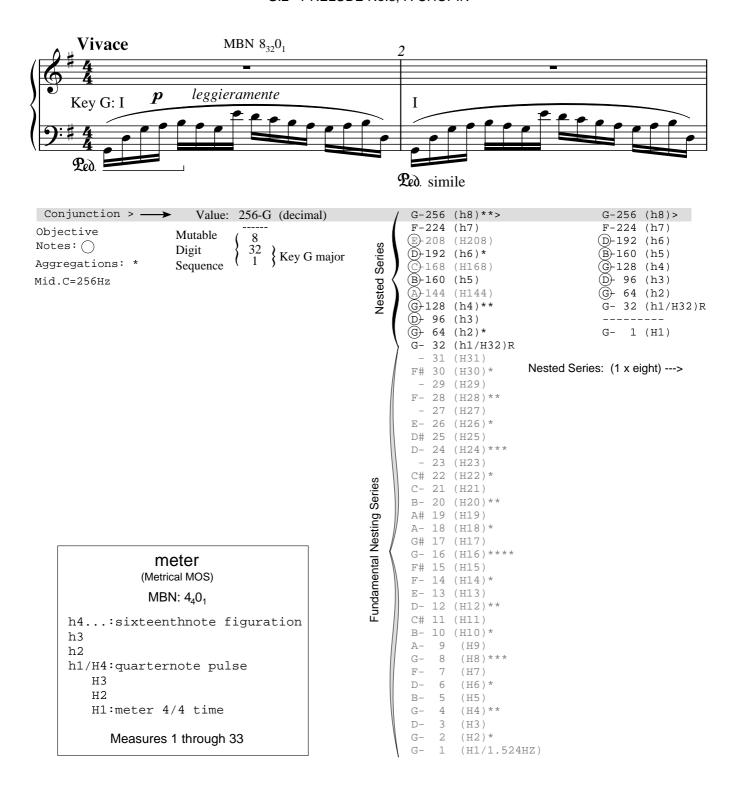
Philip Perry

Example G
Frederic Chopin – Prelude No.3, Op28
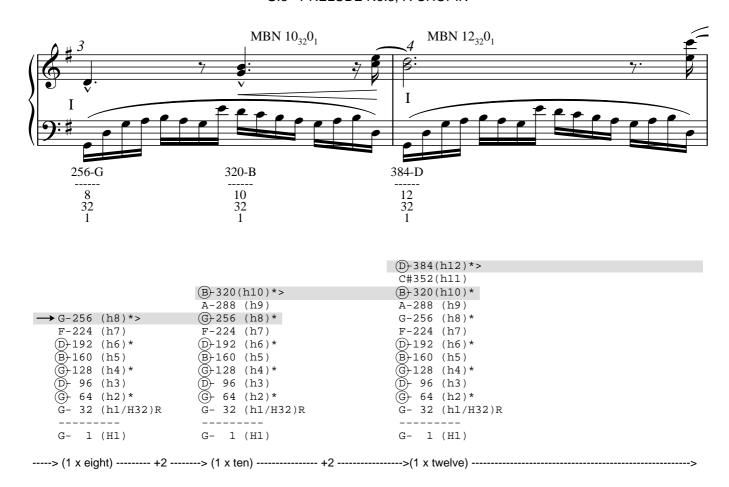
MUTABLE NUMBER ANALYSIS

The Prelude has a straightforward harmonic structure: set within extended passages of the tonic G-major chord there are two excursions, one to the dominant via the supertonic (I II^7 V$^{(7)}$, I) followed by a second excursion to the subdominant (I^7 IV, II V I). Interestingly Chopin chooses to use major chords throughout, which means the mutable base number analysis can encompass the chords within one level of nesting, that is two harmonic series one nested within the other. Fortunately this single level of nesting will allow a gentle introduction to MOS/mutable number analysis. The two series are distinguished as the upper *nested series* and the lower *nesting series*. The lower series may also be referred to as the *fundamental series* of the system: it encapsulates the sense of key. The upper nested series, in contrast, contains for the most part, the objective notes (shown circled) and movements in this more explicitly defined series imply the existence of its more fundamental nesting partner. Occasionally, objective notes which cannot find positions in the upper nested series will be relegated to this rather more shadowy background series. This placement is not unwelcome as it helps to bolster and maintain the existence of this less strongly drawn series and is a particular feature of the model's treatment of the minor chord.

In the following analysis the background nesting series is written appropriately in gray text. Under measures 1–2 the whole fundamental series has been written out once, afterward it appears abbreviated. Also the off beat non-harmonic notes in the repeated bass figure have been interpolated in between the ratios of the nested series in this first system, to illustrate how the MOS analysis accommodates them. After this first showing they are omitted.

There are elements of art (as well as much arithmetic) in making a mutable number analysis: a range of choices to confront, and on occasions, more than one logically viable sequence of computations to choose between. Here a scheme using two series has been chosen as most appropriate, more usually three nested series are required. The deciding principle is that of *least action* or maximum economy of the system taken as a whole.

Vivace MBN $8_{32}0_1$

Key G: I p *leggieramente*

I

Ped. Ped. simile

Conjunction > → Value: 256-G (decimal)

Objective
Notes: ◯
Aggregations: *
Mid.C=256Hz

Mutable
Digit
Sequence
$\left.\begin{array}{c} ------ \\ 8 \\ 32 \\ 1 \end{array}\right\}$ Key G major

Nested Series

G-256 (h8)**>	G-256 (h8)>
F-224 (h7)	F-224 (h7)
Ⓔ-208 (H208)	Ⓓ-192 (h6)
Ⓓ-192 (h6)*	Ⓑ-160 (h5)
Ⓒ-168 (H168)	Ⓖ-128 (h4)
Ⓑ-160 (h5)	Ⓓ- 96 (h3)
Ⓐ-144 (H144)	Ⓖ- 64 (h2)
Ⓖ-128 (h4)**	G- 32 (h1/H32)R
Ⓓ- 96 (h3)	---------
Ⓖ- 64 (h2)*	G- 1 (H1)
G- 32 (h1/H32)R	

Nested Series: (1 x eight) --->

Fundamental Nesting Series

```
 - 31 (H31)
F# 30 (H30)*
 - 29 (H29)
F- 28 (H28)**
 - 27 (H27)
E- 26 (H26)*
D# 25 (H25)
D- 24 (H24)***
 - 23 (H23)
C# 22 (H22)*
C- 21 (H21)
B- 20 (H20)**
A# 19 (H19)
A- 18 (H18)*
G# 17 (H17)
G- 16 (H16)****
F# 15 (H15)
F- 14 (H14)*
E- 13 (H13)
D- 12 (H12)**
C# 11 (H11)
B- 10 (H10)*
A-  9 (H9)
G-  8 (H8)***
F-  7 (H7)
D-  6 (H6)*
B-  5 (H5)
G-  4 (H4)**
D-  3 (H3)
G-  2 (H2)*
G-  1 (H1/1.524HZ)
```

---

meter
(Metrical MOS)

MBN: 4_40_1

```
h4...:sixteenthnote figuration
h3
h2
h1/H4:quarternote pulse
    H3
    H2
    H1:meter 4/4 time
```

Measures 1 through 33

The MOS analysis should clothe the objective notes and their harmonic progression in mutable numbers of the lowest value consistent with the conjunctions required by the modulation algorithm. To this end a nod in the direction of a third harmonic series is included in the analysis – by adding asterisks to the ratios of the firats full series (measure 1) and occasionally elsewhere – to indicate how the series might be broken down into simpler components. That is to say other layers of nesting. Additionally, the 'R' – root of chord marker – is placed at the bottom of the explicit outer series of the system, next to the nested fundamental (h1/H32).

G.3 - PRELUDE No.3, F. CHOPIN

MBN $10_{32}0_1$ MBN $12_{32}0_1$

256-G   320-B   384-D
------   ------   ------
  8   10   12
 32   32   32
  1   1   1

```
                                              D-384(h12)*>
                                              C#352(h11)
                        B-320(h10)*>          B-320(h10)*
                        A-288 (h9)            A-288 (h9)
→ G-256 (h8)*>          G-256 (h8)*           G-256 (h8)*
  F-224 (h7)            F-224 (h7)            F-224 (h7)
  D-192 (h6)*           D-192 (h6)*           D-192 (h6)*
  B-160 (h5)            B-160 (h5)            B-160 (h5)
  G-128 (h4)*           G-128 (h4)*           G-128 (h4)*
  D- 96 (h3)            D- 96 (h3)            D- 96 (h3)
  G- 64 (h2)*           G- 64 (h2)*           G- 64 (h2)*
  G- 32 (h1/H32)R       G- 32 (h1/H32)R       G- 32 (h1/H32)R
  ---------             ---------             ---------
  G-  1 (H1)            G-  1 (H1)            G-  1 (H1)
```

-----> (1 x eight) --------- +2 --------> (1 x ten) --------------- +2 ----------------->(1 x twelve) --->

At the beginning of measure 3 the aggregations (marked by asterisks) in the nested series having reached their most intense form (4 groups of 2), short of coalescing with the nested series which hosts them (i.e. 8 groups of 1); continue the build-up of complexity typical of the opening phrase(s) of a tonal composition, by growing upward through (5 groups of 2) to (6 groups of 2) in measure 4.

On the previous page, at the beginning of the composition, an account of the metrical aspects of the piece are given in the form of a separate modulating oscillatory system. Although this one unchanging nested scheme comprising of meter, pulse and sixteenthnote figuration remains in force throughout, the natural and expressive changes in tempo employed in performance may be factored in to the system by a judicious varying of the unit value of its fundamental frequency (i.e. H1).

MBN $20_{32}0_1$

MBN $18_{32}0_1$

640-B

20
32
1

640-B

10
2
32
1

576-A

18
32
1

```
Ⓑ-640(h20)>                        B-640(h20)*>           B-640(h20)
A#608(h19)                         A#608(h19)             A#608(h19)
A-576(h18)                         A-576(h18)*            A-576(h18)>
G#544(h17)                         G#544(h17)             G#544(h17)
G-512(h16)                         Ⓖ-512(h16)*            Ⓖ-512(h16)
F#480(h15)                         F#480(h15)             F#480(h15)
F-448(h14)                         F-448(h14)*            F-448(h14)
E-416(h13)                         E-416(h13)             E-416(h13)
Ⓓ-384(h12)                         D-384(h12)*            D-384(h12)
C#352(h11)                         C#352(h11)             C#352(h11)
B-320(h10)                         Ⓑ-320(h10)*            Ⓑ-320(h10)
A-288 (h9)                         A-288 (h9)             A-288 (h9)
G-256 (h8)                         G-256 (h8)*            G-256 (h8)
F-224 (h7)                         F-224 (h7)             F-224 (h7)
Ⓓ-192 (h6)                         Ⓓ-192 (h6)*            Ⓓ-192 (h6)
Ⓑ-160 (h5)                         Ⓑ-160 (h5)             Ⓑ-160 (h5)
Ⓖ-128 (h4)                         Ⓖ-128 (h4)*            Ⓖ-128 (h4)
Ⓓ- 96 (h3)                         Ⓓ- 96 (h3)             Ⓓ- 96 (h3)
Ⓖ- 64 (h2)                         Ⓖ- 64 (h2)*            Ⓖ- 64 (h2)
G- 32 (h1/H32)R                    G- 32 (h1/H32)R        G- 32 (h1/H32)R
---------                          ---------              ---------
G-  1 (H1)                         G-  1 (H1)             G-  1 (H1)
```

+8 ---> (1 x twenty) ---> (1 x twenty) -------- -2 --------> (2 x nine) --------8:9-------->

[secondary sesquioctava modulation]

The stress increases by a large step to a high point of (10 groups of 2). From this summit of energy and complexity in the nested series at measure 5, through a small downward adjustment to (9 groups of 2), the nested series gains access to the dominant via a sesquioctava 8:9 modulation to the supertonic chord. It might be helpful to bear in mind that the outer nested series is being driven by the changes in the objective notes (shown circled); so that although the reduction to (9 groups of 2) and the commencement of the 8:9 exchange are shown on this page, they would strictly occur at the barline (measure 7), as the new configuration of notes forces an adjustment upon the MOS (Modulating Oscillatory System).

G.5 - PRELUDE No.3, F. CHOPIN

MBN $16_{36}0_1$ MBN $24_{24}0_1$

II   V

576-A   576-A
------   ------
16   24
36   24
1   1

```
  ──→  A-576(h16)> ------------------------------> Ⓐ-576(h24)>
       G#540(h15)                                  -552(h23)
                                                 G#528(h22)
       G-504(h14) ------------------------------> G-504(h21)
       Ⓕ#468(h13)                               F#480(h20)
                                                  F-456(h19)
       E-432(h12) ------------------------------> E-432(h18)
       D#396(h11)                                D#408(h17)
       Ⓒ#360(h10)                               Ⓓ-384(h16)
       B-324 (h9)                               C#360(h15)
                                                  C-336(h14)
       Ⓐ-288 (h8) -----------------------------> B-312(h13)
       Ⓖ-252 (h7)                               A-288(h12)
                                                 G#264(h11)
       Ⓔ-216 (h6) -----------------------------> Ⓕ#240(h10)
       Ⓒ#180 (h5)                               E-216 (h9)
                                                  D-192 (h8)
       Ⓐ-144 (h4) -----------------------------> C-168 (h7)
       Ⓔ-108 (h3)                               Ⓐ-144 (h6)
                                                 Ⓕ#120 (h5)
       Ⓐ- 72 (h2) -----------------------------> Ⓓ- 96 (h4)
       A- 36 (h1/H36)R                           Ⓐ- 72 (h3)
                                                 Ⓓ- 48 (h2)
       ---------                                  D- 24 (h1/H24)R
       G-  1 (H1)                                ---------
                                                  G-  1 (H1)
```

(8 x two) ----------------------3:2----------------------> (8 x three) ----------------------2:3----------------------->

--------> (2 x eight)   [eightfold sesquialtera modulation]   [eightfold sesquialtera modulation]

The small release of energy and complexity in moving to the supertonic is followed by a large increase by means of a downward or reverse sesquialtera 3:2 modulation exchange to the dominant chord. Here arrows illustrate the precise exchange of the nested series, where eight groups of two ratios are exchanged for eight groups of three ratios. It is an eightfold or octonary sesquialtera 3:2 exchange.

The convention adopted for writing modulation exchanges is that the ratio of exchange (i.e. 8:9 or 3:2) mirrors the change in the frequency of the nested fundamental. Thus in the sesquioctava modulation the nested fundamental changes from G-32 to A-36 (i.e. 32 is to 36 as 8 is to 9) and in the sesquialtera modulation A-36 changes to D-24 (i.e. 36 is to 24 as 3 is to 2).

MBN $16_{36}0_1$

MBN $24_{24}0_1$

II

V^7

576-A

16
36
1

576-A

24
24
1

```
——→  A-576(h16)>              (A)-576(h24)>
                               -552(h23)
      G#540(h15)              G#528(h22)
      G-504(h14)              G-504(h21)
     (F)#468(h13)             F#480(h20)
                              F-456(h19)
      E-432(h12)              E-432(h18)
      D#396(h11)              D#408(h17)
     (C)#360(h10)            (D)-384(h16)
                              C#360(h15)
      B-324 (h9)              C-336(h14)
                              B-312(h13)
     (A)-288 (h8)             A-288(h12)
     (G)-252 (h7)             G#264(h11)
     (E)-216 (h6)            (F)#240(h10)
                              E-216 (h9)
     (C)#180 (h5)             D-192 (h8)
                              C-168 (h7)
     (A)-144 (h4)            (A)-144 (h6)
     (E)-108 (h3)            (F)#120 (h5)
                            (D)- 96 (h4)
     (A)- 72 (h2)           (A)- 72 (h3)
                            (D)- 48 (h2)
      A- 36 (h1/H36)R        D- 24 (h1/H24)R

      ---------              ---------
      G-  1 (H1)             G-  1 (H1)
---------> (8 x two) ------------------------3:2-------------------------> (8 x three) -------------------------------------------------------->
                          [eightfold sesquialtera modulation]
```
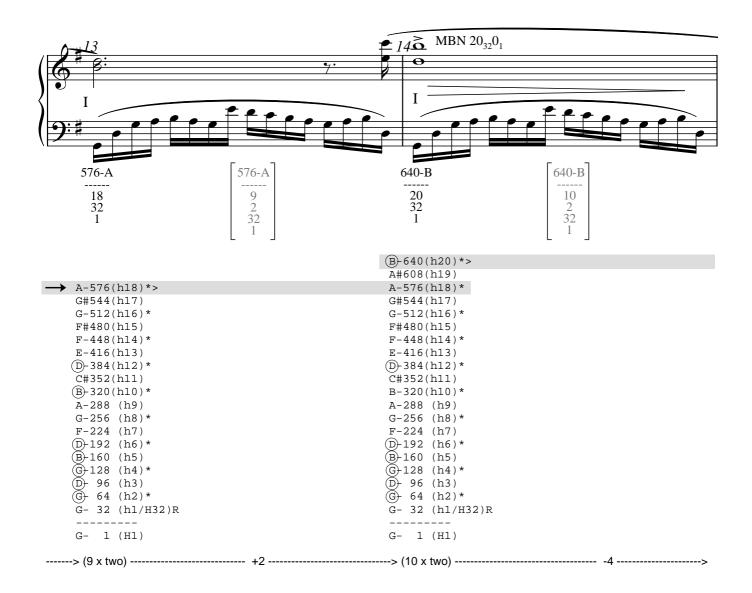
Measures 9 and 10 are almost an exact repeat of the previous page, with the extra intensification of a seventh added to the dominant chord.
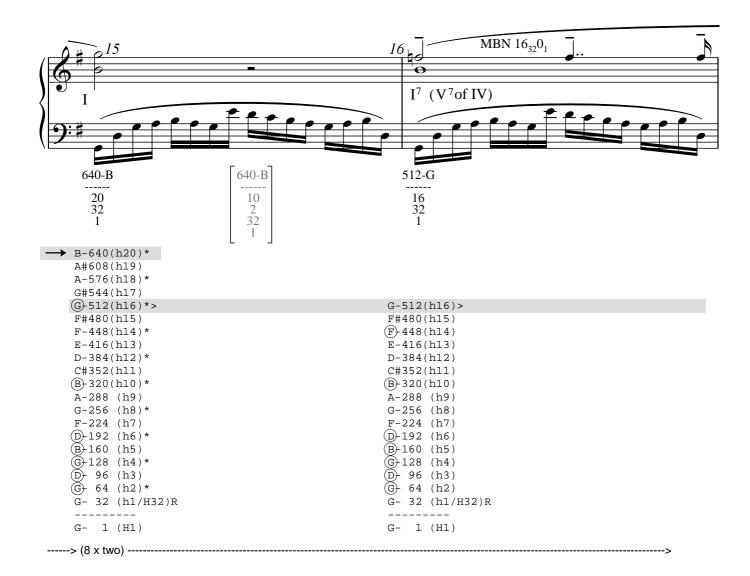
MBN $24_{24}0_1$ MBN $18_{32}0_1$

V^7 V^9_7 I

576-A

24
24
1

576-A

18
32
1

```
→     A-576(h24)> ------------------------------> A-576(h18)>        A-576(h18)>
  ⎧    -552(h23)  ------------------------------> G#544(h17)         G#544(h17)
  ⎨   G#528(h22)  ‹...                            G-512(h16)         G-512(h16)
  ⎩    G-504(h21) ------------------------------> G-512(h16)         G-512(h16)
  ⎧   F#480(h20)  ------------------------------> F#480(h15)         F#480(h15)
  ⎪    F-456(h19) ------------------------------> F-448(h14)         F-448(h14)
  ⎨   Ⓔ-432(h18)  ‹...                            E-416(h13)         E-416(h13)
  ⎩   D#408(h17)  ------------------------------> E-416(h13)         E-416(h13)
  ⎧    D-384(h16) ------------------------------> D-384(h12)         D-384(h12)
  ⎪   C#360(h15)  ------------------------------> C#352(h11)         C#352(h11)
  ⎨   Ⓒ-336(h14)  ‹...                            B-320(h10)        Ⓑ-320(h10)
  ⎩    B-312(h13) ------------------------------> B-320(h10)        Ⓑ-320(h10)
  ⎧    A-288(h12) ------------------------------> A-288 (h9)         A-288 (h9)
  ⎪   G#264(h11)  ------------------------------> G-256 (h8)        Ⓖ-256 (h8)
  ⎨   Ⓕ#240(h10)  ‹...                           Ⓖ-128 (h4)                     
  ⎩    E-216 (h9) ------------------------------> F-224 (h7)         F-224 (h7)
  ⎧    D-192 (h8) ------------------------------>Ⓓ-192 (h6)        Ⓓ-192 (h6)
  ⎪    C-168 (h7) ------------------------------>Ⓑ-160 (h5)        Ⓑ-160 (h5)
  ⎨   Ⓐ-144 (h6)  ‹...                           Ⓖ-128 (h4)        Ⓖ-128 (h4)
  ⎩   Ⓕ#120 (h5)  ------------------------------>Ⓖ-128 (h4)        Ⓖ-128 (h4)
  ⎧   Ⓓ- 96 (h4)  ------------------------------>Ⓓ- 96 (h3)        Ⓓ- 96 (h3)
  ⎨   Ⓐ- 72 (h3)  ------------------------------>Ⓖ- 64 (h2)        Ⓖ- 64 (h2)
  ⎪   Ⓓ- 48 (h2)  ------------------------------>Ⓖ- 64 (h2)        Ⓖ- 64 (h2)
  ⎩    D- 24 (h1/H24)R ------------------------> G- 32 (h1/H32)R    G- 32 (h1/H32)R
       ---------                                 ---------          ---------
       G-  1 (H1)                                G-  1 (H1)         G-  1 (H1)

  -----> (6 x four) ----------------------3:4----------------------> (6 x three) ----------------------------------------------------->
                        [sixfold sesquitertia modulation]
```

tertiary   secondary   primary

Further intensification occurs in measure 11 with the addition of a ninth to the dominant's seventh, before the return to the tonic, by means of a sesquitertia 3:4 modulation exchange. Here the notional adjustment of frequency ratios in the sixfold sesquitertia 3:4 modulation exchange is illustrated with arrows. Notionally, at the lowest primary level the fundamental of the old nested series – D-24 (h1) – is left behind (in the fundamental series) as the system moves to the new nested fundamental – G-32 (h1). Thus all the ratios in the primary group find a home. In contrast, for the secondary, tertiary and higher levels, one ratio is notionally ejected from each higher group in the system as their neighbouring ratios move closer together. For example, in the secondary group, the tritone F# - C behaves as the voice leading would suggest by resolving to the major-third G - B. By 'squeezing' five ratios out of the system (plus another one left behind) a clear reduction in energy and complexity is achieved with the return to the tonic chord.

```
576-A            [ 576-A ]              640-B            [ 640-B ]
------           [ ------ ]             ------           [ ------ ]
18               [  9    ]              20               [  10   ]
32               [  2    ]              32               [  2    ]
1                [  32   ]              1                [  32   ]
                 [  1    ]                               [  1    ]
```

```
                                        B  640(h20)*>
                                        A#608(h19)
  →   A-576(h18)*>                       A-576(h18)*
      G#544(h17)                         G#544(h17)
      G-512(h16)*                        G-512(h16)*
      F#480(h15)                         F#480(h15)
      F-448(h14)*                        F-448(h14)*
      E-416(h13)                         E-416(h13)
      D-384(h12)*                        D-384(h12)*
      C#352(h11)                         C#352(h11)
      B-320(h10)*                        B-320(h10)*
      A-288 (h9)                         A-288 (h9)
      G-256 (h8)*                        G-256 (h8)*
      F-224 (h7)                         F-224 (h7)
      D-192 (h6)*                        D-192 (h6)*
      B-160 (h5)                         B-160 (h5)
      G-128 (h4)*                        G-128 (h4)*
      D- 96 (h3)                         D- 96 (h3)
      G- 64 (h2)*                        G- 64 (h2)*
      G- 32 (h1/H32)R                    G- 32 (h1/H32)R
      ---------                          ---------
      G-  1 (H1)                         G-  1 (H1)

-------> (9 x two) ----------------------------- +2 ------------------------------> (10 x two) ----------------------------------- -4 --------------------->
```

Though the system has managed to reduce its tension somewhat with the return to the tonic, it remains relatively energetic and complex and in measure 14 even increases the level of stress a little by adding another two elements to the nested series, before sinking back to (8 groups of 2) in measure 15. Asterisks have been appended to the nested series to indicate how it might simplify its structure by breaking down into aggregates or groups of two, which could be viewed as forming an additional level of nesting.

```
         640-B                    ┌ 640-B ┐          512-G
         ------                   │ ------ │          ------
          20                      │  10    │           16
          32                      │   2    │           32
           1                      │  32    │            1
                                  └   1   ┘
```

```
──→  B-640(h20)*
     A#608(h19)
     A-576(h18)*
     G#544(h17)
    Ⓖ-512(h16)*>                         G-512(h16)>
     F#480(h15)                          F#480(h15)
     F-448(h14)*                        Ⓕ-448(h14)
     E-416(h13)                          E-416(h13)
     D-384(h12)*                         D-384(h12)
     C#352(h11)                          C#352(h11)
    Ⓑ-320(h10)*                         Ⓑ-320(h10)
     A-288 (h9)                          A-288 (h9)
     G-256 (h8)*                         G-256 (h8)
     F-224 (h7)                          F-224 (h7)
    Ⓓ-192 (h6)*                         Ⓓ-192 (h6)
    Ⓑ-160 (h5)                          Ⓑ-160 (h5)
    Ⓖ-128 (h4)*                         Ⓖ-128 (h4)
    Ⓓ- 96 (h3)                          Ⓓ- 96 (h3)
    Ⓖ- 64 (h2)*                         Ⓖ- 64 (h2)
     G- 32 (h1/H32)R                     G- 32 (h1/H32)R
     ---------                           ---------
     G-  1 (H1)                          G-  1 (H1)
```

```
------> (8 x two) ---------------------------------------------------------------------------------->
```
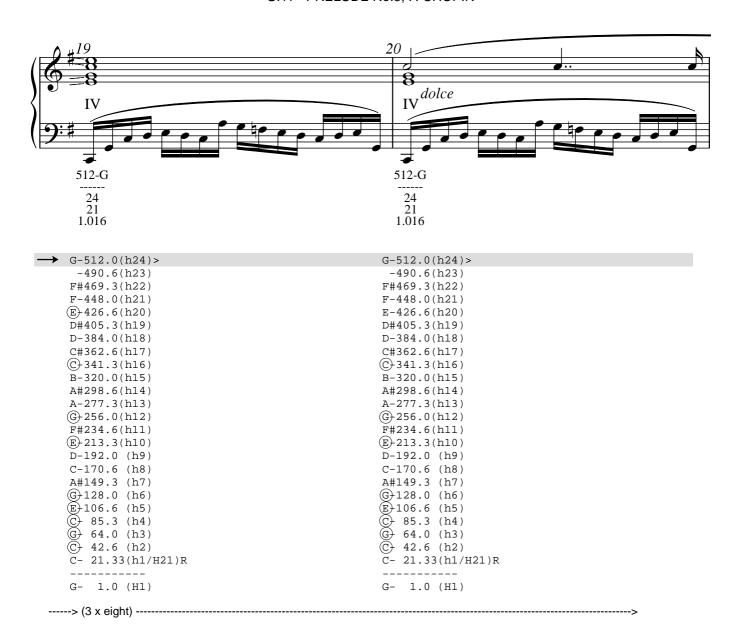
After the relaxation in the G-major harmony through the reduction to (8 groups of 2) the objective notes move to take up the more urgent configuration of I^7, the tonic-seventh, which can function as a dominant-seventh to the coming subdominant chord.
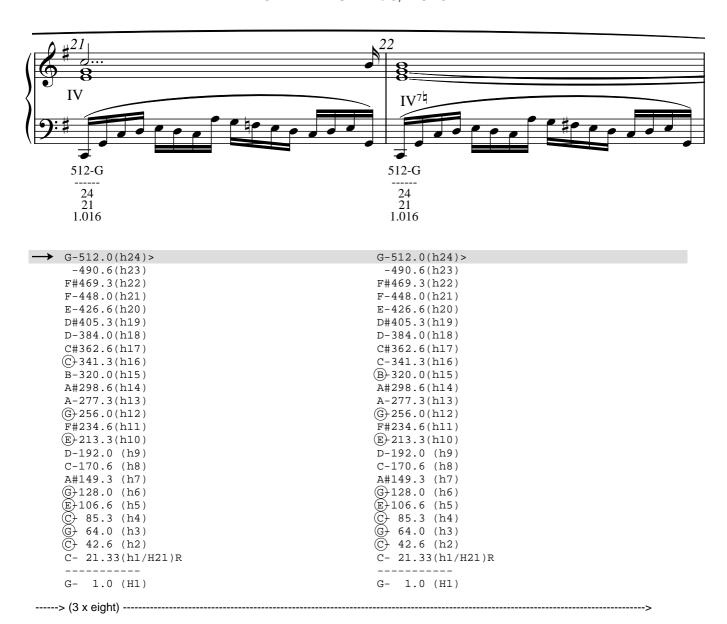
```
    → G-512(h16)>          G-512(h16)>  ---------->  G-512.0(h24)>
                                                     -490.6(h23)
      F#480(h15)           F#480(h15)   ---------->  F#469.3(h22)
     (F)-448(h14)          F-448(h14)   ---------->  F-448.0(h21)
      E-416(h13)          (E)-416(h13)  ---------->  (E)-426.6(h20)
                                                     D#405.3(h19)
      D-384(h12)           D-384(h12)   ---------->  D-384.0(h18)
                                                     C#362.6(h17)
      C#352(h11)           C#352(h11)   ---------->  (C)-341.3(h16)
     (B)-320(h10)         (B)-320(h10)  ---------->  B-320.0(h15)
                                                     A#298.6(h14)
      A-288 (h9)           A-288 (h9)   ---------->  A-277.3(h13)
      G-256 (h8)           G-256 (h8)   ---------->  (G)-256.0(h12)
                                                     F#234.6(h11)
      F-224 (h7)           F-224 (h7)   ---------->  (E)-213.3(h10)
     (D)-192 (h6)         (D)-192 (h6)  ---------->  D-192.0 (h9)
     (B)-160 (h5)         (B)-160 (h5)  --------->   C-170.6 (h8)
                                                     A#149.3 (h7)
     (G)-128 (h4)         (G)-128 (h4)  ---------->  (G)-128.0 (h6)
                                                     (E)-106.6 (h5)
     (D)- 96 (h3)         (D)- 96 (h3)  ---------->  (C)- 85.3 (h4)
     (G)- 64 (h2)         (G)- 64 (h2)  ---------->  (G)- 64.0 (h3)
                                                     (C)- 42.6 (h2)
      G- 32 (h1/H32)R      G- 32 (h1/H32)R ------>   C- 21.33(h1/H21)R
      ---------            ---------                 -----------
      G-  1 (H1)           G-  1 (H1)                G-  1.0 (H1)
```

--> (8 x two) --------------3:2-------------> (8 x three) --->

[eightfold sesquialtera modulation]

The MOS system accommodates the chord changes (I - I^7 - I^6) in measures 16 and 17 all under the roof of a single conjunction G-512, h16 in the nested series. In measure 18 the piece finally moves to the subdominant chord through a downward sesquialtera 3:2 modulation exchange. The harmonic zenith has been reached; here the composition rests for six measures. Again the arrows indicate the notional movements of the notes between chords. In this exchange energy and complexity are being injected into the system, resulting in the creation of an extra element in each of the eight groups involved in the modulation. Voice leading is as one would expect, except that in the higher groups it can occur within the range of a single note letter. Pitch, in modulating oscillatory systems is not fixed but in the ultimate analysis, just and flexible in intonation.

19   *20*

IV   IV *dolce*

512-G   512-G
------   ------
24   24
21   21
1.016   1.016

```
 →  G-512.0(h24)>                    G-512.0(h24)>
     -490.6(h23)                      -490.6(h23)
    F#469.3(h22)                     F#469.3(h22)
    F-448.0(h21)                     F-448.0(h21)
   Ⓔ-426.6(h20)                      E-426.6(h20)
    D#405.3(h19)                     D#405.3(h19)
    D-384.0(h18)                     D-384.0(h18)
    C#362.6(h17)                     C#362.6(h17)
   Ⓒ-341.3(h16)                     Ⓒ-341.3(h16)
    B-320.0(h15)                     B-320.0(h15)
    A#298.6(h14)                     A#298.6(h14)
    A-277.3(h13)                     A-277.3(h13)
   Ⓖ-256.0(h12)                     Ⓖ-256.0(h12)
    F#234.6(h11)                     F#234.6(h11)
   Ⓔ-213.3(h10)                     Ⓔ-213.3(h10)
    D-192.0 (h9)                     D-192.0 (h9)
    C-170.6 (h8)                     C-170.6 (h8)
    A#149.3 (h7)                     A#149.3 (h7)
   Ⓖ-128.0 (h6)                     Ⓖ-128.0 (h6)
   Ⓔ-106.6 (h5)                     Ⓔ-106.6 (h5)
   Ⓒ- 85.3 (h4)                     Ⓒ- 85.3 (h4)
   Ⓖ- 64.0 (h3)                     Ⓖ- 64.0 (h3)
   Ⓒ- 42.6 (h2)                     Ⓒ- 42.6 (h2)
    C- 21.33(h1/H21)R                C- 21.33(h1/H21)R
    -----------                      -----------
    G-  1.0 (H1)                     G-  1.0 (H1)

------> (3 x eight) ----------------------------------------------------------------------------------------------------->
```

The careful reader may have noticed that the slight flexing of relationships, registered through the period of the fundamental unit produced by the exchange at measure 18, is accounted for in the stacked factor format (1.016) but not in the mutable number placed above the staff (i.e. MBN $24_{21}\,0_1$ not MBN $24_{21}\,0_{1.016}$). The reason for this choice is that the use of the mutable number here represents what we understand from the exchange: That is, what is understood in the light of the tolerance with which the ear and aural cognition allows us to grasp the correct 'relational intent' from approximate intervals and periods. The stacked factor format takes the strict view of mathematical calculation, while the mutable number above the staff shows the relational essence of the exchange. Just as on hearing an equally-tempered interval of a fifth we don't recover the ratio 1:1.4983069..., but perceive pitches of ratio 2:3.

```
      512-G                        512-G
      ------                       ------
        24                           24
        21                           21
       1.016                        1.016
```

```
───▶  G-512.0(h24)>              G-512.0(h24)>
       -490.6(h23)                -490.6(h23)
      F#469.3(h22)               F#469.3(h22)
      F-448.0(h21)               F-448.0(h21)
      E-426.6(h20)               E-426.6(h20)
      D#405.3(h19)               D#405.3(h19)
      D-384.0(h18)               D-384.0(h18)
      C#362.6(h17)               C#362.6(h17)
    ⓒ-341.3(h16)                 C-341.3(h16)
      B-320.0(h15)             Ⓑ-320.0(h15)
      A#298.6(h14)               A#298.6(h14)
      A-277.3(h13)               A-277.3(h13)
    Ⓖ-256.0(h12)               Ⓖ-256.0(h12)
      F#234.6(h11)               F#234.6(h11)
    Ⓔ-213.3(h10)               Ⓔ-213.3(h10)
      D-192.0 (h9)               D-192.0 (h9)
      C-170.6 (h8)               C-170.6 (h8)
      A#149.3 (h7)               A#149.3 (h7)
    Ⓖ-128.0 (h6)               Ⓖ-128.0 (h6)
    Ⓔ-106.6 (h5)               Ⓔ-106.6 (h5)
    ⓒ- 85.3 (h4)               ⓒ- 85.3 (h4)
    Ⓖ- 64.0 (h3)               Ⓖ- 64.0 (h3)
    ⓒ- 42.6 (h2)               ⓒ- 42.6 (h2)
      C- 21.33(h1/H21)R          C- 21.33(h1/H21)R
      -----------                -----------
      G-  1.0 (H1)               G-  1.0 (H1)
```

```
------> (3 x eight) ------------------------------------------------------------------------------------------------------------>
```

From measure 18 to 21 the principal objective note on the treble staff is C-h16. Sixteen as a power of two can be broken down in to aggregated groups of two, two-by-two and two-by-two-by-two, which fit hand-in-glove with the nested series (3 x eight) structure. however at measure 22, as the period of the subdominant chord draws to a close, the note C is replaced by B-h15. Fifteen is not divisible by two but is divisible by three – h24's other factor. B-h15 is followed by A-h13 in measure 23; thirteen is an awkward prime number most foreign and irregular to twenty-four. These two objective notes B-h15 and A-h13 perhaps to some extent weaken the hold of any grouping by powers of two in the nested series, preparing the ground for a shift to groups of three in measure 24.

```
              23                              24
```

IV^6 IV$_2^6$

```
512-G          [512-G]        [576-A]        [576-A]
------          ------         ------         ------
24                8              9             12
21                3              3              2
1.016            21             21             24
                 1.016          1.016          1
```

```
                                        A-576.0(h27)*>    A-576(h24)>
                                         -554.6(h26)       -552(h23)
                                        G#533.3(h25)      G#528(h22)
→  G-512.0(h24)*>      G-512.0(h24)*     G-512.0(h24)*     G-504(h21)
    -490.6(h23)         -490.6(h23)       -490.6(h23)     F#480(h20)
   F#469.3(h22)*       F#469.3(h22)      F#469.3(h22)      F-456(h19)
   F-448.0(h21)        F-448.0(h21)*     F-448.0(h21)*     E-432(h18)
   E-426.6(h20)*       E-426.6(h20)      E-426.6(h20)     D#408(h17)
   D#405.3(h19)        D#405.3(h19)      D#405.3(h19)      D-384(h16)
   D-384.0(h18)*       D-384.0(h18)*     D-384.0(h18)*    C#360(h15)
   C#362.6(h17)        C#362.6(h17)      C#362.6(h17)      C-336(h14)
   C-341.3(h16)*       C-341.3(h16)      C-341.3(h16)      B-312(h13)
   B-320.0(h15)        Ⓑ-320.0(h15)*    B-320.0(h15)*     A-288(h12)
   A#298.6(h14)*       A#298.6(h14)      A#298.6(h14)     G#264(h11)
   Ⓐ-277.3(h13)       A-277.3(h13)      A-277.3(h13)      F-240(h10)
   Ⓖ-256.0(h12)*      Ⓖ-256.0(h12)*    Ⓖ-256.0(h12)*     E-216 (h9)
   F#234.6(h11)        F#234.6(h11)      F#234.6(h11)      Ⓓ-192 (h8)
   Ⓔ-213.3(h10)*      E-213.3(h10)      E-213.3(h10)      C-168 (h7)
   D-192.0 (h9)        Ⓓ-192.0 (h9)*    Ⓓ-192.0 (h9)*     Ⓐ-144 (h6)
   C-170.6 (h8)*       C-170.6 (h8)      C-170.6 (h8)      F#120 (h5)
   A#149.3 (h7)        A#149.3 (h7)      A#149.3 (h7)      Ⓓ- 96 (h4)
   Ⓖ-128.0 (h6)*      Ⓖ-128.0 (h6)*    Ⓖ-128.0 (h6)*     Ⓐ- 72 (h3)
   Ⓔ-106.6 (h5)       Ⓔ-106.6 (h5)     Ⓔ-106.6 (h5)      Ⓓ- 48 (h2)
   Ⓒ- 85.3 (h4)*      Ⓒ- 85.3 (h4)     Ⓒ- 85.3 (h4)      D- 24 (h1/H24)R
   Ⓖ- 64.0 (h3)       G- 64.0 (h3)*     G- 64.0 (h3)*     ---------
   Ⓒ- 42.6 (h2)*      C- 42.6 (h2)      C- 42.6 (h2)      G-  1 (H1)
   C- 21.33(h1/H21)R   C- 21.33(h1/H21)R C- 21.33(h1/H21)R
   ----------          ----------        ----------
   G-  1.0 (H1)        G-  1.0 (H1)      G-  1.0 (H1)
```

```
----> (3 x eight) ---------------------------------------------------------- +3 -----> (3 x nine) -----------8:9---- (anticipation) -->
                                                              [tertiary sesquioctava modulation]
```
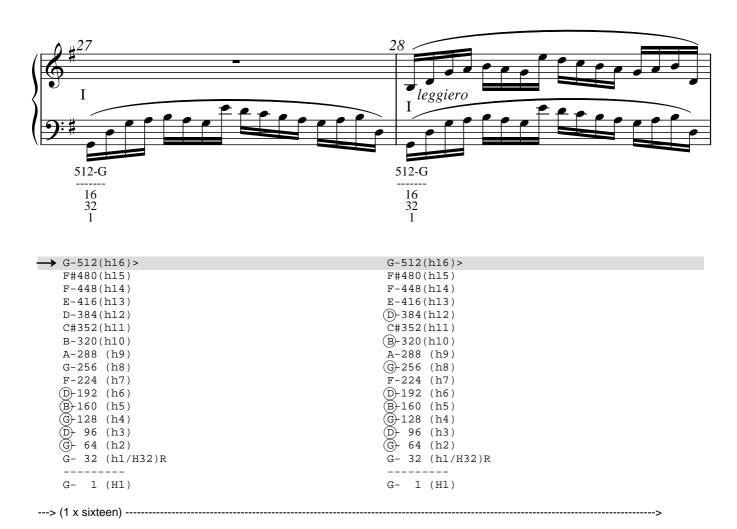
The piece now reaches a most interesting point in its harmonic progress, the return to the tonic from the position of a well established subdominant chord. The strength of the subdominant's hold has already been diluted by the introduction of objective notes which lie beyond the inner circle of any power of two aggregations which might develop in a system of twenty-four elements. The coup de grace is delivered by the cluster of objective notes at the opening of measure 24: the quarternote A and anticipatory bass eighthnotes D and A foreign to the subdominant chord C, but most particularly the semibreves G-h12 and D-h9. These notes knock aggregates in the system from any hint of groups of two in to groups of three. Significantly, groupings of three in the nested series suggest a fundamental/root note of G – heralding the returning tonic note. As with many harmonic progressions in music, time is required for a change to be fully

digested. Confusion reigns throughout measure 24. To be followed by the certainty of a dominant-seventh to tonic exchange in measures 25 and 26.


```
        MBN 24₂₄0₁              MBN 18₃₂0₁                    MBN 16₃₂0₁
25                          26
        V⁷  (II⁷ of IV)                I

        576-A                      576-A                        512-G
        ------                     -------                      -------
         24                          18                           16
         24                          32                           32
          1                           1                            1
```

```
→    A-576(h24)>              A-576(h18)
      -552(h23)               G#544(h17)
     G#528(h22)
     G-504(h21)               G-512(h16)>               G-512(h16)****>
     F#480(h20)               F#480(h15)                F#480(h15)
     F-456(h19)               F-448(h14)                F-448(h14)*
     E-432(h18)               E-416(h13)                E-416(h13)
     D#408(h17)
     D-384(h16)               D-384(h12)                D-384(h12)**
     C#360(h15)               C#352(h11)                C#352(h11)
     C-336(h14)
     B-312(h13)               B-320(h10)                B-320(h10)*
     A-288(h12)               A-288 (h9)                A-288 (h9)
     G#264(h11)               Ⓖ-256 (h8)               Ⓖ-256 (h8)***
    Ⓕ#240(h10)                F-224 (h7)                F-224 (h7)
     E-216 (h9)
    Ⓓ-192 (h8)               Ⓓ-192 (h6)               Ⓓ-192 (h6)*
    Ⓒ-168 (h7)               Ⓑ-160 (h5)               Ⓑ-160 (h5)
    Ⓐ-144 (h6)
    Ⓕ#120 (h5)               Ⓖ-128 (h4)               Ⓖ-128 (h4)**
    Ⓓ- 96 (h4)               Ⓓ- 96 (h3)               Ⓓ- 96 (h3)
    Ⓐ- 72 (h3)               Ⓖ- 64 (h2)               Ⓖ- 64 (h2)*
    Ⓓ- 48 (h2)
     D- 24 (h1/H24)R         G- 32 (h1/H32)R           G- 32 (h1/H32)R
     ---------               ---------                 ---------
     G-  1 (H1)              G-  1 (H1)                G-  1 (H1)

---8:9---> (3 x eight)
    (6 x four) --------------------------------3:4----------------------------------> (6 x three) -------------- -2 -------------> (1 x sixteen) -------->
                          [sixfold sesquitertia modulation]
```

The final home coming to the tonic chord is achieved via the standard full or perfect cadence (V⁷- I), executed by the sesquitertia 3:4 modulation exchange. The system having thus safely arrived at the tonic, is then free to reduce its level of stress by loosing two elements, taking the system down to the stable configuration of (1 x sixteen) bringing the conjunction value into line with the fundamental G-h1 of the nested series, as its fourth power of two.

```
       → G-512(h16)>                    G-512(h16)>
         F#480(h15)                      F#480(h15)
         F-448(h14)                      F-448(h14)
         E-416(h13)                      E-416(h13)
         D-384(h12)                    (D)-384(h12)
         C#352(h11)                      C#352(h11)
         B-320(h10)                    (B)-320(h10)
         A-288 (h9)                      A-288 (h9)
         G-256 (h8)                    (G)-256 (h8)
         F-224 (h7)                      F-224 (h7)
       (D)-192 (h6)                    (D)-192 (h6)
       (B)-160 (h5)                    (B)-160 (h5)
       (G)-128 (h4)                    (G)-128 (h4)
       (D)- 96 (h3)                    (D)- 96 (h3)
       (G)- 64 (h2)                    (G)- 64 (h2)
         G- 32 (h1/H32)R                 G- 32 (h1/H32)R
         ---------                       ---------
         G-  1 (H1)                      G-  1 (H1)
```

---> (1 x sixteen) --->

With all the drama of the journey behind it, the composition glides forward along the conjunction (G-512) toward a finishing flourish at measures 31 and 32 followed by the final chords.
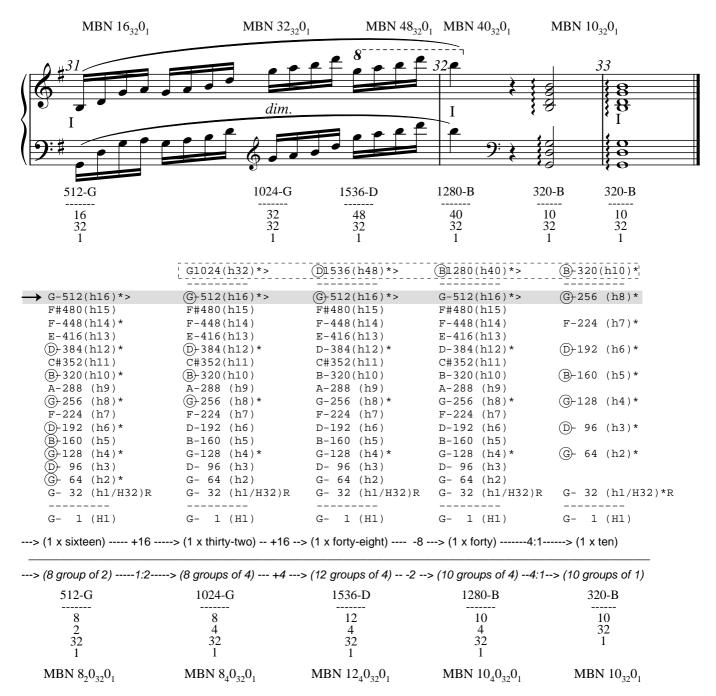
```
      G-512(h16)>              G-512(h16)>
      F#480(h15)               F#480(h15)
      F-448(h14)               F-448(h14)
      E-416(h13)               E-416(h13)
   (D)-384(h12)             (D)-384(h12)
      C#352(h11)               C#352(h11)
   (B)-320(h10)             (B)-320(h10)
      A-288 (h9)               A-288 (h9)
   (G)-256 (h8)             (G)-256 (h8)
      F-224 (h7)               F-224 (h7)
   (D)-192 (h6)             (D)-192 (h6)
   (B)-160 (h5)             (B)-160 (h5)
   (G)-128 (h4)             (G)-128 (h4)
   (D)- 96 (h3)             (D)- 96 (h3)
   (G)- 64 (h2)             (G)- 64 (h2)
      G- 32 (h1/H32)R          G- 32 (h1/H32)R
      ---------                ---------
      G-  1 (H1)               G-  1 (H1)


---> (1 x sixteen) -------------------------------------------------------------------------------------------------->
```

MBN $16_{32}0_1$ MBN $32_{32}0_1$ MBN $48_{32}0_1$ MBN $40_{32}0_1$ MBN $10_{32}0_1$

512-G	1024-G	1536-D	1280-B	320-B	320-B
-------	-------	-------	-------	------	------
16	32	48	40	10	10
32	32	32	32	32	32
1	1	1	1	1	1

```
                    ┌ G1024(h32)*> ┐ ⊙1536(h48)*>    Ⓑ1280(h40)*>   Ⓑ-320(h10)*
                    └--------------  ----------      ------------   ---------
→  G-512(h16)*>      Ⓖ-512(h16)*>    Ⓖ-512(h16)*>    G-512(h16)*>    Ⓖ-256 (h8)*
   F#480(h15)        F#480(h15)       F#480(h15)      F#480(h15)
   F-448(h14)*       F-448(h14)       F-448(h14)      F-448(h14)      F-224 (h7)*
   E-416(h13)        E-416(h13)       E-416(h13)      E-416(h13)
   Ⓓ-384(h12)*       Ⓓ-384(h12)*      D-384(h12)*     D-384(h12)*     Ⓓ-192 (h6)*
   C#352(h11)        C#352(h11)       C#352(h11)      C#352(h11)
   Ⓑ-320(h10)*       Ⓑ-320(h10)       B-320(h10)      B-320(h10)      Ⓑ-160 (h5)*
   A-288 (h9)        A-288 (h9)       A-288 (h9)      A-288 (h9)
   Ⓖ-256 (h8)*       Ⓖ-256 (h8)*      G-256 (h8)*     G-256 (h8)*     Ⓖ-128 (h4)*
   F-224 (h7)        F-224 (h7)       F-224 (h7)      F-224 (h7)
   Ⓓ-192 (h6)*       D-192 (h6)       D-192 (h6)      D-192 (h6)      Ⓓ- 96 (h3)*
   Ⓑ-160 (h5)        B-160 (h5)       B-160 (h5)      B-160 (h5)
   Ⓖ-128 (h4)*       G-128 (h4)*      G-128 (h4)*     G-128 (h4)*     Ⓖ- 64 (h2)*
   Ⓓ- 96 (h3)        D- 96 (h3)       D- 96 (h3)      D- 96 (h3)
   Ⓖ- 64 (h2)*       G- 64 (h2)       G- 64 (h2)      G- 64 (h2)
   G- 32 (h1/H32)R   G- 32 (h1/H32)R  G- 32 (h1/H32)R G- 32 (h1/H32)R G- 32 (h1/H32)*R
   ---------         ---------        ---------       ---------       ---------
   G-  1 (H1)        G-  1 (H1)       G-  1 (H1)      G-  1 (H1)      G-  1 (H1)
```

---> (1 x sixteen) ----- +16 -----> (1 x thirty-two) -- +16 --> (1 x forty-eight) ---- -8 ---> (1 x forty) -------4:1------> (1 x ten)

_____

---> *(8 group of 2)* -----1:2-----> *(8 groups of 4)* --- +4 ---> *(12 groups of 4)* -- -2 --> *(10 groups of 4)* --4:1--> *(10 groups of 1)*

512-G	1024-G	1536-D	1280-B	320-B
-------	-------	-------	-------	------
8	8	12	10	10
2	4	4	4	32
32	32	32	32	1
1	1	1	1	

MBN $8_2 0_{32} 0_1$ MBN $8_4 0_{32} 0_1$ MBN $12_4 0_{32} 0_1$ MBN $10_4 0_{32} 0_1$ MBN $10_{32} 0_1$

Although strictly the conjunction climbs rapidly through the little flourish of measures 31 and 32, the ratios have been omitted and just the conjunctions marked above a dashed line and enclosed in a dashed box. As can be seen the nested series becomes greatly extended and in this situation it would be natural for it break down into aggregated units. Like a stick supporting an ever increasing weight, sooner or later it will break under the stress into two or more parts. A possible arrangement of the nested series in two parts is illustrated at the bottom of the page below the continuous line.

The aggregated grouping is doubled to underline the expansive mood of the flourish by a dupla 1:2 modulation exchange, followed by an addition of (4 groups of 4) and a subtraction of (2 groups of 4), before the cadential relaxation of the final two chords, achieved by quadruple downward modulation exchange, which reunites the aggregates with the nested series out of which they formed at the start of the page.