Journey to the Heart of Music

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Chapter 16 – Appendix A

MUSIC THEORY TOOLKIT

Set out below are some of the basic elements of music notation and terminology which might be helpful and relevant to the reader, followed by a short discussion of the central role played by the dominant-tonic progression (chords V^7-> I) in tonal music – and, perhaps, in oscillatory systems in general. Some of the larger tables are intended principally for reference.

Ratio

The foundations of music lie in the ratios of the harmonic series, which I have argued in *Journey to the Heart of Music,* are in effect the digits of nature's own intrinsic number system.

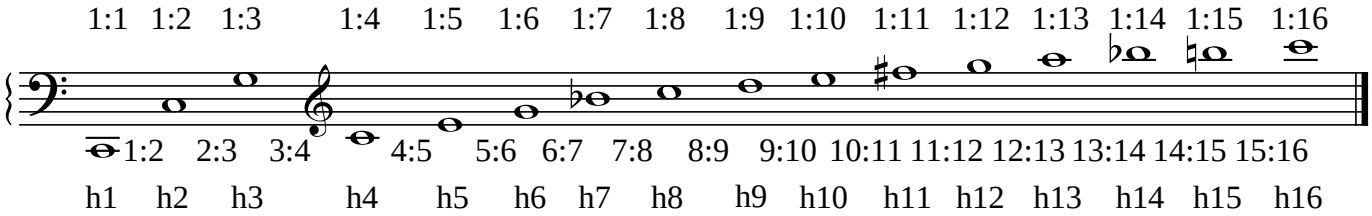


Figure 16.1 Harmonic Series: ratios of the first sixteen frequencies of the series are shown above the stave, with the ratios between adjacent harmonics marked below the stave and bottom the conventional shorthand 'h1' for the fundamental, to 'h16'.

Of nature's bountiful raw material, musician's have selected a number of basic ratios from the beginning of the harmonic series and constructed the system of tonal music out of these relationships. It has been recognised from earliest times that the basis of music lies in *ratio;* and that the units of time and pitch which form the building blocks of music are those made by durations and intervals which can be expressed in simple whole number relationships.

Notes

The principal note lengths – relative time/duration values – are found by the systematic application of the two most basic, extended, ratios (i.e. above unity, 1:1) of the harmonic series – 1:2 and 2:3.

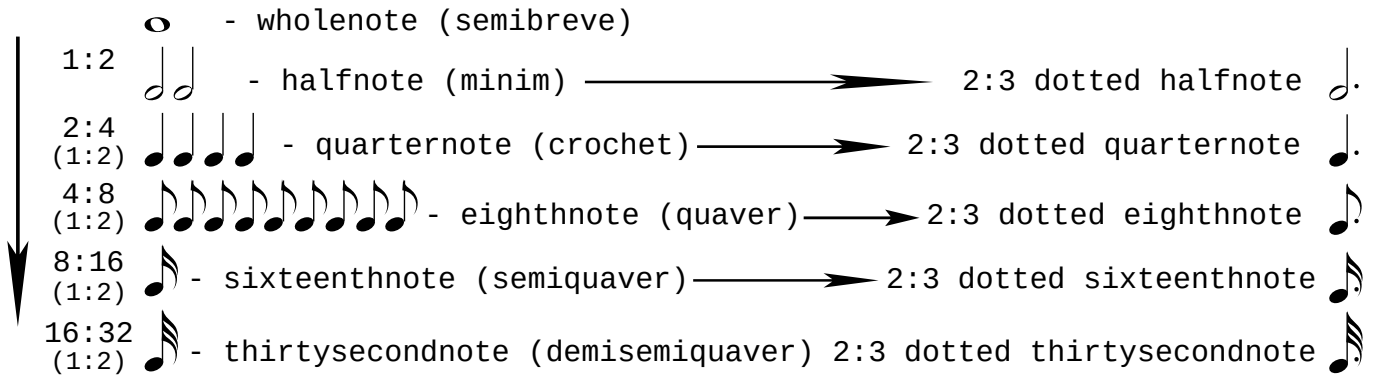


Figure 16.2 The relative note durations in music are derived from the most fundamental ratios of the harmonic series.

Notes are written on the *staff* and given the letter-names: C, D, E, F, G, A, B. The pitch or frequency of a note can be altered by an upward shift of a semitone with a sharp sign '#', for example C#, or lowered by a semitone with a flat sign.

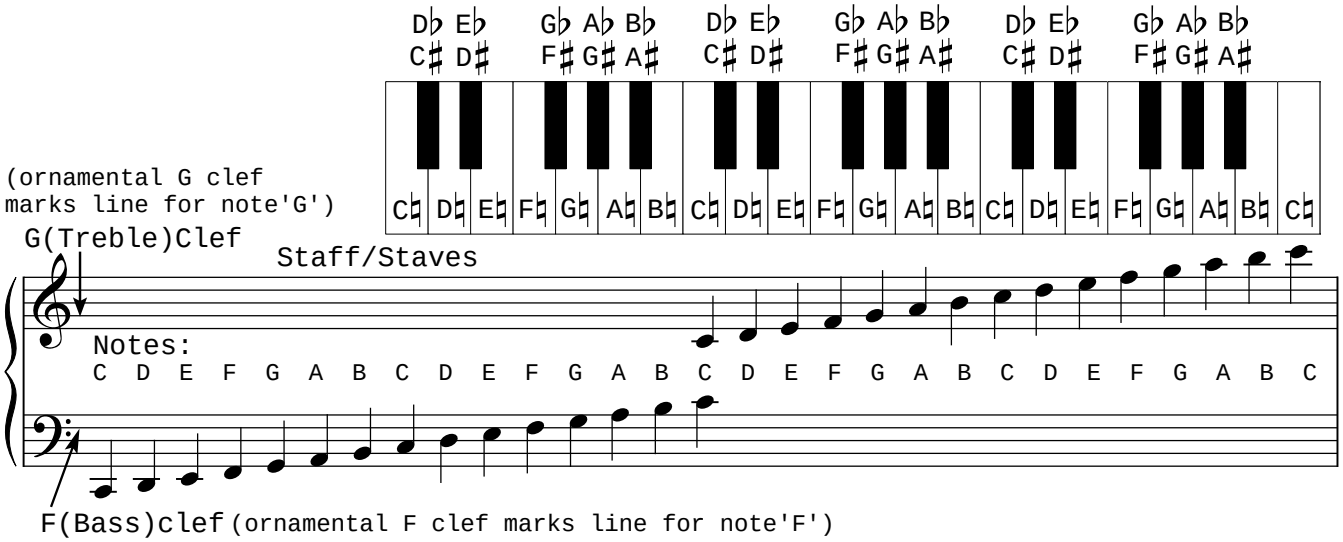


Figure 16.3 Note letter-names and their position on the staff and keyboard. The 'G' and 'F' clefs, at the beginning of the staves, fix the line used for the notes G and F.

The illustrations shown here (and generally throughout the book) focus of the *generic* key and scale of 'C'. However, scales and keys can be built on any of the notes by altering selected staff lines or spaces by applying sharps or flats as *key signatures* after the clef signs.

Figure 16.4 Three major scale and three key signatures: C-major, E-major and Eflat-major.

Scales

The scales used by musician's as the practical basis of melody are generated by manipulating the first few intervals (ratios) of the harmonic series – the octave, fifth and fourth, see Figure 16.1 – by a process of stepping up a fifth (2:3) and down a fourth (4:3), in succession, until the octave (1:2) has been filled.

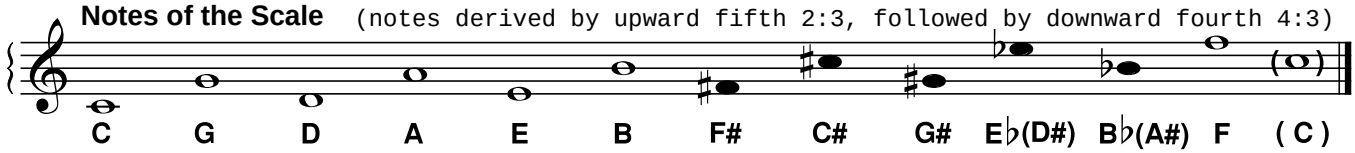
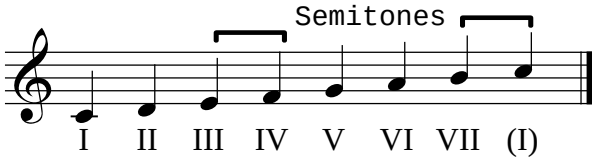


Figure 16.5 A twelve note scale (the 'chromatic' scale of semitones) with the normal major scale (the 'diatonic' scale of mostly tones) in white notes. Though predominantly whole tones, diatonic scales do contain two semitone; in major scales the semitone intervals are between the third and fourth notes E-F and the seventh and eighth B-C.

The pattern of spacing between the notes for a major scale is: (whole)tone, tone, semitone, tone, tone, tone, semitone – C D E F G A B C. And although the minor scale and others are recognised in theory, in practice music often slips back and forth between these different *modes* as if they are no more than inflections of a single tonal framework.



I - Tonic
II - Supertonic
III - Mediant
IV - Subdominant
V - Dominant
VI - Submediant
VII - Leading Note

Figure 16.6 The seven degrees of the C-major scale, plus octave. (Intriguingly, the same Roman numeral notation is used in the Periodic Table of the Elements.)

The tonic or first degree of a scale provides a tonal *center of gravity*. A scale built on C, as in Figure 16.6 is defining the key or tonal centre of C (major), it marks out the extent or area of the C major key by the closeness or distance of ratios to the 'home' frequency of the tonic note C. For example, the note G in the key of C major has a close (3:2) relationship to C, while the note D's relationship is a more distant ratio 9:8.

Intervals

In essence an interval is the space or *relational distance* between two notes (frequencies). While any sized ‘space’ is possible, in fact, only intervals of notes with (approximately) simple whole number ratios between their frequencies (rates of vibration) are used. The interval C-F# of ratio 32:45 being the most complex.

Interval-Names	Ratio	Notes	Harmonic-Series	Frequency
Octave	1:2	C-C	h1-h2	256-512Hz
Fifth	2:3	C-G	h2-h3	256-384Hz
Fourth	3:4	C-F	h3-h4	256-341Hz
Major3rd	4:5	C-E	h4-h5	256-320Hz
Minor3rd	5:6	C-Eflat	h5-h6	256-307Hz
Major2nd	8:9	C-D	h8-h9	256-288Hz
Minor2nd	15:16	C-C#	h15-h16	256-274Hz
Unison	1:1	C-C	h1-h1	256-256Hz

Figure 16.7 Basic intervals as understood by the ear, with their relationship to the ratios of the harmonic series.

Intervals are named by the number of tones of the normal diatonic scale they contain and so are classified as: Unison, Second, Third, Fourth, Fifth, Sixth, Seventh and Octave. With each interval name being qualified by the prefixes: Diminished, Minor, Major, Perfect and Augmented indicating the differing mix of component wholetones and semitones (prefixes are often omitted, e.g. Fifth for Perfect Fifth). Usually intervals are measured upward, from lower to upper note but when measured downward they are described as ‘inverted’.

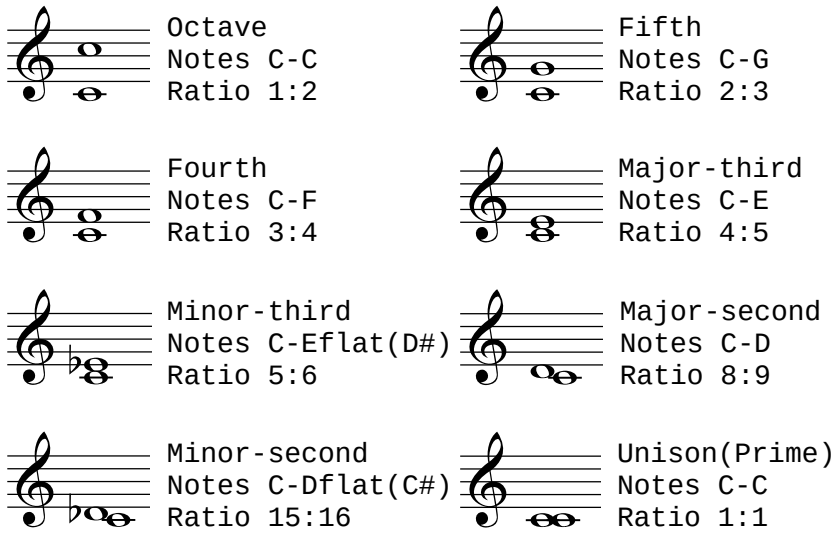


Figure 16.8 The intervals in Figure 16.7 illustrated on the staff.

Intervals greater than an octave are described as compound intervals, in that an octave component is added to the sub-octave interval, for example, octave+fifth = twelfth. As the two components overlap (1 through 8 + 8 through 12), C-C + C-G , it’s doesn’t make a thirteenth, and similarly a two octave interval C-C + C-C form a fifteenth, not a sixteenth!

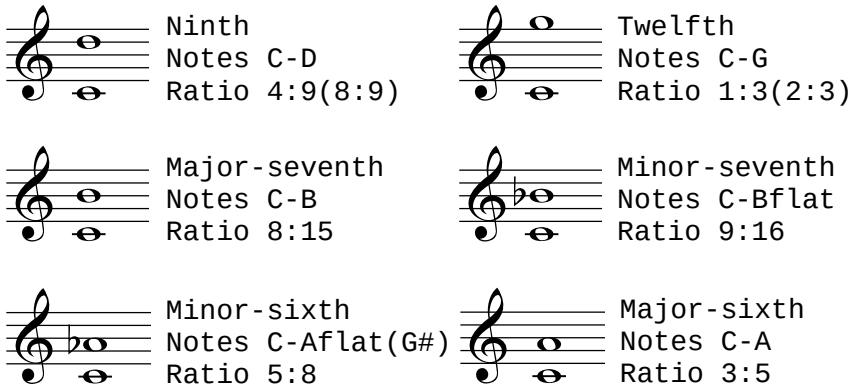
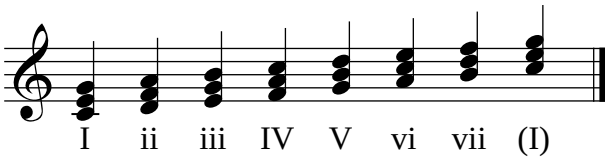


Figure 16.9 Some other intervals. The ninth and twelfth form compound intervals greater than an octave while the sixths and sevenths lie within the octave.

The information given here is presented in regard to the reference note ‘C’, the same relationships and names would be used in any key. For example, in the key of G major the interval G-B is just as much a major-third as C-E in C major. There are two interval reference charts at the end of this document.

Chords

A collection of notes sounding simultaneously forms a chord. The principle chords of a key or tonal center are the three note ‘triads’ constructed upon each note or degree of the scale. The scale note upon which the chord rests is termed the ‘root’ note of the triad or chord. On hearing these triads, and many extended chords based on them, the ear readily identifies the root note, even if it is not in the lowest position. Sometimes the ear will go even further and infer the root when it is actually not present at all.

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I   - Tonic triad         (major chord - maj3rd+min3rd)
ii  - Supertonic triad    (minor chord - min3rd+maj3rd)
iii - Mediant triad       (minor chord - min3rd+maj3rd)
IV  - Subdominant triad   (major chord - maj3rd+min3rd)
V   - Dominant triad      (major chord - maj3rd+min3rd)
vi  - Submediant triad    (minor chord - min3rd+maj3rd)
vii - Leading Note triad  (diminished  - min3rd+min3rd)
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Figure 16.10 The triads of a major scale/key. Triads with a major-third between the lowest two notes are termed major triads. A minor-third interval in this lowest position creates a minor triad. Minor chords are distinguished from major chords/triads by using lower-case Roman numerals. It is significant though rarely noted, that our ears principally interpret intervals upward from the root note and so we recover a very different quality from the major triad (h4, 5, 6) compared to the minor triad (h10, 12, 15), though both contain major and minor thirds.

As can be seen in Figure 16.10, reading upward, major triads have a major-third (four semitone interval) plus a minor-third (three semitone interval), while minor triads have a minor-third plus a major-third and the odd

one out, triad vii has two minor-thirds. However, any number of rearrangements, combinations of extra notes or permutations of the basic triads are possible, yielding a wide vocabulary of harmonic shades associated with each ‘primary color’ triad.

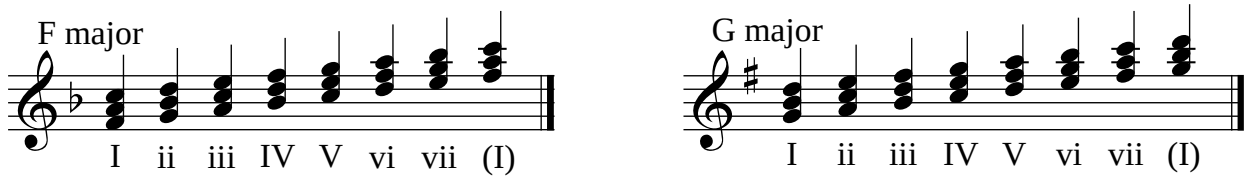

Figure 16.11 Triads of the keys of F major and G major, the two major keys/scales most closely related to C major.

The many possible rearrangements of notes in chords can be described by adding inversion letters and/or superscript numbers and signs to the Roman numerals of the chord’s root note scale degree. The superscript numbers being the interval(s) above the root/bass note. For example, a superscript ‘7’ is frequently added to a triad with an additional seventh interval (from the roots normal bottom-most position). Not all intervals are always described by this shorthand, the ‘obvious’ one are often left out and in particular a sharp or flat sign without an interval number is assumed to refer to the interval of a third above the root position or octave(s) transposition thereof. Thus a ‘#’ on its own would indicate the interval of a third above the root note of the chord was raised by a semitone, usually creating a major-third out of a what would have been a minor-third.

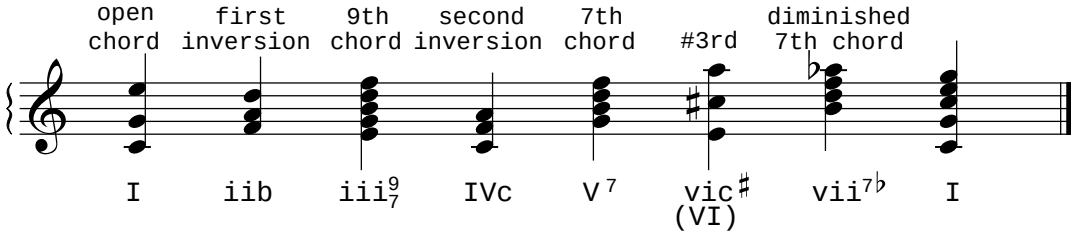

Figure 16.12 A few rearrangements of the chords of Figure 16.10, with superscripts.

Chord Inversions

Mostly chords appear in their *root position*, with the root tone the lowest note of the triad or chord as in Figure 16.10, but most often spread over a wider range of the staff. When the lowest note of a triad/chord is not the root note then the chord is said to be *inverted* or an *inversion*. If the next note up from the root note of the triad (in root position) is the lowest note in a chord, this is termed the *first inversion* and labelled (to confuse!) with the letter ‘b’ or superscript ‘6’. That is, the interval from the lowest (bass) note of the chord to the root note is a (major or minor) sixth. If the second note above the root note is placed in the lowest position, then the chord is said to be the *second inversion* and labelled ‘c’ or with superscript ‘4’. First inversions are much more common than second inversions. Second inversions usually occur at cadences and are also called *six-four chords* because they have a sixth and a fourth above the lowest note as in Figure 16.12 ‘IVc’. With a seventh chord having four notes, e.g. G B D F, there is the possibility of a third inversion (F G B D) which is labelled ‘d’. And logically a ninth chord can have a fourth inversion ‘e’ and so on.

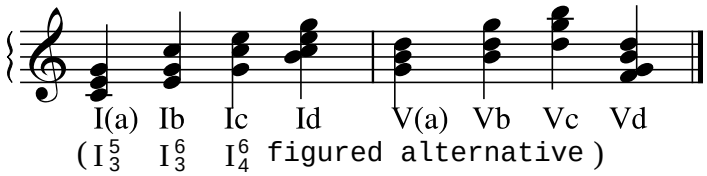

Figure 16.13 Inversion lettering for the C and G chords with the figured alternative, generally the five-three figures are left out, taken as understood and only the 6 of the six-three is written.

HARMONIC ANALYSIS

With the tools and techniques of the preceding pages it is now possible to examine a little of the harmonic architecture of a piece of music. Probably the most basic harmonic relationship in tonal music is that of the dominant-tonic chord progression – a relationship of central importance to the ideas presented in the book.

Firstly, looking at the large scale structure delineated by the three pause marks (measures 5, 10, 14 Figure 16.14), it can be seen that the first phrase begins and ends on chord I (the tonic) phrase 2 begins and ends on chord V (the dominant) and phrase 3 returns the tonal focus back to chord I. This macroscopic harmonic structure can be expressed as the ratios of tonic-dominant-tonic, 2:3:2 – or chords I-V-I, stability-change-stability.

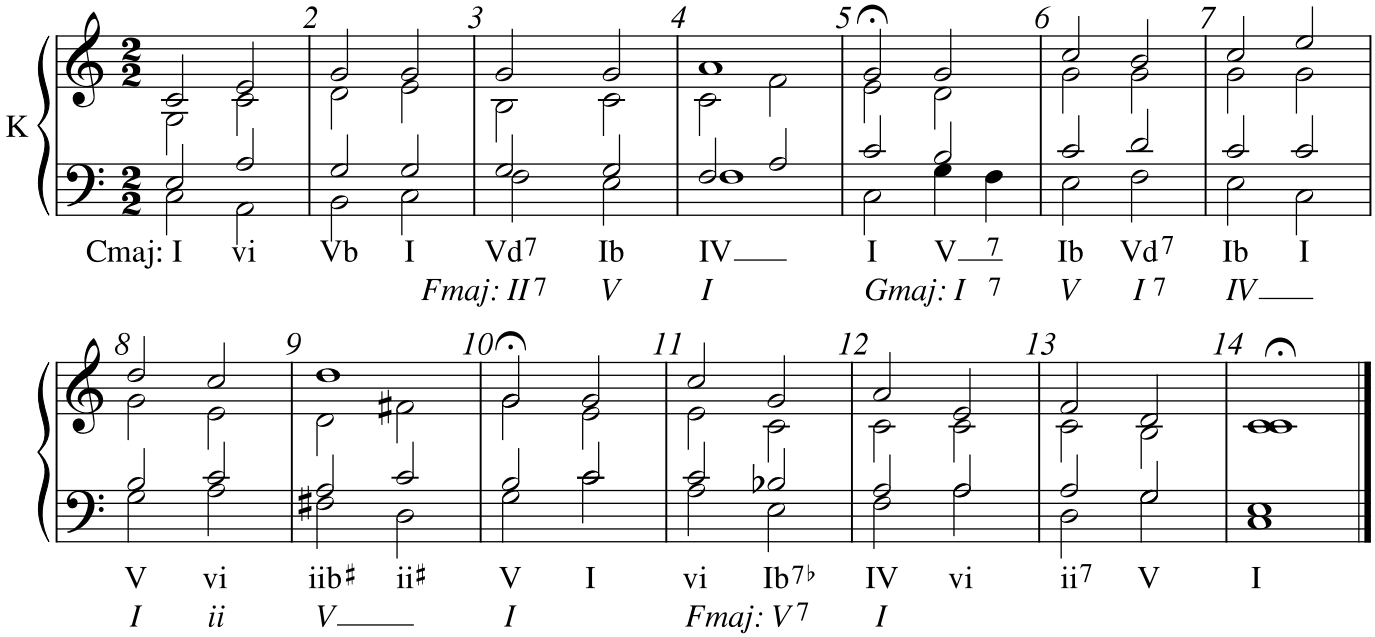

Figure 16.14 Extract from the chorale by Philipp Nicolai (1556-1608) ‘Wachet auf, ruft uns die Stimme’ (Sleeper’s Awake).

Often the most telling details of a musical phrase is how it starts and particularly how it ends, that is to say, where it has carried the listener to, in harmonic terms. Two of the phrases above end with the chord sequence ii - V - I which in the key of C major is the chord sequence D-> G-> C. Phrase two has the formula with a raised third, ii# - V - I and phrase three with an added interval of a seventh, ii^7- V - I. And if we closely examine phrase one, measures 3 and 4, the penultimate chord sequence V^7d - Ib - IV in C-major, when viewed from the perspective of the subdominant key, F-major, yields: II^7d - Vb - I - (V).

Energetics

This ‘two-five-one’ chord progression is probably the most common cadential formula in music and it provides a strong feeling of harmonic movement, direction and arrival (chord iib in the formula is on occasions ambiguous in form and misinterpreted as chord IV). Indeed, the archetypal musical phrase might be characterised as the chord pattern: I-> (wander about among loosely related chords)-> ii-V-I.

If the ratios of the principal root notes of this chord sequence, I -> (whatever) -> ii-V-I, are examined starting from chord I (notionally value 1) it is found that they are 1 × 9/8 -> 1.125 × 4/3 -> 1.5 × 4/3 -> 2. So the ‘one-> (whatever)-> two-five-one’ yields the ratios 1 : 1.125 : 1.5 : 2 or in chords C D G C, respectively. These numbers are significant in terms of stability and change in music and also provide the outline of music’s version of that most basic artistic formula: beginning, middle and end.

Looking inside the two-five-one formula reveals a nested pattern of V-I relationships (or in ratios, 3:4 relationships), in that the ‘two-five’ part of the formula viewed from the key of G major, turns into G: V-I. That is, chord ii (in C major) is *chord V of chord V* (in G major) or a dominant at one remove. So what in fact this most common cadential chord sequence represents, is a *double dominant* formula; the doubling up of the energy or motive force of the V^7-I (3:4) dominant-tonic chord relationship.

And the possibilities don’t end with a double dominant formula; the treble dominant is also a valued and much used resource – C major: vi-ii-V-I. Indeed, why not a quadruple dominant? C: iii-vi-ii-V-I. With each additional 3:4 relationship, the formula becomes evermore powerful in effect and directionality. So why not go the whole hog, as Mozart does in the well known Sonata in C major: I-IV-vii-iii-vi-ii-V-I.

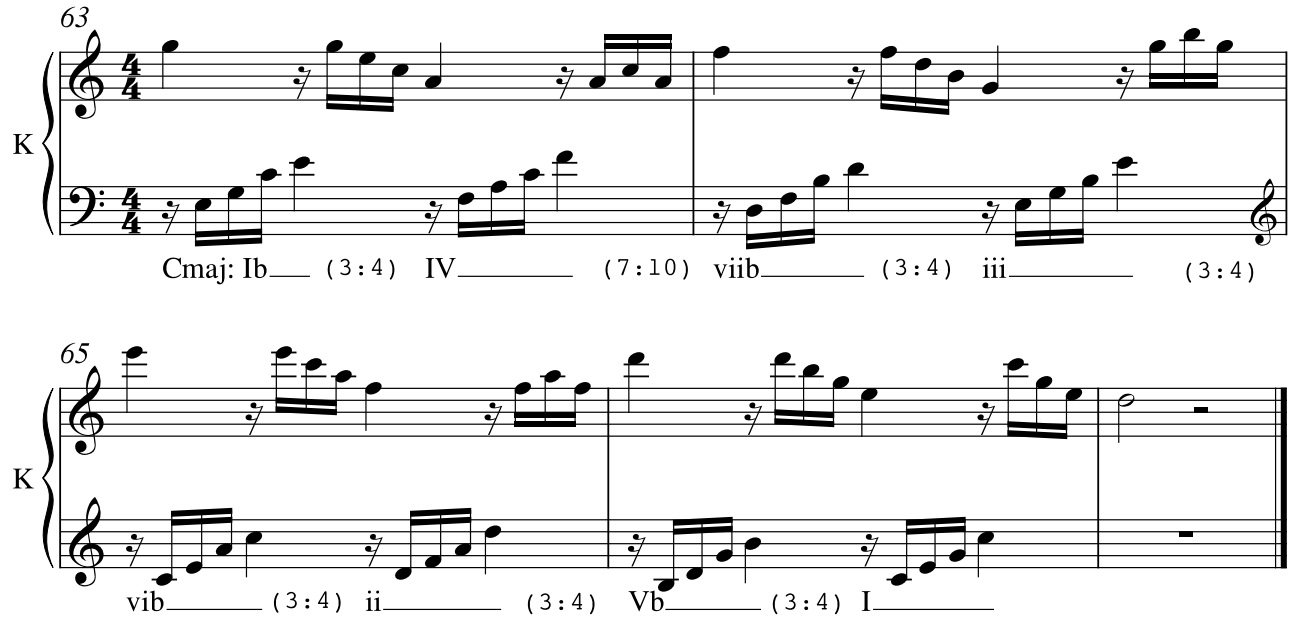

Figure 16.15 As can be seen in this example from Mozart’s Piano Sonata in C major (K545) there is one step in the CM: I-IV-vii-iii-vi-ii-V-I formula (F->B) where the relationship is only approximately 3:4; which Mozart cunningly hides from the listener, by using the first inversion viib chord with B natural not in the root position. This particular progression in the sequence provides a ‘break’ which helps to keep the tonal focus within the key of C major, whereas alternatively, a Bflat chord might encourage the ear to interpreted the sequence as moving towards the key of F major.

Stability and Change

A general pattern followed in many tonal composition, is one where once the stability of the ‘home’ key or tonal centre has been established, often with I-V-I chord sequences or with variations on it, as in Figure 16.14 ‘I-vi-Vb-I’ and Figure 16.16 – and perhaps after a more or less interesting and extended ‘whatever’ section – is to edged the system away from stability by the small ratio 8:9. This is rather like moving a super-tanker away from its mooring with a tug boat; once the vessel is in motion, it is not difficult then to accelerate the motion away from its ‘tonic moorings’, by means of a 3:4 exchange. However, with two applications of the 3:4 ratio, the system is neatly brought back to the original point of stability, chord I, ready for the next phrase to begin. Or the composer could stick with one 3:4 application, leaving the system at chord V, the dominant G; or use more than two 3:4 sequences, propelling the tonal focus toward chord IV, the subdominant F. There are, of course, an almost infinite number of shades and variations to this basic formula at the disposal of a skilled composer, as well as the possibilities of nesting the formula within itself.

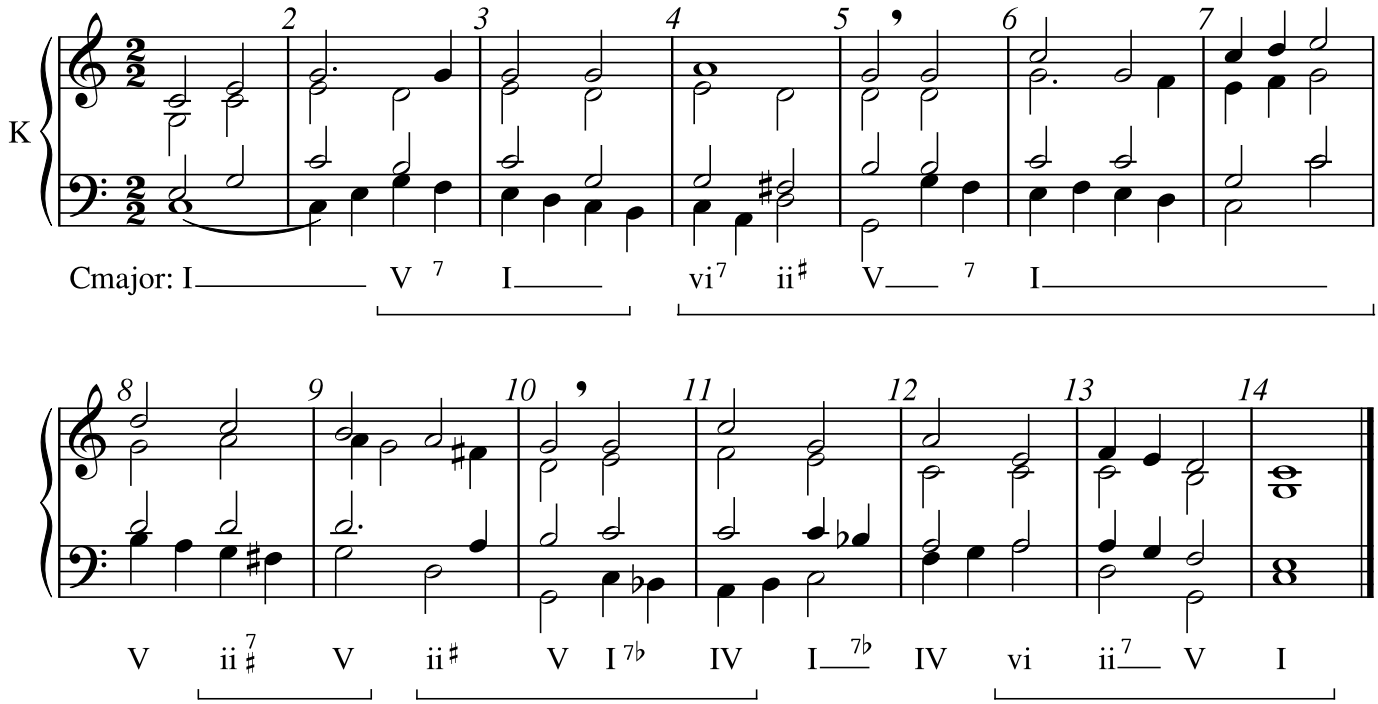

Figure 16.16 Extract from J.S.Bach’s setting of ‘Wachet Auf’ (Cantata 140), transposed from Eflat and with the phrase cadences shortened to gain a one to one match with Figure 16.14.

Concluding with J.S.Bach’s more potent setting of the chorale – which interestingly uses exactly the same chords as Nicolai’s - I, ii, IV, V & vi – it can be heard that far greater *drive* is obtained, typical of Bach's style, through a consistent use of a *chain of dominants*, the 3:4 relationship. (Marked by horizontal brackets under the score.) The repeated use of this formula, has the effect of applying a constant forward ‘acceleration’ (through the release of ‘relational energy’ by repeated 3:4 exchanges) powering the music toward the final cadence. This harmonic drive has been combined with extra motion, particularly in the bass part, complimented by other voices ‘hanging on’ (suspensions) or moving into position by steps (passing

notes) to create interesting clashes and resolutions; achieving, overall, an effect of exhilarating motion. Also, with subtle stealth, Bach places the ‘chain of dominants formula’ so that it overlaps the phrase cadences thus tying the three phrases together, into a single dynamic, arching, I-V-I span.

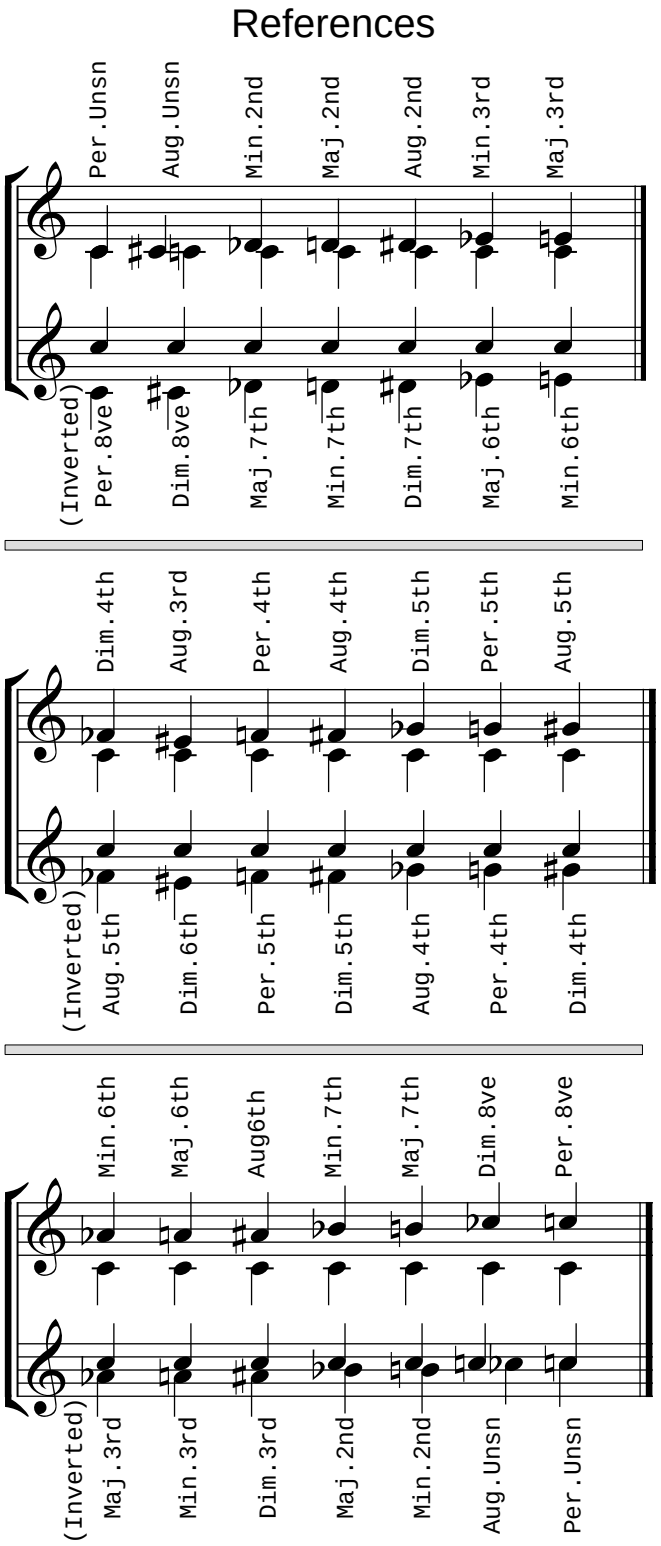

Figure 16.17 Reference table of intervals and their inversions.

Interval	Diminished	Minor	Perfect	Major	Augmented
Second	C#-D♭ 0	C-D♭1		C-D 2	C-D# 3
Third	C#-E♭ 2	C-E♭3		C-E 4	C-E# 5
Fourth	C#-F 4		C-F 5		C-F# 6
Fifth	C#-G 6		C-G 7		C-G# 8
Sixth	C#-A♭ 7	C-A♭8		C-A 9	C-A# 10
Seventh	C#-B♭ 9	C-B♭10		C-B 11	C-B# 12
Octave	C#-C 11		C-C 12		C-C# 13

Figure 16.18 Table of intervals from a diminished second to an augmented octave with the number of semitones in each to the right of the note letters.

[14/06/09]