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Appendix F
Introduction to the MOS Examples and Guide to Analysis with Mutable Numbers

MUTABLE NUMBER ANALYSIS

A full description of the MOS (Modulating Oscillatory Systems) model and the Mutable Base Position-Value Number System can be found in the main text of *Journey to the Heart of Music*. The first part of this document is an introduction to the book's attached examples, in particular Example G. Following this brief introduction there is a guide to the techniques involved in constructing MOS analyses, referral to the main chapters is also recommended.

If one were to draw an analogy between physical theory and music theory, the analysis given by the MOS approach might be likened to a form of quantum mechanics for tonal music: That is, a fundamental approach to tonal music operating at a sufficiently elemental level so as to be capable of explaining its most basic mechanisms and characteristics. The MOS model, to carry the analogy further, does not enlighten us much regarding the broader human and cultural issues or context, about the purpose of the architecture or the intent of the architect, only the underlying engineering is elucidated. The model is myopic, essentially seeing no further than the next chord. Other theories concerned with the wider perspectives of what might be termed the 'classical mechanics' of music are required to access broader vistas and higher level understandings. In particular, the MOS model might well be integrated into theories which focus on the dynamic aspect of tonal compositions. As in physics, where both the quantum and classical worlds coexist (though not always peacefully), the MOS approach does not overturn the relevance of other tonal theories but rather, for the most part, complements and underpins them; with perhaps mutable numbers playing a similar role in regard to tonal music, as that of the group of integers modulo twelve in atonal music.

At bottom the MOS model of tonal music is a means of construing the physical reality of musical sound as a representation of number. That is, by extrapolation from the frequency relationships of the objective written notes – up to common overtones and downward to a unit fundamental – a positional number system can be constructed: Mutable Base Numbers. When applying a mutable number analysis to a composition, while it is almost always possible to derive some combination of nested harmonic series able link together even the most difficult, not to say desperate, chord progressions, it may not be wise or sound analysis so to do in every case. Given the complex resources of the human mind; the ability to skip passed unexpected detail, fill in assumed 'missing' sensory information or leap across gaps of logic to form broad understandings, the MOS model should not endeavour to provide an absolutely compelling analysis for every chord sequence ever written. Rather the model should, hopefully, capture the essence of the structure generally, while also being compelling in its analysis of the detail of ‘regular’ tonal chord sequences, particularly those of core importance to the tonal structure of the composition.

Equally, the model should not be constrained by the extent of aural cognition, whatever these (as yet unknown) limits turn out to be. It would be remarkable, I think, if the ear and mind were truly able to acquire every detail of the extended family of nested harmonic series proposed in the MOS model. More likely our ‘ears’ grasp the explicit upper level *aggregated* series defined by the principal relationships of the objective musical sound and from of this network of tones (notes plus partials) aural cognition computes the essence of the mutable number digit exchanges described by the model without explicitly reaching down to the foundations of the system. Though of course, it might well turn out that the MOS approach proves to have no connection at all with the processes of aural cognition, in which case the theory simply falls back to being yet another mathematical model of tonal music, and hopefully, still a useful analytical tool.

INTRODUCTION TO THE MOS EXAMPLES

In the Example G analysis a parallel range of information is delivered in a variety of different formats, going from top to bottom of the page; 1) Mutable Base Numbers in subscript format, 2) traditional Roman numeral harmonic analysis, 3) Mutable Base Numbers in ‘stacked’ factor format, 4) explicit harmonic series and 5) additions, subtractions and ratio exchanges of the modulation algorithm of symmetrical exchange.

Above the system the current *mutable base number* may be shown in subscript format, for example,

MBN $9_2\ 0_{32}\ 0_1$

which in decimal is 576 or in a generalised positional notation, Dec $5_{10}\ 7_{10}\ 6_1$. The mutable numbers appear once, continuing in force until replaced by another mutable digit sequence – which might represent the same or a different conjunction value – as unlike fixed base number systems (i.e. Decimal, Binary, etc.) mutable base numbers will, most often, have a *range of different digit sequences* for a given value. It is this richness and variety of number representation which enables mutable base numbers to capture tonal chord progressions within the digit sequences of one value – thus rendering the progression logical and commensurable,... or incommensurable where no corresponding digit sequence exists! However, it should not be thought that mutable numbers command tonal music, rather the other way about, the chords of tonal music are (parts of) mutable numbers written in sound and the ‘rules’ or characteristics of tonal harmony are thus also those of the mutable number system. Tonal harmonic progressions which make sense aurally, will have corresponding mutable number digit sequences which ‘add up’, or in math-speak ‘mathematically exist’. For convenience and ease of reading, both the column digits and column base subscripts of mutable numbers are written in the form of plain decimal numbers. (It would be possible to use letters for extra symbols, as in the hexadecimal system and indeed any other symbols but as there is no limit to the range of either column digits or column bases in the mutable system this strategy would make the written notation extremely cumbersome.)

Between the staves a traditional Roman numeral analysis of the harmony is given, for example,

Key G: I ii V^7 I.

Below the system the current mutable base number is shown in ‘stacked’ *factor format*, with the equivalent decimal value, separated by a line of dashes, placed above it. This decimal number is the value of the conjunction which links adjacent chords together and by means of this commonality, renders the harmonic progression commensurable. The conjunction values used in factor format (and subscript format) are usually fixed at the frequency of the 'key' series, this is done so that the mutable numbers remain anchored around the unit H1, which enhances the presentational logic of being in a key/tonal center. (It is equally possible to track the precise course taken by the MOS model from start to finish, by using freely varying real values, which is what the full MOS analysis, below, does.) The stacked factor format numbers are updated with each new chord, whether they change digit sequence or not. Also, as in the example below, the note letter equivalent of the conjunction's frequency ratio is often appended to the decimal value.

576-A

9
2
32
1

Sometimes the conjunction value will run across a number of different chords linking them all by a common value; but as often as not, a new conjunction value, either higher or lower, will be required to connect to the next chord in a harmonic progression. A new conjunction is found by counting upward or downward in the ratios of the uppermost harmonic series. When this occurs the gray band running across the page is broken and a new band commenced at the appropriate level. The harmonic motion of a composition will at times lead to a 'flexing' of strict whole number relationships within a multi-column mutable number. This occurs where a modulation exchange produces a non-integer result (within the fixed value grid applied to the tonal center) and it is accommodated within the system by means of a fractional unit. That is, the unitary period of the absolute fundamental frequency (H1) is allowed a little leeway or tolerance to expand or contract as required to maintain a common conjunction value between chords, for example:

576-A

9
3
21
1.016

Although this might be viewed as something of a fudge or blemish upon the purity of a system essentially rooted in whole numbers, it mirrors the tolerance of small deviations of pitch exhibited by the human ear. Indeed, the equal-tempered scale upon which tonal music depends for access to all keys on the piano and many other instruments, would be impossible without such a tolerance. And, in the next format below this feature disappears into a continuum of smoothly variable values.

The conjunction band mentioned above actually runs through the uppermost ratios of a *nested system of harmonic series*, which are written out below the stacked mutable numbers. These series could be taken to represent the music as a 'pseudo-physical' system of relationships evolving by means of addition, subtraction and modulation exchanges (harmonic progression) in a material context. In the 'classical' material world, which is of course the environment in which music actually exists, relationships are generally taken to be smoothly variable. And so the 'flexing' of relationships here presents no obstacle, for example F-h21 is twenty-one whole steps of the harmonic series above h1 and if those steps happen to take the value of 1.016 arbitrarily defined units of frequency (rounded up from 1.015873...), then the twenty-first step will consist of F-21.333... units. In this format of extended harmonic series, the MOS model (Modulating Oscillatory System) takes the changes in the musical relationships, the modulation exchanges, as they come and computes the results in real numbers, that is, decimals with unlimited fractional expansions. Though here in these analyses, for convenience, only one decimal place is shown. (The reader may calculate to whatever accuracy they require.) Beside these real numbers the ‘logical’ number or relationship to the unit is given in brackets: h1, h2, h3, etc. these are, of course, the ratios of the harmonic series – and the digits of mutable numbers. As for the most part three nested harmonic series are present: a fundamental/nesting series (H1, H2, H3, etc.), a nested middle level series (h1, h2, h3, etc.) and a top level aggregated series marked with asterisks (h3*, h6*, h9*, etc.) – they are distinguished by the symbols given here in brackets. These three nested harmonic series correspond to the columns in mutable base numbers. However, there may be any number of columns in a mutable number or levels of nesting in a MOS analysis. Three columns is usual and convenient, two not uncommon, but if the relatively large fundamental series (e.g. typically H1 through H32, or H1 through H48) were to be broken down into their factors, then double or more columns/nestings would emerge. Such additional detail in the ‘murky depths’ are implied but not shown in most MOS analyses, beyond the connection to the middle level nested series, for example: C-h1/H32.

Finally, below the written out harmonic series is the ‘algebra’ of additions, subtractions and modulations of the MOS model’s algorithm of symmetrical exchange. Here the operations (addition, subtraction or modulation exchange) are shown with arrows linking two bracketed descriptions of a nested pair of harmonic series. In black oblique script the movement of the upper aggregated series within the middle level nested series is shown. And in a gray upright script the changes of the middle level nested series paired with the fundamental nesting series is given. Both pairs, aggregated/nested series and nested/fundamental series share the same conjunction. Indeed, there is no theoretical limit to the number of levels of nesting that could be employed, provided there are sufficient elements in the system to support the structure – yet still there would be only one, shared, conjunction. A MOS system is driven from the top by the motion of the objective notes. As the notes change from beat to beat and measure to measure, the underlying structure of nested harmonic series must respond to accommodate the harmonic motion. Sometimes only change in the upper level is required, at other times the low level nested/ fundamental series must also change along with the aggregated series to accommodate the harmonic motion of the objective notes; and where a piece changes key, the whole system of nested series must change to new ground – a new absolute fundamental tone. Finally, though in these examples the model is employed for harmonic analysis, the MOS approach is equally amenable to metrical analysis.

GUIDE TO CONSTRUCTING A MOS ANALYSIS

Heinrich Schenker famously stated that his analytical approach to music was fundamentally an art, rather than a science. Similarly, in some degree, creating modulating oscillatory system analyses also requires the intuitions of an artist – a musician’s insight, knowledge and experience. Whether or not this human element will always be required is difficult to know. An automatic – probably computer based – procedure does hold out the prospect of greater objectivity and for this reason it is to be hoped that one day it might prove to be possible. Such a procedure would no doubt require access to a considerable amount of stored data to provide the basis upon which decisions could be made – essentially supplying the component of human knowledge and experience – for this reason the Perl scripts described below are linked to database files as a first step towards building up this required body of information.

Yet still the construction of an automatic procedure remains problematic, it is by no means certain that an entirely satisfactory MOS analysis can be made without reference to how the analyst hears the music and interprets the harmonic progressions. Certainly, at points within a MOS analysis where choices and judgements have to be made, different analysts may come to a variety of conclusions, arguing for one or another route through a number of different, but equally viable, mutable number exchanges. Though, sooner or later the two (or more) variant paths will almost certainly rejoin and the analysis proceed again upon a single path. And perhaps an automatic procedure could at least find all viable paths, leaving it to the analyst to arbitrate amongst them.

Currently however, the situation is that a computer aided human approach to constructing a MOS analysis is the achievable option and maybe in the final analysis, as suggested by Schenker's aside, this will always remain the case. Overall the procedure is as follows: The strategy begins with a score, from which the music, expressed in the form of MIDI (Musical Instrument Digital Interface) data, is extracted. To this music data detailing the written notes of the composition are added the notes' constituent partials – i.e. the broad collection of harmonics swept up by the ear and transformed by aural cognition into the perception of 'note-pitch'. Then the sum of all 'notes-and-partials' is cut up into slices, with superfluous slices being discarded so as to leave only the salient harmonic progressions. Each essential chord, with its array of partials, is next tested against a simple harmonic series so as to discover the first/highest position at which the constituents of a single harmonic series will match with all the notes of the chord (and consequently all of the note-generated harmonics too). This series will become the crucial middle level *nested series* in the MOS analysis, and, as observed by James Beament in *How We Hear Music* (2005), its period is crucial to the recognition of chords: Appendix 7, "Each kind of chordal sensation which can be named: major, minor, dominant seventh and so on, has a different repetition pattern related to the repetition rate of its components... [notes]. The patterns are the only things which provide the similarities and differences [between chord types]." In addition to the structural middle level nested series, a matching *conjunction series* is derived from a selection of the nested series' harmonics which links the chord to its predecessor. That is, the frequencies of the conjunction series are common to the nested series of both the current and previous chord. One of harmonics in this conjunction series, lying at or above the highest note in adjacent chords (and normally corresponding to an audible harmonic generated by both chords), is chosen as *the conjunction*. This somewhat arbitrary choice crystallizes the relationship in a chord progression and allows a specific mutable number value to be assigned to the exchange. All of this work is done automatically by the *txt2mos* script and the results output in the form of an editable text file. Essentially, by discovering the middle level nested series (and the conjunction that allows it to be generated by a simple ratio from the previous chord's nested series), both the upper level aggregated series and the underlying fundamental series can be inferred. The editable text file containing all this information is arranged in a particularly visual way, so that when loaded into a text editor the chord progressions, each with their enfolding nested series and conjunction are displayed across the screen, one after the other. Once the automatic output of the *txt2mos* script has been checked, adjusted and fine tuned by the analyst, it is then converted by the *mos2col* script into column data, ready for entry into a MOS analysis. The music used in this example analysis is a short portion of the chorale *Ach Gott und Herr*, set by J.S. Bach.

The crucial component to find in this method, is the sequence of middle level nested harmonic series underlying the harmony – a concept very much akin to Rameau's 'basse fondamentale'. However, whereas the roots of major chords are reflected much as one would expect in this sequence, that is, as the fundamental tones of harmonic series whose higher harmonics largely encompass the objective chords as *configurations of partials*; and, that these higher harmonics will mostly possess a common factor of 2, or some higher power of 2, (e.g. the chord: C-h4, G-h6, E-h10 and C-h16). In contrast to this, for minor chords, the tones in this sequence of underlying frequencies are less intuitive, as the common factor (predominantly) bundling-up the underlying harmonic series will be 5, (e.g. chord: E-h10, B-h15, E-h20, G-h24); which leads to the fundamental tone in the underlying nested series being different from the minor chord's perceived root. In the example E-minor chord above, the fundamental tone of the nested series is 'C', C-h1 – the period of its combined component notes.

Once arrived at, this sequence of nested series may then be linked together by a chain of ratios of exchange which transform each series into the one that follows it; and, the succession of fundamental tones of these series are, effectively, themselves harmonics of a posited absolute fundamental tone 'H1' – which is taken to encapsulate the unifying sense of the tonal center or key. This chain of transformations in the middle level nested series is the backbone of a modulating oscillatory system, it is a framework articulated by the 'modulations' of the algorithm of symmetrical exchange – the *modulation algorithm*.

Some useful steps toward making a MOS analysis are:

1) Make a roman numeral analysis of the harmony,
2) Produce a MIDI file of the score,
3) Turn the MIDI data into a textual description using the *mid2txt* script,
4) Convert the text data into a visual outline of a MOS analysis using the *txt2mos* script,
5) Edit the MOS outline in the light of the roman numeral analysis,
6) Run the *mos2col* script to turn the amended MOS analysis into columns of nested harmonic series ready for entry into a printed score analysis.

After making the initial roman numeral analysis by hand, each of the remaining steps may be speeded up with the aid of a computer. In the procedure described here the MOS analysis is gradually built up through a series of steps involving the editing of the music data by the analyst and the transformation of the edited music data by four computer programs – Perl scripts: *mid2txt*, *txt2mos*, *newH1* and *mos2col*, plus the associated database files *mosmbn.dir* and *mosmbn.pag* – which should located in the same folder/directory as the scripts. The scripts, are on the attached compact disc (CHPT19 folder/directory – Appendix D) and available over the internet from *http://www.pjperry.freeuk.com/* by following the links. Some familiarity with using the command terminal of a computer is assumed, as well as access to a Perl interpreter with Sean Burke's MIDI modules installed (CHPT19 folder/directory – Appendix D or *http://www.cpan.org/modules/by-module/MIDI/*). And, as with any relatively new open source software project the scripts are supplied 'as is' without warranty or guarantee.

Step 1

Make a roman numeral analysis of the harmony. Assuming the score is accessible through some type of music editing program, it is particularly beneficial at this early stage in the procedure to make adjustments in the score so as to eliminate troublesome non-harmonic features and perhaps also strengthen weakly present harmonic elements. Guided by the roman numeral analysis, any necessary adjustments to the score to produce a reasonably clear harmonic outline can be made before exporting the MIDI file from the music editor program. While of course being careful to maintain harmonic integrity, the computer programs will give their most consistent and accurate performance when digesting chorale-like sequences of unambiguous block chords. A filigree texture of darting arpeggios over a deep pedal figure may yield problematic results leading to much time consuming editing of the computer output. Generally setting the slice width (-w switch) to the shortest harmonic period of interest and then removing all non-harmonic detail – passing notes, suspensions, etc. – can greatly reduce the burden of work in later stages. Precisely what degree of adjustment is required for any particular score will become apparent with experience. For example, in the chorale below a eighthnote slice (-w48) works well with some detail, such as the passing notes in measure 1, turned from eighth to quarternotes. (That is, unless the analyst wishes to notice a plain triad followed by a seventh chord.) Quite soon, with practice, a knack of adjusting the initial input can be developed that enables the intended analysis to emerge from the scripts in a relatively uncluttered form. Also small changes such as adding rests to the beginning of the chorale so that the score starts on a barline will avoid the use of awkward switches to inform the *txt2mos* script about the pick-up beat.

Figure F.1 A Roman numeral analysis of the chorale *Ach Gott und Herr*. (A full MOS analysis of this chorale is given in Example T; however, it should be noted that this analysis was prepared with an earlier version of the *txt2mos* script and so the example differs in some small details from the analysis presented below.)

Step 2

Produce a MIDI file of the score. This can most easily be done in a score editing program capable of generating MIDI files. MIDI data taken from live performance is unlikely to have a sufficient level of metrical regularity to enable the 'slicing' procedure used by the *txt2mos* script to work effectively, and equally, the output of the score/MIDI editor should be set to produce a ‘mechanically’ even pulse. Generally, superfluous detail such as short unaccented passing notes will be automatically eliminated by the *txt2mos* script, however, there can be situations, as mentioned above, where the harmony is obscure and difficult for the script to read and some adjustment by the analyst may be required. It is assumed that the exported data will be in the form of a standard MIDI file. There are no restrictions in regard to the number of tracks and instruments involved or upon the length of the score, however, due to the amount of textual data generated, it is probably easiest to proceed in manageable chunks of eight to sixteen measures. Again dividing up the score into units or chunks is probably best done at the music editor stage.

Step 3

Turn the MIDI data into a textual description using the mid2txt script.

```
[perl] mid2txt [-e -h] input-MIDI-file [> myfile]
```

The MIDI file is given as input to the *mid2txt* utility (MIDI-to-text) which generates from the MIDI data a *score list*, this is the default output and takes the form:

```
['note', 288, 96, 0, 72, 78],
['note', 288, 96, 0, 67, 74],
['note', 288, 96, 0, 48, 76],
['note', 288, 96, 0, 64, 78],
```

with the values: ['note', start-time, duration, channel, note-number, velocity],

These lines describe the first chord in the chorale. Start-time and duration are in midi ticks at the rate of 96 ticks per quarternote. The sixteen standard channels (instruments) are labelled 0–15. Treble C is MIDI note-number 72 out of the 128 note range C0 to G127 and the volume or velocity is 78, also taken from a zero-based range of 128 values. This score list textual output will have an entry for every note in the MIDI file plus other information concerning tempo, time signature, key signature, tick division, etc. Normally no editing of this raw information is required, however, it is worth checking through the score list by loading the text file into a text editor and scrolling down the list(s) to identify any obvious errors – e.g. my own music editor will occasionally produce a negative duration value where voices cross on a single stave! (The -e switch produces an event list and -h a short description of the scripts features.)

Step 4

Convert the MIDI text data into a visual outline of a MOS analysis using the txt2mos script. The score list text file is given as input to the *txt2mos* utility (text-to-Modulating Oscillatory System), which generates an outline MOS analysis, again in textual form, but formatted in a particularly 'visual' way. To be able to comfortably read this output in a text editor program, it is necessary to set the text editor to use a monospaced font (e.g. Courier) and a long line length (e.g. 200 or more characters). In Figure F.2 below the first chordal exchange from the example chorale, C-major to G-major, is illustrated as it appears in a text editor program. Notice that a full measure 'zero' has been added to the score.

```
000.000: redundant slice skipped, Tick: 0000000

(snip)

h8h8   :--------------------------------------------------------------------------------------
1:1    :                                    1-----------c------3----c---5--c--7-C-9-0-12-3456/
1:1    :------------------------------------------------O---------------O--O----O---+--+--+++-
0000288:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
000.750:C-maj                                           R           *      *    *   *  *  * *


000.875: redundant slice skipped, Tick: 0000336


h12h8  :------------------------------------------------O---------------O--O----O---+--+--+++-
2:3    :                                           1-----------2------3----c---5--6--7-C-9-|-1
2:3    :-------------------------------------------------------O------O----O---O--+----+-+-+--
0000384:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
001.000:G-maj                                                  R           *      *    *   *
```

Figure F.2a The opening chord progression of the chorale *Ach Gott und Herr* as displayed in the textual output generated by the *txt2mos* script. (To the right the output has been truncated and continued in Figure F.2b.)

The output from the *txt2mos* script is divided into ‘slices’. The slice width here equals one eighthnote, the default (48 ticks) but may be configured differently by using the -s switch in the *txt2mos* command. However, so as not to clutter the screen with duplicate slice information for chords of long duration, only the salient slices where the harmony is changing are displayed in full and the redundant slices simply noted in passing. The central task performed by the script is to find the sequence of first/highest harmonic series that contain partials matching all the notes in each successive chord, plus, the conjunctions between each chord and its predecessor; that is, the common frequencies (objective harmonics arising in both chords) linking the succession of harmonic progressions together.

In the lower slice in Figure F.2, the top line shows the previous chord (C-major), the third line down the present chord (G-major) – ‘O’ signify notes and ‘+’ the partials arising from these notes. Sandwiched between the previous and present chords is a nested harmonic series matching the present chord. The fourth line marks the ascending note letters of the chromatic scale with one character position for each MIDI note number. Thus, the note C# is marked by the sharp sign, alone, following C. In a similar single-character shorthand, the ascending harmonics of the nested series are labelled by only their unit column: [h]1 through [h]9 followed by 0 for ten, 1 for eleven, 2 for twelve, etc.; and eventually, beyond [h1]6 the harmonics are denoted by slashes ‘/’, or vertical bars ‘|’ where they form potential conjunctions between each pair of chords. On close inspection of the nested series in the last slice in Figure F.2 (second line of slice) a letter appears between (h)7 and (h)9: ‘C’, once only. This is the particular conjunction singled out by the utility to link the chord progression. (Some of the aforementioned symbols may be obscured by vertical bars and ‘c/C’ conjunction markers.) The final line of a slice contains the current chord’s name/type plus its nominal root ‘R’ and the groupings ‘*’ of its aggregated series. A full anatomy of a slice is provided in Figure F.7a/b.

To the left of a full slice in Figure F.2a there are five information fields, reading from top to bottom: 1) the harmonic value of the current conjunction ‘C’ in terms of the previous and current nested series – e.g. h12h8, 2) the ratio of exchange between previous and current nested harmonic series – e.g. 2:3, 3) the ratio of exchange between previous and current aggregated series –i.e. the heard harmonic progression, 4) the MIDI tick of the present chord and 5) the measure/slice number of the present chord with position in the measure expressed as a decimal fraction – e.g. 0.750. To the right of a full slice in Figure F.2b there are further information fields detailed later, but including: conjunction harmonics with note letter, mutable base numbers, nested series degree with ratios of exchange – e.g. (4xthree –> 4xtwo), aggregated series exchanges – e.g. (6 groups of 2 –> 4 groups of 2) and finally chord name/types with inversion.

```
------------------------------------------:h8h8-C:MBN 1x24x2x4 -> 1x24x2x4
//////////////////////////////////////////:(8xone)->(8xone)
+-+--++--++-+-+++++++++-+-+--+----+-------:(4 grps of 2)->(4 grps of 2)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h2h2-C:MBN 1x24x2x1 -> 1x24x2x1
* * ** *********                          :C-major (a)
```

```
+-+--++--++-+-+++++++++-+-+--+----+-------:h12h8-G:MBN 1x24x2x6 -> 1x36x2x4
|-345|///||/|/||||/|/|//|////|////////////:(4xthree)->(4xtwo)
+--+++-+-++-++++++-+-+-+++--++---+--------:(6 grps of 2)->(4 grps of 2)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h6h4-G:MBN 1x24x2x3 -> 1x36x2x2
*  * * * * ** *********                   :G-major (a)
```

Figure F.2b Continued across the page from Figure F.2a, the opening chord progression of the chorale *Ach Gott und Herr* as displayed in the textual output generated by the *txt2mos* script.

This then is the raw *txt2mos* output ready for the analyst to mold into a meaningful interpretation. There are a number switches that may be used with the *txt2mos* command:

```
[perl] txt2mos [-a<n> -b<n> -h -d<n> -k<n> -m<n> -r -s -w<n>] mid2txt_list [> myfile]
```

Examples

```
 -a288  adjustment in ticks from start of measure zero to pick-up note, default 0
 -b0    first bar/measure number, default 1
 -d120  tick division, default 96
 -k7    specify fundamental key/tonal center period (H1), default first/highest
 -m288  measure length in ticks when no time-sig in MIDI data, default 384
 -r     restrict display of redundant slices, default display all slices
 -s     turn on smoothing of nested and aggregated series, default off
 -w96   slice width in ticks, default 48
```

Taking the more useful switches in order: The -a switch is used to tell the *txt2mos* script how many ticks to add to the beginning of the score (which presumably has the pick-up beat as delta-time zero) to make a complete measure, so that the measure count is correctly aligned. In this situation the script actually makes up the measure with negative ticks, thus maintaining the tick alignment of the original MIDI file. Using the -a switch will probably require the -b switch as well, because the default first measure is set to 1 and having used the -a switch, the first measure is nominally measure zero. If the score begins at measure 1, none of the above is necessary. However, the -b switch is also useful for preserving the measure numbering where the score or segment of score begins beyond m1. Assuming time signature information is included in the MIDI file, the *txt2mos* script will automatically set the measure length to begin with and adjust the measure count to accommodate for changes of time signature encountered within the score.

The granularity of tick division is read automatically from the MIDI file and if this information is missing from the file it is set by default to the common 96 ticks per quarternote.[1] The -k switch sets an arbitrary fundamental period (H1), specified by pitch-class C0 through B11 or negative MIDI note number, details below. If there is no time signature information the -m switch is used to inform the script of the measure length. The default is 4/4 common time – 384 ticks at 96 ticks per quarternote. There is no facility for making the -m switch effect a time signature change within the body of the score, although this could be achieved by inserting time signature information directly into the score list beforehand. The -s switch introduces a degree of smoothing-out of the intervals between adjacent nested harmonic series and also between adjacent aggregated series where the ratios of the enfolding nested series allow. Because the *txt2mos* script finds the first/highest appropriate nested harmonic series and preserves these positions because of their importance for chord recognition (Beament, 2005), this can produce a jagged profile between adjacent series. Thus, a simple smoothing algorithm is used to double or occasionally quadruple the period of nested series so as to produce a more coherent flow of middle level exchanges. These octave extensions to the nested series are marked ‘h===...’. Finally, the -w switch sets the slice width, the default is 48 ticks which is equivalent to an eighthnote at a tick division of 96. A judgement about the appropriate slice width would depend upon the harmonic rhythm and notation of the score in question.

Step 5

Edit the MOS outline in the light of the roman numeral analysis. The *txt2mos* utility is not a so called ‘expert system’, although it is making some progress in that direction. That said, the script does produce a general framework for a whole section or score ready for further development – a first approximation. At present, the script sets up the chord progression data with the smoothed out middle level nested harmonic series ready for editing as required. Chords, roots and conjunctions are identified with their requisite nested and aggregated ratios of exchange. Nevertheless the initial roman numeral study, based on the analyst's own hearing of the harmonic progressions, should not be lost or overridden by the mass of data produced by the script. Making a MOS analysis still requires considerable care, effort and attention to detail – principally applied to the close scrutiny and editing of this *txt2mos* output – though when complete, I find, there is a degree of satisfaction in having successfully working out the 'turning wheels within wheels' of a complex nested system.

Figure F.3a/b is an example of typical output from the *txt2mos* script. It requires a careful eye to check that the computer generated analysis is consistent with the analyst's own interpretation. This is a two-sided process, most editing involves correcting shortcomings in the automatic analysis but occasionally the script will throw up ideas that make you think again.

Typical editing procedures are, for example, that sometimes the initial position of the nested series – shown on the second line of the five line slice data – will yield the chord interpretation the analyst requires and at other times not. Changing the position of this nested series, and thus the harmonic interpretation, is done by placing the text editor cursor below (to the left of) the fundamental h1 of the nested series and moving the series to the desired position by adding or removing spaces. Usually spaces are removed and the series shifted to the left; after which, the conjunction marker 'C' will also probably need moving too. Place a new conjunction in the position required and replace the old one with an appropriate symbol: forward slash, vertical bar or number. Once the nested series is in its new position, the conjunction harmonic and the ratio of exchange, in the top and second top left margin will also need changing to reflect the new situation. As will the textual descriptions of the exchanges on the right-hand side. It is necessary that all of these changes are made as the *mos2col* script will need this information to generate the formatted lists of harmonic series. Figures F.4 through F.6 further illustrate these procedures.

Although the script automatically smoothes out the nested series exchanges, once a chord re-interpretation edit has been made (which is done by changing the position of the nested series) the analyst may also need to change some aspects of adjacent chords, such as the octave positioning of the nested harmonic series and conjunction. This might be done in order to obtain a more logical overall trajectory for the sequence of slices. A typical situation could be in the approach to an amended minor chord that requires an extended nested series to accommodate the relatively large aggregations of groups of five. In *Ach Gott und Herr* at slice 2.125, Figure F.3a, the *txt2mos* script has interpreted the chord as C-major with added sixth but this chord could be construed as A-minor seventh. To change the interpretation the nested series needs to be edited down to a low F, so as to accommodate the bass note E as the fifth (h15) of the A-minor chord – illustrated in Figures F.4. Interestingly this new interpretation reveals a connection between chords IV and vi through the MOS analysis (i.e. F-major/A-minor), in that they share a common nested series so flow easily from one to the other – a connection, below, extended to the relative, C-major.

The necessary alterations to slice 2.125 and the consequent effects upon slices 2.000 and 2.250 are illustrated in Figures F.4–6 and Figure F.10. First the 2.125 nested series is lowered to MIDI note number F5 which positions the bass note of the chord at E-h15, and to smoothly match the lower 2.125 nested series, the nested series in slices 2.000 and 2.500 are octave doubled. This shows up a dichotomy: octave leaps don't change the chord type and so the original position of the nested series is not changed (i.e. 1--2--3-4 etc.) but when the interpretation of the chord changes then also must the position of the nested series – as in slice 2.125. Thus the link between the period of the nested series and the periods of the chords' component notes is maintained in line with James Beament's observation quoted earlier.

```
h6h12  :------------------------------------------------O-----------O---O--O----+---+--+--+++-
2:1    :                        1-----------2------3----c---5--6--7-c-9-0-1C-345|///|//|//|/|/
2:1    :------------------------------------------------O---------O---O----O--+-+-+-++-+-++-+-
0000720:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
001.875:C-Maj9                              R           *      *    *   *  *  * * * * ** *****

h16h12 :------------------------------------------------O---------O---O----O--+-+-+-++-+-++-+-
3:4    :                             1-----------2------3----4---5--c--7-8-9-0-1C-34||/|/|//|/
3:4    :-----------------------------------------------------O---O--O--------O--+---++-+-+--++
0000768:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.000:F-Maj                                    R           *      *    *   *  *  * * * * **

h12h16 :-----------------------------------------------------O---O--O--------O--+---++-+-+--++
4:3    :                        1-----------2------3----4---5--6--7-c-9-0-12-|45C///|//|/|//|/
4:3    :----------------------------------------------------O--O----O--------O-++-+-+--+++-++-
0000816:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.125:C-Maj6                              R           *      *    *   *  *  * * * * ** *****

h18h24 :----------------------------------------------------O--O----O--------O-++-+-+--+++-++-
4:3    :                   1-----------2------3----4---5--6--7-c-9-0-12-345c///||/C//////|/||/
4:3    :--------------------------------------------------O--O--------O--------O+-+--++--+-++-
0000864:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.250:G-Maj7                         R           *      *    *   *  *  * * * * ** *********

002.375: redundant slice skipped, Tick: 0000912

h24h18 :--------------------------------------------------O--O--------O--------O+-+--+++-+-++-
3:4    :                        h===========1-----------2------c----4---5--c--7-|-C-0-12-34||/
3:4    :------------------------------------------------O------O--------O-------O-+-+--+--+++-
0000960:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.500:C-Maj                               R           *      *    *   *  *  * * * * ** *****

002.625: redundant slice skipped, Tick: 0001008

h18h24 :------------------------------------------------O------O--------O-------O-+-+--+--+++-
4:3    :                   h===========1-----------2------3----c---5--6--7-c-9-0-1C-345|///|//
2:3    :-------------------------------------------O---------------O-------O------O--+++-+-+--
0001056:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.750:G-Maj                                      R           *      *    *   *  *  * * * * *
```

Figure F.3a Measure1.875 through 2.750 of *Ach Gott und Herr*, as output by *txt2mos* and viewed in a text editor. The period doubling effect of smoothing are clearly marked in slices 2.500 and 2.750.

The next task is to attend to the root ‘R’ of the new A-minor chord. Similarly to the nested series, the root marker and its associated grouping of nested series harmonics ‘*’ is re-positioned from C to A-h5 by removing three spaces from the line – compare Figure F.3a with F.4. The root markers of slices 2.00 and 2.500 remain in their original positions as the chord interpretation has not changes for them but the groupings they now represent are double from what they were before – despite the figures 1---2--3-4 etc. in the nested series not being changed to reflect this fact – à la Beament.

Calculating the new conjunctions is now possible. In slice 2.000 the conjunction remains as before, but due to the doubling of the period of the nested series its value changes from h12 to h24 in terms of that nested series. For slices 2.125 and 2.250 completely new conjunctions must be calculated and this can most easily be done by counting up, in parallel, the root-ratio ‘R’ groups of each pair of chords until they coincide – illustrated in Figure F.4. In slice 2.125, counting in groups of four and five respectively, we count 1, 2, 3, 4, 5 against 1, 2, 3, 4, 5, 6, 7, 8. Five groups of four coincides with eight groups of five at A the top note in each chord. Therefore the modulation exchange that will carry slice 2.000’s aggregated series built on F-h4 (F-major chord) to slice 2.125’s aggregated series on A-h5 (A-minor7th chord) is 8:5.

```
+-+--++-+++-+-++++++-++-+-+--+------------:h6h12-G:MBN 1x24x2x3 -> 1x12x2x6
|/|//||/|||/|/||||/|/|//|////|////////////:(6xone)->(6xtwo)
+-+++++++++-+-++++-+++--+-----+-----------:(3 grps of 2)->(6 grps of 2)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h2h4-C:MBN 1x24x2x1 -> 1x12x2x2
****                                      :C-dominant ninth no third (a)
```

```
+-+++++++++-+-++++-+++--+-----+-----------:h16h12-C:MBN 1x12x2x8 -> 1x16x2x6
//||/|/||||/|/||/|/|/|////////////////////:(4xfour)->(4xthree)
-+++-+-++++++-++-+-+-+++--+----+----------:(8 grps of 2)->(6 grps of 2)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h8h6-C:MBN 1x12x2x4 -> 1x16x2x3
*********                                 :F-major (a)
```

```
-+++-+-++++++-++-+-+-+++--+----+----------:h12h16-C:MBN 1x16x2x6 -> 1x12x2x8
//||/|/||||||/|//|/|/|||//|////|//////////:(4xthree)->(4xfour)
+-+++++++++++-+--+-+-+++--+----+----------:(6 grps of 2)->(8 grps of 2)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h6h8-C:MBN 1x16x2x3 -> 1x12x2x4
****                                      :C-major added sixth (b)
```

```
+-+++++++++++-+--+-+-+++--+----+----------:h18h24-D:MBN 1x12x2x9 -> 1x9x2x12
|/||||/|/||/|/|////|/|/|//////////////////:(6xthree)->(6xfour)
++++++-+-++-+++++--+-+-+++--+----+--------:(9 grps of 2)->(12 grps of 2)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h6h8-G:MBN 1x12x2x3 -> 1x9x2x4
                                          :G-rootless dominant seventh (c)
```

```
++++++-+-++-++++++-+-+-+++--++---+-------+:h24h18-D:MBN 1x9x2x12 -> 1x12x2x9
|/||/|/|/||/|/|/|////|//|////////////////:(6xfour)->(6xthree)
+-++-+++-++-+-+-+++-+++-+-+--+----+-------:(12 grps of 2)->(9 grps of 2)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h8h6-G:MBN 1x9x2x4 -> 1x12x2x3
****                                      :C-major (a)
```

```
+-++-+++-++-+-+-+++-+++-+-+--+----+-------:h18h24-D:MBN 1x12x2x9 -> 1x9x4x6
|////|/|/|//|///||///||/|/|//|////////////:(6xthree)->(6xfour)
++--++-+-+-+++-+++-+-++-+-+-++-+----+-----:(9 grps of 2)->(6 grps of 4)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h6h8-G:MBN 1x12x2x3 -> 1x9x4x2
* *********                               :G-major (a)
```

Figure F.3b Continued across the page from Figure F.3a, measure 1.875 through 2.750 of *Ach Gott und Herr*, as output by *txt2mos* and viewed in a text editor.

Rather miraculously, this conjunction will also transform the nested series in an equally coherent manner, through a dupla 2:1 exchange, from F to its octave below – which, counting in individual nested harmonics yields the two series F-h1 through A-h20 and F-h1 through A-h40 – written on the right-hand side of the nested series as (20×one)–>(20×two) in Figure F.6b.

Finding the conjunction for slice 2.250 proceeds in the same way, however, it introduces an added complication, too many conjunctions! Again counting up from the root-ratios, this time from A-h5 in slice 2.125 and G-h2×2 in slice 2.250, in groups of five and four respectively, we find three adjacent coincidences at: G-7:8, A-8:9 and B-9:10 – illustrated in Figure F.4. The narrowness of the exchange negotiating the interval of a tone introduces a range of possibilities focused on the ratio 8:9, but with 7:8 and 9:10 also covered by the ‘penumbra’ of the modulation. (For a minor-third modulation interval there are two – 5:6 and 6:7, and for the semitone step there are four ratios – 14:15, 15:16, 16:17 and 17:18.) The central ratio 9:8 appears the natural choice, but before finally deciding on which of multiple conjunctions to use it is necessary to check how these aggregated exchanges will interact with the underlying nested series exchange. The aggregated and the nested series exchanges need to compatible.

```
h16h24 :------------------------------------------------O---------O---O----O--+-+-+-++-+-++-+-
3:2    :                 h===========1-----------2------3----4---5--c--7-8-9-0-1C-34||/|/|//|/
3:4    :-----------------------------------------------------O---O--O--------O--+---++-+-+--++
0000768:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.000:F-Maj                                    R           *      *    *   *  *  * * * * **
                                                                             |
                     Counting in groups of four: 1-----------2------3----4---5    ratio 5:8
                                                                             |
h12h40 :-----------------------------------------------------O---O--O--------O--+---++-+-+--++
2:1    :     1-----------2------3----4---5--6--7-8-9-0-12-3456///////////////C//|///|//|/|//|/
8:5    :----------------------------------------------------O--O----O--------O-++-+-+--+++-++-
0000816:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.125:A-min7                           R                  *    *   *  *  * * * * ** *****
                                                                           | | |
             Counting in groups of five: 1-----------2------3----4---5--6--7-8-9  ratio ?
                                                                           | | |
h45h40 :----------------------------------------------------O--O----O--------O-++-+-+--+++-++-
8:9    :       h===========1-----------2------3----4---5--6--7-8-9-0-12-3456///C|/|//////|/|+/
10:9   :--------------------------------------------------O--O--------O--------O+-+--++--+-++-
0000864:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.250:G-dom7                         R                  *    *   *  *  * * * * ** *********
                                                                           | | |
           Counting in groups of four: 1-----------2------3----4---5--6--7-8-9-10 ratio ?
```

Figure F.4 Editing slice 2.125 to transform it into an A-minor seventh chord and counting the aggregated groups to calculate the conjunction.

In contrast to the root movement of top level aggregated series (in this example edit) the middle level nested series exchange is moving by a tone in the opposite, ascending, direction from F-h1 in slice 2.125 to G-h1 in slice 2.250. And again, though rather less neatly, the conjunctions arrived at by counting up in aggregated groupings also provides the basis for the transition between the nested series. However, there is a problem, a technicality revealed by re-applying the simple parallel counting technique at the level of individual nested series harmonics – illustrated in Figure F.5. Counting up from the fundamental nested tone in slices 2.125 and 2.250 (labelled ‘1’ and ‘h’, and remembering that in slice 2.250 the numbers have been doubled) we find that the point of coincidence occurs between G-h9 and G-h8 (h4×2) – marked with vertical bars in Figure F.5. But just as with aggregated series there are two other conjunctions h8–h7 and h10–h9. This indicates that the underlying nested exchange is also seeking to step from low F to G, ideally, by a sesquioctava 8:9 modulation exchange. So, the question arises: Is the contrary motion nested series sesquioctava 8:9 exchange compatible with the existing aggregated series sesquioctava 9:8 modulation exchange? The way to resolve this question is to keep on counting.

The nested series conjunctions that have been found so far are too low, in fact, lower than all the notes in the two chords. Indeed what has been found are the (possible) ‘primary’ conjunctions for two nested series separated by a tone; and, by continuing to count further some rather less plausible conjunctions emerge – such as h11–h10 and h13–h12 – before the doubled trio of h16–h14, h18–h16 and h20–h18 reappear. These three double value conjunctions set an octave above the primary conjunctions would form secondary modulation exchanges, for example, a secondary sesquioctava 8:9 (16:18) exchange. Does the pattern continue? Indeed it does. Now just concentrating on the central sesquioctava exchange, and counting on a further eight/nine harmonics, we find another, tertiary, sesquioctava conjunction at h27–h24. Yet another appears at h36–h32 and a fifth at h45–h40 – marked by vertical bars in Figure F.5. The pattern continues without end and the pattern created by two adjacent harmonic series stepping in groups of nine harmonics and groups of eight harmonics, respectively. It is the pattern of a ‘conjunction’ series, yet another harmonic series nested within the system picking out the valid common frequencies that bind a tonal progression together. The question of which harmonic in a conjunction series best represents the connection between two harmonic series is open to debate: the fundamental or some higher harmonic. Conjunction series in *txt2mos* files are marked with the letter ‘c’ and capitalised ‘C’ for the chosen conjunction. Normally conjunction series for aggregated exchanges only are shown in the slice data. There are conjunction series for the sesquiseptima 7:8 and the sesquinona 9:10 exchanges too:

Modulation Exchanges	Primary	Secondary	Tertiary	Quarternary	Five-fold
Sesquiseptima 7:8	Fh8–h7	Fh16–h14	Ch24–h21	Fh32–h28	Ah40-h35
Sesquioctava 8:9	Gh9–h8	Gh18–h16	Dh27–h24	Gh36–h32	Bh45–h40
Sesquinona 9:10	Ah10–h9	Ah20–h18	Eh30–h27	Ah40–h36	

What emerges from this table is a form of exclusivity. For example, if the sesquioctava 8:9 exchange is used to govern the upward step of a tone from slice 2.125 to 2.250 for the nested series, then, when this is extrapolated up to the level of the aggregated exchange (in the form of a five-fold exchange) it dictates that for the aggregated series only the sesquinona 10:9 modulation exchange (Bh45–h40 –i.e. 9×5–10×4) is compatible with the underlying nested series’ configuration of conjunctions – illustrated in Figure F.5.

```
h12h40 :-----------------------------------------------------O---O--O--------O--+---++-+-+--++
2:1    :     1-----------2------3----4---5--6--7-8-9-0-12-3456//////|////////C//|///|//|/|//|/
8:5    :----------------------------------------------------O--O----O--------O-++-+-+--+++-++-
0000816:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.125:A-min7                           R           *      *    *   *  *  * * * * ** *****

                                                 | | |         |      |      | |
  Parallel:  1___________2______3____4___5__6__7_8_9_10________18_____27____36_45
  Counting:    1           2      3    4   5  6  7 8 9         16     24    32 40
                                                 | | |         |      |      | |
  Nested exchange conjunction series:              c           c      c      c C

h45h40 :----------------------------------------------------O--O----O--------O-++-+-+--+++-++-
8:9    :       h===========1-----------2------3----4---5--6--7-8-9-0-12-3456///C|/|//////|/||/
10:9   :--------------------------------------------------O--O--------O--------O+-+--++--+-++-
0000864:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.250:G-dom7                         R           *      *    *   *  *  * * * * ** *********
```

Figure F.5 Slices 2.125 and 2.250, sesquioctava 8:9 exchange counting in single nested series harmonics.

The basic constraint highlighted here is that the conjunction series of both the underlying nested series and the top level aggregated series need to dovetail well together – as in Figure F.5 where in slice 2.250 both series converge with groupings of nine. However, if both nested and aggregated levels used the sesquioctava 8:9 and 9:8 exchanges respectively in slice 2.250, groupings of nine would grate against groupings of eight and the conjunctions would not match exactly.[2]

But what of the sesquiseptima 8:7 exchange (Gh36–h32)? Unfortunately it is not viable because the A-minor chord in slice 2.125 demands groups of five and 7×5–8×4 yields only Gh35–h32. However, if the roles in Figure F.5 are reversed and an aggregated exchange of 9:8 is employed, this is only possible if the underlying nested series modulation is converted to a sesquinona 9:10 exchange, then its quarternary conjunction (Ah40–h36 –i.e. 8×5–9×4) fits the aggregated sesquioctava exchange perfectly with both groupings of five and four respected – Figure F.6. In practice though, this conjunction is too low – below the note B in slice 2.250 – but its octave transposition (Ah80–h72) is ideal as it picks out significant partials generated by both chords: the second harmonic of top note A in slice 2.125 and the third, fifth and sixth harmonics of notes in slice 2.250. This conjunction position is marked with ‘C’ in Figure F.6.

Yet again the sesquiseptima 8:7 exchange (Ah40–h35) is a near miss because this time the G-major chord in slice 2.250 demands groups of four and 8×5–9×4 yields Ah40–h36. (Though it is not clear if such small discrepancies would disturb the ear’s appreciation of a commensurable relationship.) However, having found the compromise of sesquinona exchange for the nested series and sesquioctava exchange for the top level aggregated series – which places the central ratio in the foreground (as shown in Figure F.6a) – the left-hand fields must be updated to reflect this. The top field in each slice now contains the values of the new conjunctions (in nested series harmonics) for both the previous and current nested series in slices 2.125 and 2.250. And the two fields below have there updated ratios of exchange entered. All of these changes can be compared with the original *txt2mos* analysis given in Figure F.3a/b.

```
h6h12  :-----------------------------------------------O----------O---O--O----+---+--+--+++-
2:1    :                       1-----------2------3----c---5--6--7-c-9-0-1C-345|///|//|//|/|/
2:1    :-----------------------------------------------O---------O---O----O--+-+-+-++-+-++-+-
0000720:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
001.875:C-Maj9                             R           *      *    *    *   *   * * * * ** *****

               Parallel: _______1___________2___________|______|____|___|__|__|_|8fold×two
               Counting: 1           2      3           |      |    |   |  |  | |8fold×three

h16h24 :-----------------------------------------------O---------O---O----O--+-+-+-++-+-++-+-
3:2    :                h===========1----(h3)---2------3----4---5--c--7-8-9-0-1C-34||/|/|//|/
3:4    :----------------------------------------------------O---O--O-------O--+---++-+-+--++
0000768:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.000:F-Maj                                   R                *      *    *   *  *  * * * * **


h20h40 :----------------------------------------------------O---O--O-------O--+---++-+-+--++
2:1    :     1-----------2------3----4---5--6--7-8-9-0-12-3456/////|///////C//|///|//|/|//|/
8:5    :---------------------------------------------------O--O----O-------O-++-+-+--+++-++-
0000816:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.125:A-min7                          R           *      *    *   *  *  * * * * ** *****

                                                    |           |      |    |   |  |  | |
  Parallel:  1___________2______3____4___5__6__7_8_9_10_________20_____30___40__50_60_7080
  Counting:    1           2      3    4   5  6  7 8 9           18     27   36  45 54 6372
                                                    |           |      |    |   |  |  | |
  Nested exchange conjunction series:               c           c      c    c   c  c  c C

h80h72 :---------------------------------------------------O--O----O-------O-++-+-+--+++-++-
9:10   :      h===========1-----------2------3----4---5--6--7-8-9-0-12-3456/c/||/|/|////C/||/
9:8    :-------------------------------------------------O--O--------O--------O+-+--++--+-++-
0000864:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.250:G-dom7                        R           *      *    *   *  *  * * * * ** *********
```

Figure F.6a Slices 2.000 through 2.250 with all edited values in place. Slices 2.125 and 2.250: sesquinona 9:10 exchange counting in single nested series harmonics.

Now turning to the right-hand fields shown in Figure F.6b describing the nature of the exchanges undertaken by both the nested and the aggregated series – the original values are displayed in Figure F.3b. Ignoring for the moment the first/top right field and beginning with the second describing the ‘degree’ of the nested exchange that carries the previous nested series into the current nested series (i.e. primary, secondary, ..., nth-fold) plus recording the ratio of exchange in words. (This is the reverse of the ratio of exchange given on the extreme left of the same line in figures.) Slice or measure 2.000 is the first requiring amendment and to ascertain the degree of the nested exchange the technique of parallel counting is used between the nested series of slice 1.875 and 2.000. This is shown between the slices in Figure F.6a and involves counting up 1, 2 in slice 1.875 against 1, 2, 3 in slice 2.000 – h3 is explicitly marked in brackets being obscured by doubling. This is the primary conjunction and dividing the conjunction harmonic (top left field) by its primary conjunction value yields the degree of the exchange –i.e. 16/2 = 8 and 24/3 = 8. Alternatively, counting up in primary conjunction steps to the current chord’s conjunction marker ‘C’ as illustrated in Figure F.6a produces the same result. Therefore the previous nested series (8xtwo) is transformed into the current nested series (8xthree), by the downward step of a fifth, through an eight-fold sesquialtera 3:2 modulation exchange. The two values in brackets are entered into the top right-hand field. (The brackets and arrows in this field and the field below are mandatory because they are used by the *mos2col* script; also, the letter ‘x’ is used rather than a multiplication sign to remain within the standard ASCII range.) The degree of the downward octave step through a dupla 2:1 exchange (20xone)–>(20xtwo) by the nested series from slice 2.000 to slice 2.125 is straightforward and the upward step of a tone via a sesquinona 9:10 exchange (8xten)–>(8xnine) has been explained above.

```
+-+--++-+++-+-++++++-++-+-+--+------------:h6h12-G:MBN 1x48x2x3 -> 1x24x2x6
|/|//||/|||/|/||||/|/|//|////|////////////:(6xone)->(6xtwo)
+-+++++++++-+-++++-+++--+----+------------:(3 grps of 2)->(6 grps of 2)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h2h4-C:MBN 1x48x2x1 -> 1x24x2x2
****                                      :C-dominant ninth no third (a)


                                                        eight-fold sesquialtera 3:2
+-+++++++++-+-++++-+++--+----+------------:h16h24-C:MBN 1x24x2x8 -> 1x16x4x6
//||/|/||||/|/||/|/|/|////////////////////:(8xtwo)->(8xthree)
-+++-+-++++++-++-+-+-+++--+----+----------:(8 grps of 2)->(6 grps of 4)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h8h12-C:MBN 1x24x2x4 -> 1x16x4x3
*********                                 :F-major (a)
                                                         secondary sesquitertia 3:4

                                                             twenty-fold dupla 2:1
-+++-+-++++++-++-+-+-+++--+----+----------:h20h40-A:MBN 1x16x4x5 -> 1x8x5x8
//||/|/||||||/|//|/|/|||//|////|//////////:(20xone)->(20xtwo)
+-+++++++++++-+--+-+-+++--+----+----------:(5 grps of 4)->(8 grps of 5)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h20h40-A:MBN 1x16x4x5 -> 1x8x5x8
****                                      :A-minor seventh (c)
                                                              primary 8:5 exchange


                                                         eight-fold sesquinona 9:10
+-+++++++++++-+--+-+-+++--+----+----------:h80h72-A:MBN 1x8x5x16 -> 1x9x4x18
|/||||/|/||/|/|////|/|/|//////////////////:(8xten)->(8xnine)
++++++-+-++-+++++--+-+-+++--+----+--------:(16 grps of 5)->(18 grps of 4)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h40h36-A:MBN 1x8x5x8 -> 1x9x4x9
                                          :G-rootless dominant seventh (c)
                                                        secondary sesquioctava 9:8
```

Figure F.6b Continued across the page from Figure F.6a, the edited right-hand fields of slices 2.000 through 2.250. (The descriptive italic script on the right margin is not part of the editing process.)

The third right-hand field describes how the top level aggregated series, encapsulating the harmonic gist of the chord, navigates from the previous chord to the present chord. The elemental units of these aggregated series are formed from groupings of nested series harmonics and the freedom to use various aggregates – groups of two, three, four, five, etc. – allows some flexibility of manoeuvre within the confines of the underlying nested series. Though often the nested and aggregated series will move in step, at other times the motion may be independent in both direction and/or interval. Calculating these aggregated exchanges simply involves parallel counting of the aggregate markers on the fifth line of the previous and current slice – R, *, *, etc. – up to the current slice’s conjunction marker ‘C’. To ascertain these values for slice 2.000 (third right field), first count the aggregate markers on the fifth line of the previous slice 1.875 up to the ‘C’ in slice 2.000 (8 groups of 2) and then count the aggregate markers in slice 2.000 up to the selfsame ‘C’ in slice 2.000 (6 groups of 4) – not ignoring the effect of doubling. Alternatively the previous and present conjunction values – top left field in slice 2.000 – may be divided by the root-ratios ‘R’ of slices 1.875 and 2.000 respectively: 16/2 yields (8 groups of 2) while 24/4 yields (6 groups of 4). These values are entered in the third right field of slice 2.000 as shown in Figure F.6b. The right-hand field values for slices 2.125 and 2.250 many be obtained by applying the same process.

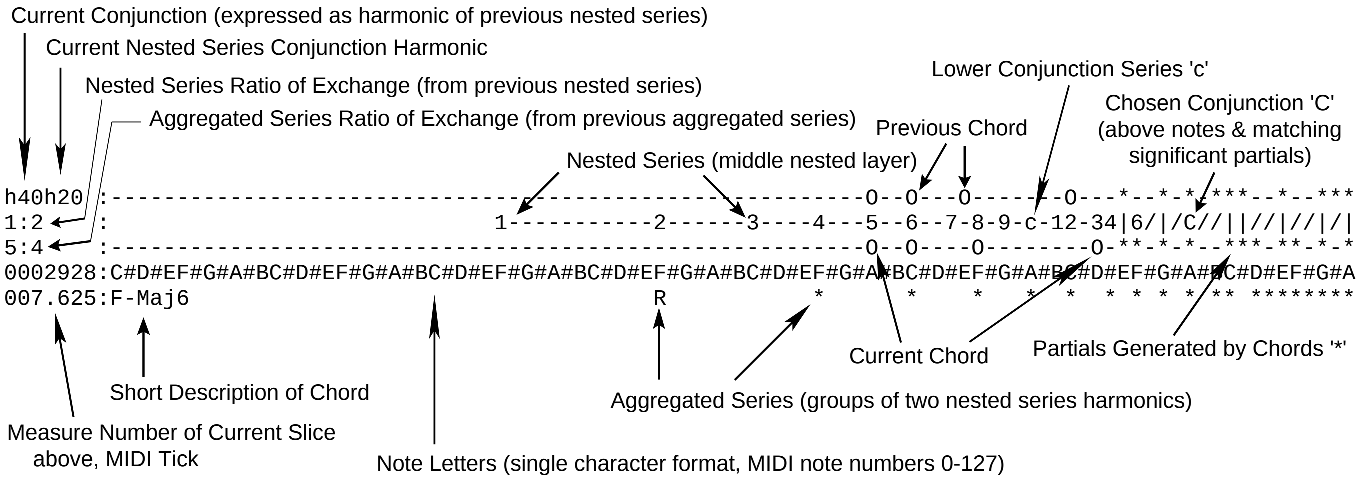


Figure F.7a The anatomy of a slice. Slice 7.625 with all fields labelled.

Actually moving the position of the aggregate markers and/or nested series produces a different harmonic interpretation of the chord, as discussed earlier in the re-interpretation of slice 2.125 from added sixth C-major to A-minor seventh. However, a less drastic and occasionally useful editing of the aggregate markers can be obtained by doubling or halving the value of the root-ratio ‘R’. For example, in slice 2.000 the root-ratio could easily be halved from 4 to 2 by drawing down the aggregate markers – R, *, *, etc. – by twelve spaces thereby converting the aggregated exchange from an upward sesquitertia 3:4 step into a downward sesquialtera 3:2 motion (8 groups of 2)–>(12 groups of 2).

The bottom right-hand field contains a full chord type description with inversion letter appended in brackets – ‘a’ for root position, ‘b’ first inversion, etc. This fills out the abbreviated description given next to the measure number, and like the duplicated conjunction values in the top left-hand field they are provided to cut down left-right scrolling, given the line lengths of *txt2mos* data files.

The top right-hand fields and the fourth right-hand fields are concerned with conjunctions and the mutable base numbers (MBN) they define. Handling these fields and the associated ‘-k’ switch require some thought and care. As described earlier the technique of parallel counting can be used to discover the points of conjunction between two harmonic series, and that this process reveals a unending sequence of alignments defined as a ‘conjunction series’. The *txt2mos* script seeks the most appropriate conjunction series for the surface level aggregated series (which encapsulates the harmony) and records it in the nested series field (second line) as a sequence of zero or more lower-case ‘c’ terminating in an upper-case ‘C’ – the chosen conjunction. Thus if the chosen conjunction is the fundamental of the conjunction series, only a ‘C’ will be present and if not, one or more ‘c’ will precede it.

The first of the two top right fields contains the chosen conjunction for the current slice expressed in terms of the previous nested series and the current nested series with note letter appended – i.e. all reflect the position of ‘C’ in the current slice. This is followed by the two mutable numbers (in factor format) that encapsulate the whole exchange from the absolute fundamental period (H1) of the system, up to the chosen conjunction (e.g. `MBN 1x16x2x6 -> 1x12x2x8`). The structure of the first mutable number expresses the conjunction value in terms of the previous chord and the structure of the second mutable number expresses the same value in terms of the current chord. In this way the change in these two mutable number digit sequences, expressing one self-same value, encompasses both the coherence and variety of harmonic progression – two different structures or inflections of musical sound apprehended by the ear but capable of being interpreted by aural cognition as a unity through their common conjunctions.

Current Conjunction (counted in previous and current nested series harmonics, plus note letter)

Partials Generated by Chords '*'

Mutable Base Numbers of Whole Exchange (factor format)

```
*****-*-*******-*-*--*----*-------:h40h20-A:MBN 1x8x5x8 -> 1x16x2x10
|||||/|/||/|//|/|/|///////////////:(20xtwo)->(20xone)
**********-*--*-*-***--*----*-----:(8 grps of 5)->(10 grps of 2)
#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h20h10-A:MBN 1x8x5x4 -> 1x16x2x5
*                                 :F-major added sixth (b)
```

Nested Series Exchange (by degree)

Aggregated Series Exchange
(in groups of nested series harmonics)

Mutable Base Numbers of Lowest
Conjunction Exchange (factor format)

Conjunctions Between Partials '|'

Long Chord Description

Lowest Conjunction (counted in previous and current nested series harmonics, plus note letter)

Figure F.7b Slice 7.625 continued, the anatomy of a slice, all fields labelled.

The fourth down right-hand fields match the top fields and perform the same functions as described in the previous paragraph but in terms of the first occurring conjunction, the fundamental of the conjunction series. On occasions the top and fourth right fields will be identical, where the chosen conjunction ‘C’ is the fundamental of the conjunction series, but mostly they will be different. The reason for including the fundamental as well as the chosen conjunction is that the fundamental conjunction is solely determined by the dynamics of the exchange itself and thus is independently derived. However, occasionally a frequency matching the fundamental conjunction will not actually be present in the objective musical sound, and under such circumstances it becomes something of an abstraction. While in contrast, though somewhat more arbitrary, the chosen conjunction seeks out an objective partial (or top note) and so is always real and concrete, as well as being more encompassing – an outgrowth of the chords involved in the harmonic exchange. Ultimately of course for the ear, the whole conjunction series within aural range is available to link together succeeding chords, there is no reason for limiting the selection to any one particular frequency – excepting that these frequencies should exist in the sound (or perhaps occasionally be subjectively contrived in the form of combination tones, etc.) – and equally, be capable of detection by the hearing mechanism. Here also the effect of mutual interference on the ear’s detector membrane is perhaps a significant factor in limiting the useful range of the theoretically unending sequence of conjunction.

A close inspection of the amended mutable number values in Figure 6a/b reveals that they are now much larger than in Figure F.3a/b, this is a ramification of the extension of the nested series in slice 2.125 down to MIDI note number F5. The *txt2mos* script automatically calculates the shortest absolute fundamental period (H1) of the entire system – the ‘natural period’ – and records it, with other information, in a ‘header’ section. The original calculation for the chorale *Ach Gott und Herr* yields a fundamental period equivalent to MIDI note number F–19, that is F negative 19. The script calculates all the mutable numbers in terms of this natural period unless instructed otherwise with the -k switch.

The MIDI note F5 in slice 2.125, the amended h1 tone of the nested series, lies two octaves above F–19, and so in terms of the fundamental series (i.e. F-H1, F-H2, C-H3, F-H4, ..., Hn) constructed upon this absolute unit, the nested series can be considered as being built on F-H4(h1). This is all well and good for slice 2.125, but in the next slice 2.250 the nested series steps a tone up to MIDI note G7, which expressed in terms of the fundamental series yields a non-whole harmonic H4.5(h1). This is incompatible with a nested system built exclusively on whole numbered relationships. The solution to this conundrum is to adjust the absolute fundamental unit (H1) down to a level at which it regains its whole numbered relationship with the all elements within the system. This status is achieved at MIDI note F–31, F negative 31, one octave lower; and a knock-on effect of defining a lower fundamental unit, while maintaining or raising the positions of the conjunctions in the analysis, as illustrated in Figure F.6, is to greatly inflate the values represented by the mutable base numbers. This new unit (H1), F–31, applies to all numbers from start to finish – as all are ruled by, and united in, that single absolutely fundamental period.

The -k switch allows an arbitrary fundamental unit to be chosen. Although the *txt2mos* script will always calculate and record the 'natural period' of the input music data, often this will not be the same as the key/tonal center. For example, the input music data may have an untypical harmonic profile compared to a clearly defined key overall or editing may have been carried out, as described for slice 2.125. There are four possibilities:

1) If the -k switch is absent from the *txt2mos* command the natural period is used as the unit (H1) fundamental.
2) If the -k switch is used without a parameter attached, the *txt2mos* script will read the last occurring key signature data in the file and set the unit (H1) fundamental to the key letter (pitch class) for major keys or to a major-third below the key letter for minor keys. An appropriate frequency level for the unit fundamental will be chosen so as to maintain whole numbered relationships throughout.

3a) The -k switch can also be used with either of two parameters: a positive number in the pitch class range C–B, 0 to 11 or a negative MIDI note number. The former positive numbers 0 through 11 can be used where no key signature data is in the file, or alternatively, to override key signature data that is in the file. Care is needed: all key-tones are viewed as single acoustic entities (e.g. Gflat major is -k6, i.e. F#) and accommodation should normally be made for the minor key nesting in a series a major-third below its key-letter pitch class (e.g. G minor is -k3, i.e. D#).

3b) Negative MIDI note numbers in the range –1 through –60 (written -k-n, hyphen for minus sign and no white space) are used to explicitly set the key-letter pitch class and frequency level of the unit fundamental (e.g. -k-24, sets the unit to C two octaves below MIDI note number zero). Count down chromatically from C zero to find the number required: C0, B–1, A#–2, A–3, etc. In the highly unlikely situation that a non-negative MIDI note level is required one of the earlier options should achieve this.

Unfortunately, determining the right setting for this switch will often only become clear after considerable editing of the *txt2mos* output and re-running *txt2mos* with a new -k setting will erase many of the changes that have been made. In this situation the script *newH1* can be used to set a new unit H1 and automatically recalculate the absolute fundamental series as represented in the factor format mutable base numbers in the *txt2mos* file. The -k switch as above and a help option are available with the *newH1* command:

```
[perl] newH1 [-h -k<n>] amended_txt2mos_output [> myfile]
```

Example

```
-k-24  reset absolute fundamental H1 and recalculate MBN, default natural period
```

If the negative MIDI note number parameter is not low enough to allow for whole numbered relationships the script exits after printing a warning message quoting the lowest nested series h1 that is incompatible with the chosen absolute fundamental H1.

Step 6

Run the mos2col script to turn the amended MOS analysis into columns of nested harmonic series ready for entry into a printed score analysis. This last step is routine; having made all the desired changes to the *txt2mos* output file, the amended text file is turned into columnar text, as shown in Figure F.8.

There are two options and a help switch used with the *mos2col* command:

```
[perl] mos2col [-f -h -p<n>] amended_txt2mos_output [> myfile]
```

Example

```
-f        fixed equal-temperament frequency grid
-p1.022   adjust the pitch standard to A=440Hz, default pitch middle C=256Hz
```

The -p switch is used to adjust the written pitch in the column data to whatever level is required. The default (or with the switch -p1) sets a pitch level of middle C=256Hz. By giving the -p switch a decimal fraction greater or lesser than 1, the pitch level may be adjusted up or down. As in the example, 256 × 1.022 = 261.6, approximately the pitch of middle C at A=440Hz. Longer fractional expansions may be used if greater accuracy is required, however, it should be noted that quite soon any normally complex harmonic progression will ‘wander’ away from its opening pitch level as the algorithm of symmetrical exchange computes the whole number ratios. If on the other hand a fixed frequency grid is desired, the -f switch can be used to force the output to adhere to equal-temperament at whatever pitch level is chosen.

In Figure F.8 the nested harmonic system found by the *txt2mos* script and illustrated in Figure F.6 are presented, as generated by the *mos2col* script, in the form of lists of frequencies with ideal harmonics attached. Additionally, asterisks to the left and right mark the incoming and outgoing modulation exchanges of the aggregated series, and hyphens chart the degree of exchanges for the the nested series –i.e. primary, secondary, tertiary, etc. The chosen conjunction frequencies are marked by an arrow ‘>’. Below each chord list, the slice/measure number is recorded, followed by the exchange that generated the current aggregated series from the previous aggregated series and the current nested series from the previous nested series. In this regard each slice list (like the slice rows generated by *txt2mos*) are backward looking, that is, the information within the first brackets refers to the previous slice. At the bottom the slice tick, chord description and two mutable number exchanges are given. The slices are produced, in order, in the form of a continuous text file. From this *mos2col* output file each individual slice list and other information may be copied and pasted into a score analysis, as illustrated in Figure F.9.

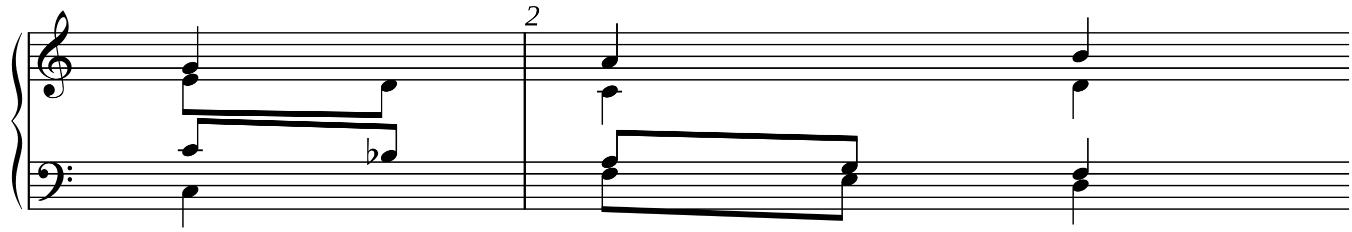
From this point on, the final assembly of the MOS analysis will depend to a considerable extent upon the analyst’s particular mode of working, whether by hand or by computer, and if by computer what score editor and graphics/dtp program(s) are being used and the facilities they offer. However, before adding any refinements to an interpretation, it is a good idea to enter all the lists into the score and roughly work through the analysis, checking aggregate groups, conjunctions, exchanges, etc. In this way a ‘sketch’ analysis may be relatively quickly produced, printed off and studied at leisure. Almost inevitably many shortcomings will become apparent: positions where the fundamental tone (h1) of the nested series needs to fall instead of rise or vice versa, better conjunctions more appropriate to the overall logical flow of the analysis, narrow exchanges that need fine tuning and all manner of other refinements. After this first work through, a re-run of Steps 5 and 6 is usually required, and often several more! Under these circumstances the edited *txt2mos* file gradually diverges from the original *txt2mos* output as it comes more and more to reflect the analyst’s own thoughts and hearing of the piece. *Beware*, all this work can be lost if the edited file is not stored safely.

```
-* 384.0:G-(h06)*->
   320.0:E-(h05) -
 * 256.0:C-(h04)*-
-  192.0:G-(h03) -
 R 128.0:C-(h02)R-
    64.0:C-(h01/H48) -
 Measure:001.750

(4 grps of 2)-3:4->(3 grps of 2)
(2xfour)-3:4->(2xthree)

Tick:0000672
C-major (a)
MBN 1x36x2x4 -> 1x48x2x3
MBN 1x36x2x4 -> 1x48x2x3
------------------------


   512.0:C-(h16)*->
   480.0:B-(h15)
   448.0:A#(h14)*-
   416.0:A-(h13)
-* 384.0:G-(h12)*-
   352.0:F#(h11)
-* 320.0:E-(h10)*-
   288.0:D-(h09)
-* 256.0:C-(h08)*-
   224.0:A#(h07)
-* 192.0:G-(h06)*-
   160.0:E-(h05)
-* 128.0:C-(h04)*-
    96.0:G-(h03)
-R  64.0:C-(h02)R-
    32.0:C-(h01/H24)
 Measure:001.875

(3 grps of 2)-2:1->(6 grps of 2)
(6xone)-2:1->(6xtwo)

Tick:0000720
C-dominant ninth no third (a)
MBN 1x48x2x3 -> 1x24x2x6
MBN 1x48x2x1 -> 1x24x2x2
------------------------


-* 512.0:C-(h24)
   490.7:--(h23)
   469.3:B-(h22)
-  448.0:A#(h21)
 * 426.7:A-(h20)*->
   405.3:G#(h19) -
-  384.0:G-(h18) -
   362.7:F#(h17) -
 * 341.3:F-(h16)*-
-  320.0:E-(h15) -
   298.7:D#(h14) -
   277.3:D-(h13) -
-* 256.0:C-(h12)*-
   234.7:B-(h11) -
   213.3:A-(h10) -
-  192.0:G-(h09) -
 * 170.7:F-(h08)*-
   149.3:D#(h07) -
-  128.0:C-(h06) -
   106.7:A-(h05) -
 R  85.3:F-(h04)R-
-   64.0:C-(h03) -
    42.7:F-(h02) -
    21.3:F-(h01/H16) -
 Measure:002.000

(8 grps of 2)-3:4->(6 grps of 4)
(8xtwo)-3:2->(8xthree)

Tick:0000768
F-major (a)
MBN 1x24x2x8 -> 1x16x4x6
MBN 1x24x2x4 -> 1x16x4x3
------------------------
```

Figure F.8 Measure/Slice 1.750 through 2.250 of *Ach Gott und Herr*, output by *mos2col* and viewed in a text editor.

```
   853.3:A-(h80)*->
   842.7:--(h79)
   832.0:--(h78)
   821.3:--(h77)
   810.7:G#(h76)
   800.0:G#(h75)*
   789.3:--(h74)
   778.7:--(h73)
   768.0:G-(h72)
   757.3:--(h71)
   746.7:G-(h70)*-
   736.0:--(h69)
   725.3:F#(h68)
   714.7:--(h67)
   704.0:--(h66)
   693.3:--(h65)*
   682.7:F-(h64)
   672.0:F-(h63)
   661.3:--(h62)
   650.7:--(h61)
   640.0:E-(h60)*-
   629.3:--(h59)
   618.7:--(h58)
   608.0:D#(h57)
   597.3:D#(h56)
   586.7:--(h55)*
   576.0:D-(h54)
   565.3:--(h53)
   554.7:D-(h52)
   544.0:--(h51)
   533.3:C#(h50)*-
   522.7:--(h49)
   512.0:C-(h48)
   501.3:--(h47)
   490.7:--(h46)
   480.0:B-(h45)*
   469.3:B-(h44)
   458.7:--(h43)
   448.0:A#(h42)
   437.3:--(h41)
-* 426.7:A-(h40)*-
   416.0:--(h39)
-  405.3:G#(h38)
   394.7:--(h37)
-  384.0:G-(h36)
 * 373.3:G-(h35)*
-  362.7:F#(h34)
   352.0:--(h33)
-  341.3:F-(h32)
   330.7:--(h31)
-* 320.0:E-(h30)*-
   309.3:--(h29)
-  298.7:D#(h28)
   288.0:D-(h27)
-  277.3:D-(h26)
 * 266.7:C#(h25)*
-  256.0:C-(h24)
   245.3:--(h23)
-  234.7:B-(h22)
   224.0:A#(h21)
-* 213.3:A-(h20)*-
   202.7:G#(h19)
-  192.0:G-(h18)
   181.3:F#(h17)
-  170.7:F-(h16)
 * 160.0:E-(h15)*
-  149.3:D#(h14)
   138.7:D-(h13)
-  128.0:C-(h12)
   117.3:B-(h11)
-* 106.7:A-(h10)*-
    96.0:G-(h09)
-   85.3:F-(h08)
    74.7:D#(h07)
-   64.0:C-(h06)
 R  53.3:A-(h05)R
-   42.7:F-(h04)
    32.0:C-(h03)
-   21.3:F-(h02)
    10.7:F-(h01/H8)
 Measure:002.125

(5 grps of 4)-8:5->(8 grps of 5)
(20xone)-2:1->(20xtwo)

Tick:0000816
A-minor seventh (c)
MBN 1x16x4x5 -> 1x8x5x8
MBN 1x16x4x5 -> 1x8x5x8
```

```
-* 853.3:A-(h72)
   841.5:--(h71)
   829.6:A-(h70)
   817.8:--(h69)
 * 805.9:G#(h68)
   794.1:--(h67)
   782.2:--(h66)
   770.4:--(h65)
 * 758.5:G-(h64)
-  746.7:G-(h63)
   734.8:--(h62)
   723.0:--(h61)
 * 711.1:F#(h60)
   699.3:--(h59)
   687.4:--(h58)
   675.6:F-(h57)
 * 663.7:F-(h56)
   651.9:--(h55)
-  640.0:E-(h54)
   628.1:--(h53)
 * 616.3:E-(h52)
   604.4:--(h51)
   592.6:D#(h50)
   580.7:--(h49)
 * 568.9:D-(h48)*->
   557.0:--(h47)
   545.2:--(h46)
-  533.3:C#(h45)
 * 521.5:C#(h44)*-
   509.6:--(h43)
   497.8:C-(h42)
   485.9:--(h41)
 * 474.1:B-(h40)*-
   462.2:--(h39)
   450.4:A#(h38)
   438.5:--(h37)
-* 426.7:A-(h36)*-
   414.8:A-(h35)
   403.0:G#(h34)
   391.1:--(h33)
 * 379.3:G-(h32)*-
   367.4:--(h31)
   355.6:F#(h30)
   343.7:--(h29)
 * 331.9:F-(h28)*-
-  320.0:E-(h27)
   308.1:E-(h26)
   296.3:D#(h25)
 * 284.4:D-(h24)*-
   272.6:--(h23)
   260.7:C#(h22)
   248.9:C-(h21)
 * 237.0:B-(h20)*-
   225.2:A#(h19)
-  213.3:A-(h18)
   201.5:G#(h17)
 * 189.6:G-(h16)*-
   177.8:F#(h15)
   165.9:F-(h14)
   154.1:E-(h13)
 * 142.2:D-(h12)*-
   130.4:C#(h11)
   118.5:B-(h10)
-  106.7:A-(h09)
 *  94.8:G-(h08)*-
    83.0:F-(h07)
    71.1:D-(h06)
    59.3:B-(h05)
 R  47.4:G-(h04)R-
    35.6:D-(h03)
    23.7:G-(h02)
    11.9:G-(h01/H9)
 Measure:002.250

(16 grps of 5)-9:8->(18 grps of 4)
(8xten)-9:10->(8xnine)

Tick:0000864
G-rootless dominant seventh (c)
MBN 1x8x5x16 -> 1x9x4x18
MBN 1x8x5x8 -> 1x9x4x9
------------------------
```



KEY

Written notes underlined

Aggregated Series marked by * asterisks.

Nested Series given as full lists; the degrees of exchages (i.e. primary, secondary, ..., nth-fold) are indicated to left and right by - hyphens

The Fundamental Nesting Series is not shown, except for the connections (/Hn) to the Nested Series. Fundamental F-H1 = 1.3Hz (approx), MIDI –31.

Conjunctions marked in gray with arrows '->' and chord roots indicated by the letter 'R'

```
-* 384.0:G-(h06)*------->
   320.0:E-(h05) -
 * 256.0:C-(h04)*-
-  192.0:G-(h03) -
 R 128.0:C-(h02)R-
    64.0:C-(h01/H48)-
 Measure:001.750
```

```
    512.0:C-(h16)*------->
    480.0:B-(h15)
    448.0:A#(h14)*-
    416.0:A-(h13)
-*  384.0:G-(h12)*-
    352.0:F#(h11)
-*  320.0:E-(h10)*-
    288.0:D-(h09)
-*  256.0:C-(h08)*-
    224.0:A#(h07)
-*  192.0:G-(h06)*-
    160.0:E-(h05)
-*  128.0:C-(h04)*-
     96.0:G-(h03)
-R   64.0:C-(h02)R-
     32.0:C-(h01/H24)
 Measure:001.875
```

```
-*  512.0:C-(h24)
    490.7:--(h23)
    469.3:B-(h22)
-   448.0:A#(h21)
 *  426.7:A-(h20)*------->
    405.3:G#(h19) -
-   384.0:G-(h18) -
    362.7:F#(h17) -
 *  341.3:F-(h16)*-
-   320.0:E-(h15) -
    298.7:D#(h14) -
    277.3:D-(h13) -
-*  256.0:C-(h12)*-
    234.7:B-(h11) -
    213.3:A-(h10) -
-   192.0:G-(h09) -
 *  170.7:F-(h08)*-
    149.3:D#(h07) -
-   128.0:C-(h06) -
    106.7:A-(h05) -
 R   85.3:F-(h04)R-
-    64.0:C-(h03) -
     42.7:F-(h02) -
     21.3:F-(h01/H16)-
 Measure:002.000
```

```
    853.3:A-(h80)*------>
    842.7:--(h79)
    832.0:--(h78)
    821.3:--(h77)
    810.7:G#(h76)
    800.0:G#(h75)*
    789.3:--(h74)
    778.7:--(h73)
    768.0:G-(h72)
    757.3:--(h71)
    746.7:G-(h70)*-
    736.0:--(h69)
    725.3:F#(h68)
    714.7:--(h67)
    704.0:--(h66)
    693.3:--(h65)*
    682.7:F-(h64)
    672.0:F-(h63)
    661.3:--(h62)
    650.7:--(h61)
    640.0:E-(h60)*-
    629.3:--(h59)
    618.7:--(h58)
    608.0:D#(h57)
    597.3:D#(h56)
    586.7:--(h55)*
    576.0:D-(h54)
    565.3:--(h53)
    554.7:D-(h52)
    544.0:--(h51)
    533.3:C#(h50)*-
    522.7:--(h49)
    512.0:C-(h48)
    501.3:--(h47)
    490.7:--(h46)
    480.0:B-(h45)*
    469.3:B-(h44)
    458.7:--(h43)
    448.0:A#(h42)
    437.3:--(h41)
-*  426.7:A-(h40)*-
    416.0:--(h39)
-   405.3:G#(h38)
    394.7:--(h37)
-   384.0:G-(h36)
 *  373.3:G-(h35)*
-   362.7:F#(h34)
    352.0:--(h33)
-   341.3:F-(h32)
    330.7:--(h31)
-*  320.0:E-(h30)*-
    309.3:--(h29)
-   298.7:D#(h28)
    288.0:D-(h27)
-   277.3:D-(h26)
 *  266.7:C#(h25)*
-   256.0:C-(h24)
    245.3:--(h23)
-   234.7:B-(h22)
    224.0:A#(h21)
-*  213.3:A-(h20)*-
    202.7:G#(h19)
-   192.0:G-(h18)
    181.3:F#(h17)
-   170.7:F-(h16)
 *  160.0:E-(h15)*
-   149.3:D#(h14)
    138.7:D-(h13)
-   128.0:C-(h12)
    117.3:B-(h11)
-*  106.7:A-(h10)*-
     96.0:G-(h09)
-    85.3:F-(h08)
     74.7:D#(h07)
-    64.0:C-(h06)
 R   53.3:A-(h05)R
-    42.7:F-(h04)
     32.0:C-(h03)
-    21.3:F-(h02)
     10.7:F-(h01/H8)
 Measure:002.125
```

```
-*  853.3:A-(h72)
    841.5:--(h71)
    829.6:A-(h70)
    817.8:--(h69)
 *  805.9:G#(h68)
    794.1:--(h67)
    782.2:--(h66)
    770.4:--(h65)
 *  758.5:G-(h64)
-   746.7:G-(h63)
    734.8:--(h62)
    723.0:--(h61)
 *  711.1:F#(h60)
    699.3:--(h59)
    687.4:--(h58)
    675.6:F-(h57)
 *  663.7:F-(h56)
    651.9:--(h55)
-   640.0:E-(h54)
    628.1:--(h53)
 *  616.3:E-(h52)
    604.4:--(h51)
    592.6:D#(h50)
    580.7:--(h49)
 *  568.9:D-(h48)*---------->
    557.0:--(h47)
    545.2:--(h46)
-   533.3:C#(h45)
 *  521.5:C#(h44)*-
    509.6:--(h43)
    497.8:C-(h42)
    485.9:--(h41)
 *  474.1:B-(h40)*-
    462.2:--(h39)
    450.4:A#(h38)
    438.5:--(h37)
-*  426.7:A-(h36)*-
    414.8:A-(h35)
    403.0:G#(h34)
    391.1:--(h33)
 *  379.3:G-(h32)*-
    367.4:--(h31)
    355.6:F#(h30)
    343.7:--(h29)
 *  331.9:F-(h28)*-
-   320.0:E-(h27)
    308.1:E-(h26)
    296.3:D#(h25)
 *  284.4:D-(h24)*-
    272.6:--(h23)
    260.7:C#(h22)
    248.9:C-(h21)
 *  237.0:B-(h20)*-
    225.2:A#(h19)
-   213.3:A-(h18)
    201.5:G#(h17)
 *  189.6:G-(h16)*-
    177.8:F#(h15)
    165.9:F-(h14)
    154.1:E-(h13)
 *  142.2:D-(h12)*-
    130.4:C#(h11)
    118.5:B-(h10)
-   106.7:A-(h09)
 *   94.8:G-(h08)*-
     83.0:F-(h07)
     71.1:D-(h06)
     59.3:B-(h05)
 R   47.4:G-(h04)R-
     35.6:D-(h03)
     23.7:G-(h02)
     11.9:G-(h01/H9)
 Measure:002.250
```

Aggregated Series

(3 groups of 2) ----2:1----> (6 groups of 2)

(8 groups of 2) ----3:4----> (6 groups of 4)

(5 group of 4) --8:5--> (8 groups of 5)

(16 groups of 5) ---9:8---> (18 groups of 4)

Nested Series

(6×one) --------2:1-------> (6×two)

(8×two) --------3:2--------> (8×three)

(20×one) ------2:1------> (20xtwo)

(8×ten) -------9:10------> (8×nine)

MBN $3_2 0_{48} 0_1$ -----------------> $6_2 0_{24} 0_1$

$8_2 0_{24} 0_1$ --------------------> $6_4 0_{16} 0_1$

$5_4 0_{16} 0_1$ ------------------> $8_5 0_8 0_1$

$16_5 0_8 0_1$ --------------------> $18_4 0_9 0_1$

Figure F.9 MOS analysis.

General Comments and Conclusion

Returning to the topic of the absolute fundamental period, earlier in the text it was noted that the MOS approach crystallized a connection between chord IV and chord vi, which in the key of C major would be between the F-major chord and the A-minor chord. Effectively, in the MOS approach an ‘A-minor’ aggregated series nests, in groups of five, within a ‘F-major’ nested series – see amended slice 2.125. Also, intriguingly, it has emerged from the description of mutable numbers in relation to the fundamental period of the whole system, that the example chorale in the key of C major has a ‘natural’ fundamental period equivalent to the pitch class F. Indeed, any normal or average composition in C major will tend to a F pitch class fundamental period because the subdominant scale degree’s nearest equivalent lies so far up a C-based harmonic series – C-h1 through F-h21 – that it pushes the natural fundamental period down the cycle of fifths toward F. That is to say, the shortest fundamental period is generally obtained by positioning the three principal scale degrees thus: IV-h1, I-h3 and V-h9 (rather than I-h1, V-h3, IV-h21). Although it may seem odd and difficult to accept that the shortest fundamental period, the intrinsic unit of a composition, is often not the key pitch class, this disturbing cloud has a silver lining, *the normal fundamental period of the major key is the same as the natural fundamental period of its relative minor*. Here there is unity, symmetry. But the underlying symmetry between the major and its relative minor encapsulated in a single fundamental period is broken at higher levels of nesting, when the harmonics within the middle level nested series break down into groups of four to encompass the major chord or groups of five for the minor chord. Overall, unity in maintained at the deep level of the fundamental series in a modulating oscillatory system but at the surface level of harmonic progression, it is somewhat obscured.

Upon the related topic of the fourth scale degree, while it is certainly open to disputed whether or not the ‘tolerance of the ear’ allows the subdominant to appear within the harmonic series, this difficulty is swept aside by the dynamic nature of modulating oscillatory systems. The elements or values within a MOS analysis are taken to be in a permanent state of flux, dependent only upon the outcome of the last executed modulation exchange. A modulating oscillatory system lives in the present moment, knowing only its own, most simply ordered, relationships and nothing of scales or other external metrics. The conjunction(s) – more precisely the conjunction series – that govern the exchange between the previous and present state of the system are the only elements that remains fixed during the course of an exchange, while all other relationships adjust to accommodate this rule. (This is somewhat analogous to the fixed speed of light in relativity theory enforcing a flexibility upon spacetime.) For example, in an exchange stepping from tonic to subdominant, from C to F, the value of the conjunction C (512Hz) remains constant,

```
512Hz:C-(h4)     ————————>  512.000Hz:C-(h3)
384Hz:G-(h3)                341.333Hz:F-(h2)
256Hz:C-(h2)            ->  170.666Hz:F-(h1/H21)
128Hz:C-(h1/H16)  3:4       ........:.....
.....:.....                 ........:.....
.....:.....                 ........:.....
  8Hz:C-(H1)                  8.127Hz:C-(H1)
```

while the value of the fundamental C (8Hz/8.127Hz) must flex to accommodate the newly minted relationships (see factor format numbers below Figure F.10a/b). Transposed to the realm of hearing, perhaps aural cognition is anchored and guided by the unchanging conjunction tone(s) in exchanges while equally the ear’s obliging tolerance accepts the flexing of the implied fundamental without demur.

Some further points:

1) The ratios of exchange shown on the extreme left of the second and third lines of a slice are expressed in the order denominator : numerator, in the sense that the frequency of the fundamental of the current nested series and aggregated series is obtained by multiplying the fundamental of the previous series by the numerator and dividing that product by the denominator.

2) In the upper reaches of the nested series, where the harmonic numbering disappears in the second line of a slice in the *txt2mos* output file, the number may be found by doubling from the same note-letter lower down the series.

3) Also, these higher note-letter positions may encompass more than one harmonic (e.g. B-h44/h45 D-h52/h54), where this occurs the *txt2mos* script will automatically choose the divisible number.

4) If a non-wholly divisible conjunction is selected when editing the *txt2mos* file the output from the *mos2col* script will show up the fault by producing decimal fractions for the nested series exchanges. For example:

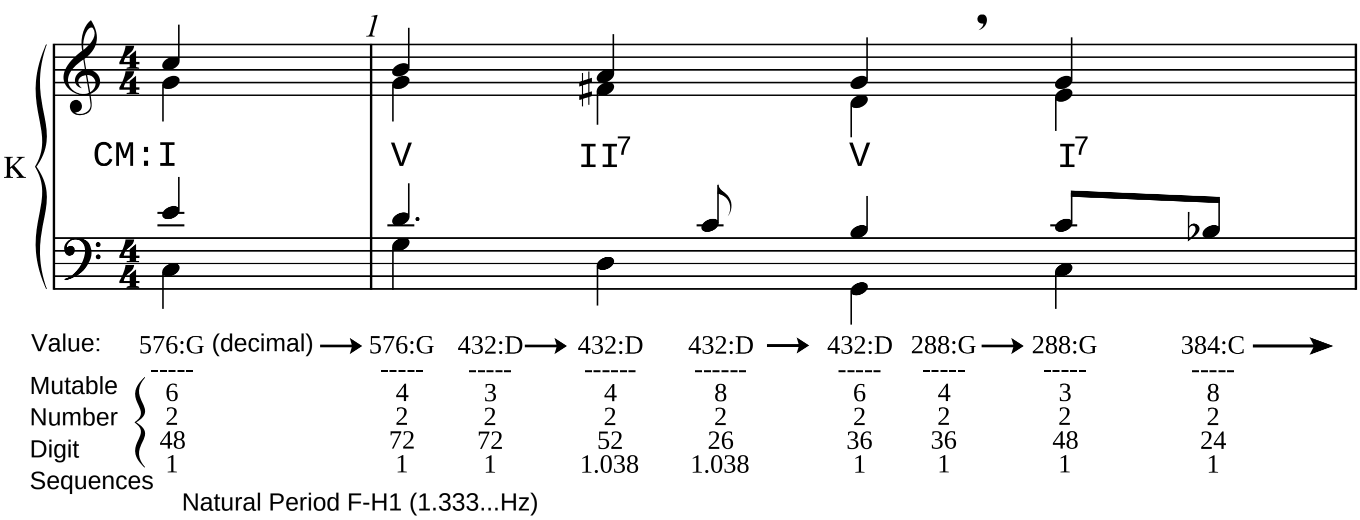
```
(5.33333333333333xfour)--3:4-->(5.33333333333333xthree)
```

5) Comments may be freely added to a *txt2mos* output file on lines above any slice but always beginning on the left margin with a hash '#' sign. This will ensure that the *newH1* script will preserve the comments and also that they will not interfere with the production of column data by the *mos2col* script. Additionally, an overview of the chords and conjunctions processed by the *txt2mos* script will be printed at the end of the file as a succession of comments if an -o switch is included in the command.

In conclusion, as mentioned before, constructing a MOS analysis does require considerable effort and attention to detail, however, once satisfactorily achieved, such an analysis reveals a broadly unique,[3] objective and mathematically consistent reading of the music – from which no little gratification may be drawn. There are, of course, many subtle limits and nuances in the structure of nested systems which must be respected if a truly rigorous and coherent account of tonal music is to be achieved. Yet notwithstanding these disciplines, the simple procedure of parallel counting between pairs of harmonic series is able to reconcile and elucidate the difficulties in determining which are, and which are not, consistent conjunctions. Although the example editing of the *txt2mos* file is undoubtedly rather intense and myopic – slice 2.125 could easily have been classified as passing notes lacking harmonic significance – as the MOS model is capable of penetrating to the lowest level of harmonic detail, it seemed desirable to demonstrate this capacity. Also, notwithstanding that the mountainous list of harmonics generated by slice 2.125 might appear excessive (displayed in Figures 8 and 9), from the ear's point of view, the conjunction sitting upon its apex is no more that the second harmonic of the top note in the slice 2.125 chord – often the most energetic harmonic in a complex tone. Hearing takes and processes only what evolution has determined it needs from objective sound sources. The detailed mathematical underpinning provided by the MOS model's 'acoustic radar' complements the ear's probably more intuitive and efficient flight path through the environment created by the performance of a musical composition. Finally, the complete edited section, matching the original output shown in Figure F.3a/b, is given below in Figure F.10a/b. However, to get fully to grips with MOS analysis the best tutor of all is simply doing it. Working at analyses until the ratios and conjunctions all slot into place and mostly could not be otherwise.

```
h6h12  :------------------------------------------------O-----------O---O--O----+---+--+--+++-
2:1    :                        1-----------2------3----c---5--6--7-c-9-0-1C-345|///|//|//|/|/
2:1    :------------------------------------------------O---------O---O----O--+-+-+-++-+-++-+-
0000720:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
001.875:C-Maj9                              R           *      *    *   *  *  * * * * ** *****


h16h24 :------------------------------------------------O---------O---O----O--+-+-+-++-+-++-+-
3:2    :                 h===========1----(h3)---2------3----4---5--c--7-8-9-0-1C-34||/|/|//|/
3:4    :-----------------------------------------------------O---O--O--------O--+---++-+-+--++
0000768:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.000:F-Maj                                    R           *      *    *   *  *  * * * * **


h20h40 :-----------------------------------------------------O---O--O--------O--+---++-+-+--++
2:1    :     1-----------2------3----4---5--6--7-8-9-0-12-3456//////|////////C//|///|//|/|//|/
8:5    :----------------------------------------------------O--O----O--------O-++-+-+--+++-++-
0000816:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.125:A-min7                           R           *      *    *   *  *  * * * * ** *****


h80h72 :----------------------------------------------------O--O----O--------O-++-+-+--+++-++-
9:10   :       h===========1-----------2------3----4---5--6--7-8-9-0-12-3456/c/||/|/|////C/||/
9:8    :--------------------------------------------------O--O--------O--------O+-+--++--+-++-
0000864:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.250:G-dom7                         R           *      *    *   *  *  * * * * ** *********

002.375: redundant slice skipped, Tick: 0000912

h48h36 :--------------------------------------------------O--O--------O--------O+-+--+++-+-++-
3:4    :            h===========h===========1-----------2------c----4---5--c--7-|-C-0-12-34||/
3:4    :------------------------------------------------O------O--------O-------O-+-+--+--+++-
0000960:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.500:C-Maj                               R           *      *    *   *  *  * * * * ** *****

002.625: redundant slice skipped, Tick: 0001008

h18h24 :------------------------------------------------O------O--------O-------O-+-+--+--+++-
3:4    :                   h===========1-----------2------3----c---5--6--7-c-9-0-1C-345|///|//
2:3    :-------------------------------------------O---------------O-------O------O--+++-+-+--
0001056:C#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#
002.750:G-Maj                                      R           *      *    *   *  *  * * * * *
```

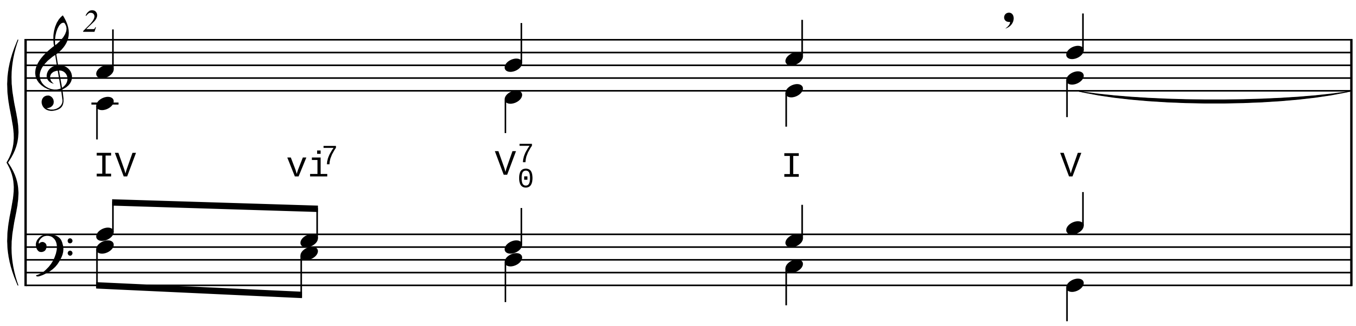
Figure F.10a The complete edited section, compared to the original shown in Figure F.3a/b. Changes to slice 2.125 have further knock-on effects for the position of the nested series in slices 2.500 and 2.750 before the analysis returns to that output by the *txt2mos* script.



```
+-+--++-+++-+-++++++-++-+-+--+------------:h6h12-G:MBN 1x48x2x3 -> 1x24x2x6
|/|//||/|||/|/||||/|/|//|////|////////////:(6xone)->(6xtwo)
+-+++++++++-+-++++-+++--+----+------------:(3 grps of 2)->(6 grps of 2)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h2h4-C:MBN 1x48x2x1 -> 1x24x2x2
****                                      :C-dominant ninth no third (a)


+-+++++++++-+-++++-+++--+----+------------:h16h24-C:MBN 1x24x2x8 -> 1x16x4x6
//||/|/||||/|/||/|/|/|////////////////////:(8xtwo)->(8xthree)
-+++-+-++++++-++-+-+-+++--+----+----------:(8 grps of 2)->(6 grps of 4)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h8h12-C:MBN 1x24x2x4 -> 1x16x4x3
*********                                 :F-major (a)


-+++-+-++++++-++-+-+-+++--+----+----------:h20h40-A:MBN 1x16x4x5 -> 1x8x5x8
//||/|/||||||/|//|/|/|||//|////|//////////:(20xone)->(20xtwo)
+-+++++++++++-+--+-+-+++--+----+----------:(5 grps of 4)->(8 grps of 5)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h20h40-A:MBN 1x16x4x5 -> 1x8x5x8
****                                      :A-minor seventh (c)


+-+++++++++++-+--+-+-+++--+----+----------:h80h72-A:MBN 1x8x5x16 -> 1x9x4x18
|/||||/|/||/|/|////|/|/|//////////////////:(8xten)->(8xnine)
++++++-+-++-+++++--+-+-+++--+----+--------:(16 grps of 5)->(18 grps of 4)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h40h36-A:MBN 1x8x5x8 -> 1x9x4x9
                                          :G-rootless dominant seventh (c)


++++++-+-++-++++++-+-+-+++--++---+-------+:h48h36-D:MBN 1x9x4x12 -> 1x12x4x9
|/||/|/|/||/|/|/|////|//|/////////////////:(12xfour)->(12xthree)
+-++-+++-++-+-+-+++-+++-+-+--+----+-------:(12 grps of 4)->(9 grps of 4)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h16:h12-G:MBN 1x9x4x4 -> 1x12x4x3
****                                      :C-major (a)


+-++-+++-++-+-+-+++-+++-+-+--+----+-------:h18h24-D:MBN 1x12x4x9 -> 1x18x4x6
|////|/|/|//|///||///||/|/|//|////////////:(12xthree)->(12xtwo)
++--++-+-+-+++-+++-+-++-+-+-++-+----+-----:(9 grps of 4)->(6 grps of 4)
D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G#A#BC#D#EF#G:h12:h8-G:MBN 1x12x4x3 -> 1x18x4x2
* *********                               :G-major (a)
```

Figure F.10b Continued across the page from Figure F.10a, the edited right-hand fields of slices 1.875 to 2.750.

Below: pages 26 and 27, measures 1 and 2 of *Ach Gott und Herr* with factor format mutable numbers appended.



→ 384:C	320:A→	320:A	640:A→	640:A	432:D →	432:D →	432:D	640:A →	
-----	-----	-----	-----	-----	-----	-----	-----	-----	
6	5	8	16	18	12	9	6	9	
4	4	5	5	4	4	4	4	4	
16	16	8	8	9	9	12	18	18	
1	1	1	1	0.998	1	1	1	0.998	

Notes

1. In the rare situation where a different division (e.g. 120 ppq) has been used but is not recorded in the MIDI data the -d switch can be used.

2. An exact conjunction could be manufactured between a sesquioctava 9:8 exchange in the aggregated series and near sesquioctava primary 80:72 (8.888...:8) exchange in the underlying nested series or perhaps the slightly misaligned conjunction employed at 81:72 (9:8) for the nested series. Both of these solutions lack elegance.

3. By ‘broadly unique’ I mean that two or more MOS analyses of unambiguously tonal compositions, worked out independently, would inevitably arrive at essentially the same configurations of mutable number digit sequences overall – nothing else will fit. However, the analyses would probably produce occasionally different readings for ambiguous chord progressions, but such deviations would soon re-combine once the flow of the harmony again becomes clear.