15

Wider Horizons

FROM PARTICLES TO PLANETS

I begin this closing chapter of *Journey to the Heart of Music* with some trepidation and in a spirit of exploration, knowing full well that I am venturing far beyond the reasonable competence of a piano tuner and student of music. Indeed, I must confess to being in two minds over whether to include this chapter at all. However, if in these documents any progress has been made at all, the principal method by which this has been achieved is through the comparative study of organisation and structure in tonal music and natural systems, side by side. Though often conceptualized as inhabiting its own universe, music does not exist in splendid isolation but finds life and voice in the material world; and so ideally, by implication, music theory should not only explain its subject, music, but also place it meaningfully within the broader sweep of human knowledge and understanding. To put it bluntly, without the guidance afforded by the detailed and objective information available about atomic and nuclear structure wrestling with the far less objective and tractable structural imperatives of tonal music would have been very much more difficult. Basically, by studying the two systems in tandem (with occasional help from other well documented physical systems), when the path became difficult and uncertain in music, with luck, the structures and processes apparently evident in the periodic table in particular offered help or inspiration. It is for this reason, so as to tell the full story of the *Journey to the Heart of Music*, that I venture upon such dangerous territory for the amateur. It is absolutely not my intention or purpose to offer explanations or solutions to difficult and arcane problems in twenty-first century physics – which lie far beyond my understanding – but rather to ask questions and to point to interesting features arising out of the musical perspective taken in this narrative, which, perhaps others more knowledgeable in these scientific fields, might consider worthy of further investigation.

In the early part of the last century, the influential Danish physicist Niels Bohr constructed a new atomic model, which led him to confront a difficult problem: according to James Maxwell’s theory of electromagnetism – a crowning achievement of late nineteenth century science – oscillating electrons should radiate energy and quickly collapse into the nucleus of atoms. This clearly doesn’t happen. Borrowing an idea pioneered by Max Planck, Bohr made a quantized model of the atom that used ratios of the harmonic series to confine electrons to discrete energy levels, a structural device providing for the atomic stability which is observed in nature. Together with Planck, Bohr was laying the foundations of quantum mechanics, a difficult to comprehend but very successful description of the physical world at the fine scale of atoms. As the 20th century progressed toward its end, a similar problem remained unresolved, at the larger ‘classical’ scale of human perception, and beyond. Knowledge of the wider cosmos reveals that it too possesses a varied and stable structure, which according to the logic of thermodynamics, should collapse to a state of equilibrium – but it doesn't, or at least, hasn't yet. One of the many (speculative) proposals being put forward to explain this puzzling, yet most welcome situation, is the idea that some type of *discrete computational procedure*, fuelled by fundamental processes, lies embedded within our familiar physical world of space and time – a mechanism to account for the structure, variety and stability we see about us: a dynamical self-organizing universe, built on quantum foundations of tiny integer units.

Picture courtesy Wikipedia

Niels Henrik David Bohr (1885–1962) was born into a cultivated and well to do family living in Copenhagen, Denmark. His father was a professor of physiology and his mother, of Jewish extraction, came from a family of bankers. A keen sportsman (as was his brother Harald an eminent mathematician), Bohr studied at Cambridge and later took his doctorate at Copenhagen University. At Cambridge he studied under Ernest Rutherford and used his teacher's discoveries and theories, along with Max Planck's recent insight, to construct a quantized model of the electronic structure of the atom. Bohr postulated that electrons could jump between different fixed energy levels and thereby absorb or radiate discrete amounts of electromagnetic energy – photons. The young Einstein also made contributions in this area, for which he was, officially, awarded the Nobel prize. Bohr's theory assigned energy levels to the hydrogen atom in the ratios of the harmonic series as: principal quantum numbers 'n' = 1, 2, 3, 4, etc. Still in his early thirties, in 1916, Niels Bohr was appointed professor at Copenhagen and then director of the Institute of Theoretical Physics in 1920. Under his direction the Institute became a leading center for advanced atomic research. During the second world war Denmark was occupied by German troops and in 1943 Bohr, being half Jewish, fled to Sweden and later went to the USA where he worked on the atomic bomb. Like many scientists, he was deeply uneasy and ambivalent about the Manhatten Project, and the legacy they were creating for humanity. Bohr was married and had six children, one of whom, Aage Niels became a physicist too and, like his father, won a Noble Prize.

Musicians also inhabit a quantized world. A universe described primarily by discrete units of rhythm and pitch (and volume – though the quantized units of energy which Max Planck postulated in 1900 are far too small to be aurally perceived). Glissando and free rhythm excepted, for the most part our music proceeds by quanta, little steps and jumps of pitch and duration from note to note and chord to chord. This amazing graininess appears to present no obstacle to our perception of a musical flow, which we recover at units of larger scale – motives, phrases, melodies etc. Many experimental attempts have been made to create music out of a continuum of pitch and duration: like the machine which now resides in the Percy Granger Museum in Melbourne and the many electro-acoustic devices that have followed in its path; however, to date, they have shown little sign of replacing the traditional methods of 'quantum music'.

Over many years, indeed centuries, musicians have learnt some interesting tricks about how to work such a system of discrete units: how to create structure, to integrate detail, and make the many components hang together as a coherent whole. Our rich musical heritage is the result. But it is a reward that had to be worked for, over a considerable time, with contributions from across Europe: the creation of composers, musicians and not least their patrons, the audience – by exercising choice and selection. In particular, the system of keys (tonal centers) was not invented overnight but rather grew up slowly, as each generation of composers explored the boundaries, until, moving forward through ever more keys, eventually brought the system back to where it had started from! This remarkable structural device, the cycle of key centers which ‘returns to itself’, and the mechanism of modulation which allows music to step from one tonal center to another, was a collective artistic achievement, an evolutionary development over an extended period of time.

The evolutionary trajectory taken by western music was, I suspect, beyond the overall control of individuals and groups. Once musicians strode forth in harmonic steps of whole number relationships, they became unknowing players in an evolutionary game destined to find an organisational solution for music which follows fundamental principles that apply to perhaps all oscillatory systems. Essentially, it could be argued, rather than creating some arbitrary scheme of musical organisation, over the centuries, western musicians evolved or developed a rough facsimile of nature’s own organisational solution, a solution which perhaps almost inevitably emerges from the combination of simple (oscillatory) whole number relationships, and thus forms a class of phenomena in the material world with probably quite extensive coverage. For indeed, everything with a temperature above absolute zero is involved at some level, in forms of periodic behaviour – vibration, rotation, revolution: oscillation.

The idea, the thread running through *Journey to the Heart of Music* is that by closely examining the structures of our musical system and the organisational principles of tonal music which evolved during the period of its great flowering (roughly 1500 to 1900 AD) – principles and techniques for the manipulation of quantized sound, regulated by the whole number relationships of modulation – some interesting and useful insights and general perspectives might be gleaned, and perhaps, subsequently, applied to illuminate the nature of other oscillatory systems.

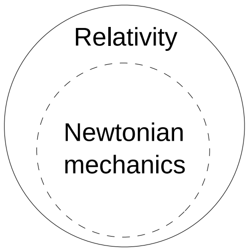
One of the great ironies of music history is, I feel, that just at the point where musicians were beginning to discover this powerful principle of organisation, their position within the medieval quadrivium of arithmetic, geometry, astronomy and music – the disciplines concerned with *ratio* – was being lost. As the new Renaissance learning blossomed into the scientific revolution in the seventeenth century, music became increasingly side-lined, viewed more as art than science, and the organisational scheme that musicians were to come to understand viscerally, and implicitly explain in their music theory, was to go relatively unregarded. Certainly, from quite an early date, the organisational principles which underpin tonal music were available for mathematical and scientific enquiry and analysis. Quite how they came to be so neglected is, I think, a great mystery. Perhaps, with the developments in the techniques of calculus, configuration spaces, etc., and the *principle of the continuum* which they imply, the mindset of mathematicians and scientists became skewed away from the older ideas of ratio and proportion, in favour of an idealised system of smooth curves and transitions containing an infinity of points. Only now, at the beginning of the twenty-first century, with the digital computer a commonplace artifact and string theory plus other similar approaches, serious and productive branches of enquiry in physics, is attention being drawn back to the study of systems, that evolve by discrete units.

The connections between music’s implicit organisational model and recent developments in theoretical physics are interesting, even tantalising. In particular the unifying conception of symmetry employed in mathematical group theory, which runs through so much of contemporary physics and cosmology. Equally, as touched on in previous chapters, symmetry extends its sway to music – as viewed from the perspective of the MOS model – as well as many other theories and models constructed by music theorists in recent years. Perhaps, I wonder, might future generations of music theorists again have cause to discuss with their colleagues in mathematics and the sciences, matters of serious concern and relevance to all parties? As was the situation at the start of the scientific revolution, in conversations between

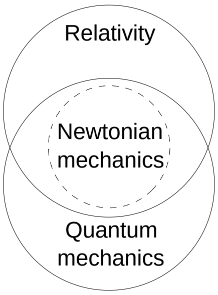
Vincenzo Galileo, the music theorist and his famous son; or later, during the Enlightenment, with Rameau addressing the French Royal Academy of Sciences in 1749 and collaborating with Rousseau, Diderot and d'Alembert. Indeed, this chapter is to some extent an exploration of what today's connections could, or might, be – a pathfinding reconnaissance – and, as far as I know, the ideas in the form presented in *Journey to the Heart of Music* have not appeared elsewhere. Though, of course, it must be acknowledged that the works of a vast range of individuals inevitably become combined in the mind over time, and in particular, I should record again the indelible mark left by R.F. Goldman's portrayal of the dominant-seventh acting dynamically, forcefully driving toward the tonic – encountered so many years ago, in his wonderful book *Harmony in Western Music*. An acknowledgement that might also be read as a proclamation of the fundamentally musical perspective which has guided this narrative throughout.

Looking back over the broad development of science since music's departure from the fold of the disciplines of ratio, the basic concept of a wholly relational world system was perhaps first put forward by Gottfried Leibniz during 'the great debate' between his and Issac Newton's followers. However, a truly relational approach proved too difficult to develop at that time, and had to wait for twentieth century relativity and gauge (symmetry) principles. Newton, however, found that by setting matter, and tracing motion, against an absolute fixed background of (infinitely divisible) space and time, spectacular progress could be made. His dynamical system swept all before it and dominated thinking for more than two hundred years, until an unknown patent clerk from Berne wrote a challenging paper in 1905, examining the implications of combining the fixed speed of electromagnetic waves (including light) found by Maxwell, with the effects of relative motion – special relativity. In the long run, the approach of Leibniz, won through: he had seen that the explanation of physical reality had, ultimately, to be relational. Einstein praised Newton for his courage in making an essential 'staging-post' on the mountain – but read the relational authors Leibniz and Ernst Mach for inspiration and guidance on a route ahead.

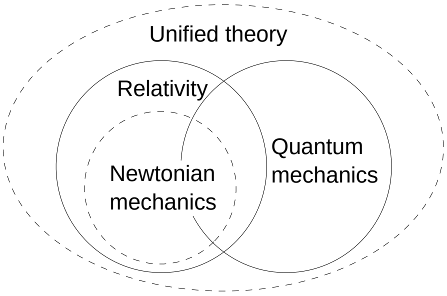
Each of these steps, though, has been more an *extension* of the area of application, than an overturning of previous work, so that Newton's mechanics *fits within* the broader scope of later work, as a perfectly good approximation (in normal everyday conditions) to relativity's more subtle interpretation. Also over time the various branches of scientific knowledge have tended to join up and overlap, bring about a deepening integration and coherence to scientific knowledge. As for example has happened with the application of thermodynamics to fields far removed from the original study of steam engine efficiency: a trend which has allowed the disparate patterns of reality to be viewed from an ever more unified standpoint.



The great problem which emerged during the early twentieth century, however, is that science developed two fundamental theories, which don't fit wholly together – relativity and quantum mechanics – explanations of the workings of the large scale and the small: the classical domain of human perception (extending outward to the cosmological scale) and the domain of atomic phenomenon.



Considerable time and effort has been expended in the hunt for a comprehensive synthesis, a unifying principle or perspective capable of accommodating what is known of nature, on both the large and small scales. At the heart of the problem lies the question of whether the fundamental units of the material world are interpreted as being point (zero size) particles with intrinsic qualities occupying a continuum or spatially extended waves with their natural integral relationships. After much effort and many earlier failures, string theory and other similar approaches hold out the promise of progress, based on the simple unifying principle, *that all matter and forces arise out of the relationships between the different resonances (notes) of one fundamental entity*. This is equally a musical conception as a scientific postulate, an idea grounded in the *harmonic series* and one that befits Rameau's epithet: *nature's gift*.



String theory to a degree marks a return to the situation of ancient learning and philosophy, where without the tools and techniques of measurement and observation developed during the seventeenth through twentieth centuries – which allowed experimental science to *lead* theory – scientific advance is again more dependent on the exercise of logic, mathematics and philosophy to probe regions far beyond the direct reach of today's technology. String theorists, like music theorists, are searching for a logically consistent fundamental theory, based on a few simple assumptions, which can be naturally and convincingly extrapolated to encompass, perhaps even explain, all the phenomena of their respective worlds, whether musical or material. This is of course an ambitious undertaking, both in science and music, but ultimately, there is nowhere else to go, no other destination so worthy of our striving. Though it remains an open question as to precisely which course offers the best prospects of success, the 'string' approach both in music and physics, overall, is now probably the most promising general direction to follow. Essentially, the *standard model* of particle interactions developed in the second half of the twentieth century, provides a hugely successful predictive description of the atomic world, but explains very little. String theory attempts to rectify this shortcoming through an analytic, rather than synthetic, approach. Similarly in tonal music, the traditional 'Roman numeral' approach (see Chapter 16) provides a comprehensive description of harmonic progression, without enlightening us much as to why chords are arrayed in the particular ways that we find them. The numeral approach provides a very useful synthetic description but not an analytic understanding of tonal music. The route taken in *Journey to the Heart of Music* has been not unlike that of laying out a 'string theory' for tonal music, an attempt to probe tonal music's most fundamental nature and attributes, so as to be able to construct a coherent model capable of linking these foundations to the superstructure of observed musical practice. And, on a less serious level, might it not be that a piano tuner is rather well placed to take on the task of concocting such a 'string' theory for music?

As discussed in Chapter 9, if you remove a note, any note, from a piece of music, what do you get? A frequency, an amplitude and a duration – a few bare facts from which little meaning can be extracted. It is the relative relationships within a composition that carry the significant information, not the absolute qualities of notes in isolation. Which is to say the nature of music is fundamentally relational. What a note is – its *identity* in a piece of music – is the sum of its relationships with the other notes in the piece, and that, fundamentally, a musical composition is a sequence of changes to these note relationships – a scheme of musical *events*. Significantly, all of these relationships, and changes in relationships, can in principle be expressed in *whole numbers*, and, as documented in earlier chapters, be manipulated via the workings of the modulation algorithm of symmetrical exchange.

Richard Feynman wrote in his book: *The Character of Physical Law*:

> "... I have often made the hypothesis that ultimately physics will not require a mathematical statement, that in the end the machinery will be revealed, and the laws will turn out to be simple, like a chequer board with all its apparent complexities."

Overall, each step along the road of scientific discovery and understanding has also been, paradoxically, a stepping back or away from the (apparent) complexities of reality as we experience and see them, so that as wider horizons are encompassed, the categories and principles needed to describe them actually become simpler and more universally applicable. For example, as scientists probe back towards the origins of the material universe they find that both the conditions, and the laws which governed those conditions, become simpler, more uniform, wide ranging, and increasingly symmetrical. Logically, this process is tending toward just one category and just one principle: a description of *identity*, 'what things are', and an account of how things change – *events*. This view was first clearly expressed by Gottfried Leibniz: that, ultimately, the description of the physical world would, necessarily, have to be made in such wholly relational terms, that *identity* finds its source in relationships, and that *events* are changes in these connections.[1]

Picture courtesy Wikipedia

Gottfried Wilhelm Leibniz (1646–1716) was born in Leipzig, the son of Friedrich Leibniz a professor of philosophy at Leipzig University who died when Gottfried was only six years old. Leibniz, like his father, had the gift of great intellect; and from his mother, Catherina, he inherited a deep religious conviction. Perhaps significantly, his early education had some degree of autodidactism based on the extensive library of texts left by his father. Gottfried Leibniz entered Leipzig University at fourteen to study law and philosophy, graduating in 1666. Failing to find a teaching position at Leipzig, Leibniz switched to the University at Altdorf; and then, when offered a position there, decided instead to work in the service of Johann Christian von Boineburg at the Court of the Elector of Mainz. His duties included diplomatic negotiations and overseeing legal reforms. Leibniz was appointed to the Appeal Court of the Electorate of Mainz and travelled to Paris and London in his diplomatic role, which allowed him to meet many of the leading mathematicians and scientists of the age. At the Royal Society in London, Leibniz demonstrated a calculating machine of his own invention. Much of Leibniz's work was to foreshadow later developments: as with this calculator and Babbage's mechanical computers, in his ideas concerning a universal 'algebra of thought' and Boole's work on binary logic (Leibniz invented the binary number system), and in his concept of fundamental entities – monads – and contemporary string theory. A significant amount of Leibniz's vast output, across a wide range of fields, was to lie undiscovered in letters and private papers. Also his contributions were underestimated because his conclusions clashed with those of Newton, and, it was not until twentieth century science 'caught up' with his relational view of the world that his reputation as a natural philosopher was to be rehabilitated. In 1674 the Elector of Mainz died and Leibniz had to find another patron; luckily he had been in contact with the Duke of Brunswick for some time; and, in 1676, he took up a position in the duke's service. Leibniz was to remain with the family for the rest of his life as counsellor, historian and archivist. Leibniz cultivated and corresponded with a wide range of friends and admirers, which went some way to soften the isolation he felt in being mostly kept away from the great cosmopolitan centers of learning and fashion. Perhaps the greatest mathematical achievement of Gottfried Leibniz was his independent discovery of calculus in the 1670s (Issac Newton developed a similar system of fluxions). Indeed, Leibniz introduced a superior method of notation for calculus, which remains in use down to the present day. In 1711 an argument erupted as to whether he or Newton was the inventor of calculus, the implication being that Leibniz had stolen Newton's then unpublished ideas, while in London. Historians generally discount this version of events, and although the accusation darkened his last years and dimmed his reputation after his death, yet today, the light of this thought shines forth clearer than ever. Leibniz did not marry, and many distrusted the mixture of charm and guile he had acquired as a career diplomat. Perhaps his greatest admirer was the Electress Sophia of Hanover, whose support he repaid with sincere friendship and advice to her, and many of her female relatives. Gottfried Leibniz died at Hanover in 1716, out of favor with his patron and unregarded by the world. His insights, in so many fields were profound and prescient, but often had to wait decades or centuries to be appreciated; no more so than in music:

> "Music is the pleasure the human mind experiences from counting without being aware that it is counting."

The modern scientific tradition grew up on foundations laid in the renaissance – the rediscovery and re-evaluation of ancient learning. In the middle of the sixteenth century the Franciscan scholar and composer Gioseffo Zarlino, 1517–1590 (later to be maestro da cappella at St Mark's, Venice), was busily engaged in this enterprise of unearthing the old knowledge; and importantly, interpreting and extending what he found. *Le Istitutioni Harmoniche* of 1558, by Zarlino stands at the beginnings of modern music theory. In this work Zarlino attempts to combine a synthesis of the ancient learning with new ideas, so as to incorporate the contemporary musical practice of using intervals of a third and sixth, within an all-embracing system that also incorporated the older schema of the Pythagorian scale. Zarlino (and others) were not content simply with relearning the old knowledge; they wanted to use it as a foundation and stepping stone to a new and fuller understanding of present musical practice. Like Rameau, Zarlino was a practical musician: theorist, composer and teacher. Significantly perhaps, amongst his pupils was Vincenzo Galilei, whose son, Galileo, was destined to become the 'father' of modern science. Indeed, as a young man, Galileo, an able musician himself, assisted his father in his acoustic experiments to determine the nature and benefits of different scale temperaments, thus providing Galileo with an early experience of empiricism and mathematical explanation – the hallmark of modern scientific method. This makes Gioseffo Zarlino, and his pupil Vincenzo Galilei, in some degree, 'grandfathers' of modern science. Music theory was a participant in the beginnings of the scientific revolution, and though subsequently left trailing in the wake of the scientific juggernaut, a connection has been maintained: with Kepler's celestial harmonics in the seventeenth century, through Rameau's interest in Newtonian theory in the eighteenth century to the thoroughly scientific approach of Helmholtz and Oettingen in the nineteenth. At the start of twentieth century, the development of the quantum principle by Planck and Bohr began to draw science and music theory closer again; and, at the beginning of the twenty-first century, string theory is continuing the trend. Modern music theory and modern physical theory began together, children of the Renaissance hand-in-hand, and perhaps, some day in their maturity, they will link arms again.

There are many excellent books describing the development of science in general and string theory in particular; this chapter is not intended to be another in that honourable but crowded line. *Journey to the Heart of Music* takes a radically different approach in this chapter, turning the tables about and viewing mathematical and scientific concepts and hypotheses from the perspective of *Music Theory*! By looking through the lens of music and essentially asking the questions: Does the study of music have anything to offer science? Could the organisational principle discovered or evolved by musicians through their music-making, actually contain hints of an approach capable of shedding light upon, or even unlocking, the workings of other (natural) systems? Ridiculous, maybe. But it is perhaps salutary to remember that Max Planck, the father of the quantum, was himself a talented pianist who only narrowly chose science over music, as a career. One wonders: were his musical instincts, as much as his scientific intuition, involved in his great discovery? This is certainly an adventurous proposition to put forward – to suggest that music might have something to contribute to twenty-first century mathematics and science – but I believe it is a proposition worth considering, though of course, it is most definitely a long shot, and probably, a shot in the dark.

Picture courtesy Wikipedia

Max Karl Ernst Ludwig Planck (1858–1947) was born into a scholarly and conservative family of lawyers and theologians from the university town of Gottingen. His father was Professor of Law at Kiel and after 1867 at Munich. At school Planck excelled in academic subjects and music, he played many instruments and perhaps significantly, dabbled in composition. Against the advice of his physics teacher, he chose science over music as a career, because he felt he was more likely to be able to make a contribution in that field. In 1877 Planck went to Berlin to study under Helmholtz, who was to become a firm friend (they played music together) and by 1880 he had completed his academic and teaching qualifications. His chosen branch of physics was to be heat theory and general thermodynamics. In 1885 Planck became an associate professor at Kiel and in 1889 was working back in Berlin, becoming a professor there in 1892, where he staying until retirement in 1926. Planck was offered but turned down Boltzmann’s position in Vienna in 1907. Married twice, first to Marie Merck (the sister of an old friend) who died in 1909 and later in 1911 to Marga von Hoesslin; Planck was sadly fated to see all four children of his first marriage die, the two girls in childbirth and the two boys in wars. Before these tragedies, the Planck family lived happily in Berlin, their home a center for musical and social gatherings. Einstein and Helmholtz both enthusiastic amateur musicians at different times swelling the family band. Planck was right in his choice of vocation for he truly did make a contribution, arguably one of the greatest of the twentieth century, that of founding quantum theory. Through his radiation law, published in 1901, he was able to answer a long standing problem concerning the emission of radiation, by assuming that it could only be released or absorbed in discrete units. Like the music he played, he postulated that the radiation of electromagnetic energy proceeded by steps and jumps and was not smoothly continuous. Planck, at least at first, viewed his quantum theory more as a mathematical construct than as a description of reality at the smallest scale. Einstein took the theory more literally and used the idea of discrete particles in his papers on electromagnetism. Planck supported his precocious younger colleague, despite doubts, which enabled the concepts of relativity to become established much sooner than would otherwise have been the case. Max Planck returned to his family’s home town of Gottingen, the once but by then no longer, center of European mathematical endeavour, where he died in 1947.

ELEMENTS OF MUSIC?

The information presented below is certainly of a speculative nature and the question mark in the title is more than decoration. The intention here is exploratory: to ask questions, follow intuitions, rather than providing an accepted scientific exposition or explanation. Indeed, in applying musical principles to the electronic configuration of the atom, my aim is to draw attention to a remarkable congruence between the two systems, rather than actually provide a scientifically rigorous description. The story of atomic structure, given in terms of the underlying principles of tonal music, is told as a means of highlighting the parallels displayed between the outward appearances of the two systems. Whether this congruence points to some degree of common ground – an inner unity of organisational principle – is a question worth asking, but the answer I leave to others better qualified to judge. Though at times challenging, I hope the reader will find a musical journey through the periodic table of elements interesting and perhaps informative.

To recap, the argument is: That rather than creating some arbitrary scheme of musical organisation, over the centuries western musicians evolved through tonal musical practice, a rough facsimile of a ‘natural’ oscillatory organisational solution for systems based on integral relationships. A schema perhaps to be found in a broad range of physical systems. And that modulation, the algorithm of symmetrical exchange, is the mechanism by which these physical systems achieve that organisation – compute structure or its equivalent, process information. Despite appearances to the contrary, the ultimate subject matter being presented is not music or physics but mathematics: computation by mutable numbers. The ideas put forward below – which are obviously extremely speculative and unsubstantiated – are developed only as far as to cover the opening section of the periodic table of elements, as this is sufficient to demonstrate the principles involved.

In his book *Nature’s Building Blocks* (2001) John Emsley records a number of attempts to draw parallels between schemes of organisation founded on music and the regularities evident amongst the elements – the basic constituents of our familiar material world. As early as 1829 J.W. Dobereiner proposed a *Law of Triads* linking elements in groups of three with the middle element the mean of the outer two atomic weights, e.g. lithium 6.9, sodium 22.9 and potassium 39.1. And Leopold Gmelin extended the idea further to encompass ten triads, three tetrads and a pentad. Later J.A.R. Newlands (1837–1898) presented a paper to the London Chemical Society in 1864 entitled *The Law of Octaves* which noted the recurring pattern of eight in the properties of the then known elements – coming very near to preempting Dimitri Mendeleyev’s (and J.L. Meyer’s) invention of the periodic table. Newlands’ ideas were ridiculed at the time by his peers as unscientific. What Newlands missed for the most part, but Mendeleyev realised, was that spaces for elements as yet undiscovered needed to be left in the table. It was the rapid discovery of many of these predicted, but then unknown elements, that confirmed Mendeleyev’s discovery of the periodic law – the full working out of Newlands’ law of octaves. It is this octave feature, a pattern which repeats itself after eight steps, that provides the basis for making a link between tonal music, the algorithm of symmetrical exchange and the structure of the periodic table.

So what follows is another (brief) attempt in a long, if not hallowed, tradition. However, being clear that this is not an attempt to make the patterns in the periodic table follow the rules of music, though often for convenience and clarity musical terms and analogies are used. But rather, this is an application of what might possibly be a general principle of integral self-organisation which informs both systems, based on modulating oscillatory systems, acting under the influence of the second law of thermodynamics.

The Periodic Table

If all the elements, the basic ingredients of matter – carbon, oxygen, etc., are formed into a chart, running from the lightest and simplest, hydrogen, up to the heavy complex atoms like uranium; it was noted by Mendeleyev that a repeating pattern of properties emerge. And with a particular arrangement of seven horizontal rows or periods, a repeating pattern of eight[2] vertical families can be discerned, which in the older format of the table used here are marked by Roman numerals I to VIII.

Atoms consist of a small dense electrically positive nucleus surrounded by a cloud of negatively charged electrons. It is the arrangement of the electrons, particularly those in the outer layer of electronic structure, that principally determine the chemical properties of an element, and so, to which family it belongs. As the number of electrons (plus balancing protons and neutrons in the nucleus) increases through the table, the structure of the outer layer of electrons repeats, and so produces regular likenesses amongst the elements – the similarity of properties displayed by the vertically arrayed families. With the above tabulation, the Roman numerals above each column, indicates how many electrons are in the outer layer and the family to which each element belongs (e.g. carbon, silicon, germanium, tin, etc., family IV, has four outer layer electrons and fluorine, chlorine, etc., the halogen family VII, seven outer electrons).

Broadly, elements toward the left-hand side of the table have metallic qualities and toward the right-hand side are non-metals. The elements in the right-most column, family VIII, are special – the noble gases – they are almost totally unreactive, rarely forming associations with other atoms. And when atoms other than noble gases do form molecular associations – molecules – the principal attraction of the association is that, in combination, their total complement of six outer layer electrons is equal to that of the outer layer of a noble gas. In effect the atoms cluster together as molecules so as to emulate the outer layer electronic structure of the members of family VIII. The noble gas configuration marks a point of structural repose in the evolution of atoms – sheltered, dipping, valleys on an otherwise relentless climb up the mountain range of ever heavier elements in the table.

After hydrogen and helium in Period 1, the next two periods consist of eight elements (shown in full in Figure 15.4), followed by two periods of eighteen elements, one period of thirty-two elements and a last incomplete period of nearly thirty-two elements. The complete table is given at the end of the chapter in Figure 15.15, however, the discussion here will be limited to covering only Periods 1 and 2, which will suffice for demonstrating the application of the overall concept of modulating oscillatory systems and mutable numbers to the structures in periodic table of the elements. The elements of the first two periods, followed by their chemical symbols, are: hydrogen H, helium He, lithium Li, beryllium Be, boron B, carbon C, nitrogen N, oxygen O, fluorine F and neon Ne.

To digress for a moment: one remarkable coincidence that emerges from the crossover between music and physical theory is the similarity of symbolic nomenclature adopted by chemists in the periodic table, physicists for the fundamental particles of nature and musicians in their Roman numeral analysis of harmony. For example, what interpretation should be placed on the symbol I^-, is it the tonic chord in a major key with a lowered third or an Iodine ion with an extra electron? Does IVb refer to a family of elements in the transition metals or a subdominant in first inversion? Does B^0 indicate a B-zero meson, a boron atom with its rightful complement of five electrons or a B-major chord without its root note present?

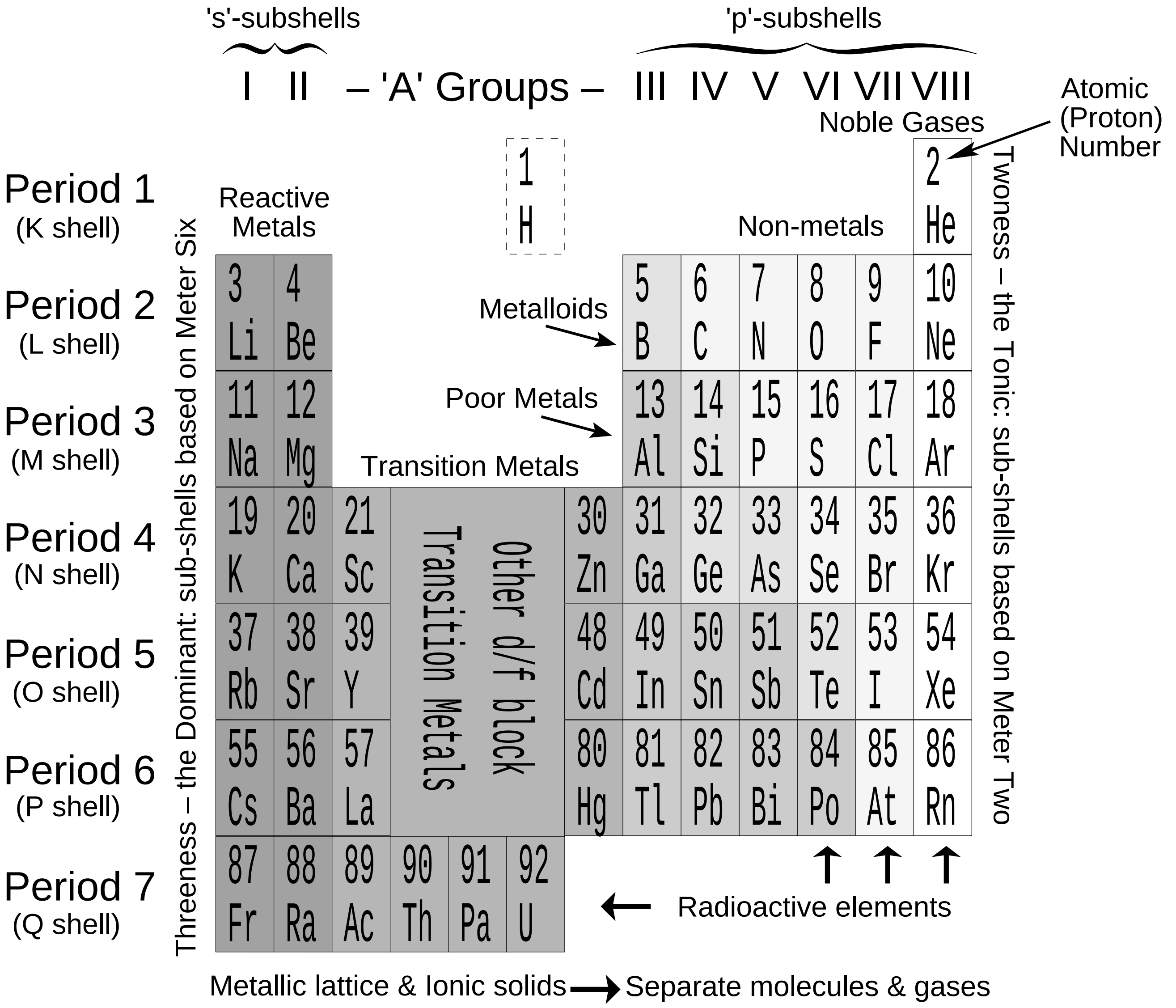


Figure 15.4 Shading in the (truncated) periodic table illustrates how ‘threeness’, that is the metallic characteristics associated with subshells built on a Metric of Six foundations (the dominant relationship in music) progressively spread across the periods, until eventually overturning the whole system at the close of Period 6.

The structures formed by the electrons as they cluster around the nucleus are regular (Figure 15.5), with two electrons combining to create an *orbital* and various numbers of orbitals combining to create *subshells*. The horizontal rows in the periodic table, the Periods 1, 2, 3, etc., form complete structural *shells* created from various combinations of subshells. Each period contains one or more subshells; they are named ‘s’ for sharp (two electrons), ‘p’ principal (six electrons), ‘d’ diffuse (ten electrons) and ‘f’ fundamental (fourteen electrons). These descriptive names originate from the appearance of the characteristic spectral emission lines – discrete frequencies of electromagnetic radiation – released by each element and related to that element’s particular arrangement of electrons. Every period begins with an ‘s’ subshell. The Period 1 atoms, hydrogen and helium, have a shell consisting of only the 1s-subshell. Period 2 has a 2s-subshell and 2p-subshell: the period begins with the reactive metals lithium and beryllium, their electrons forming the 2s-subshell structure, and then the period goes on to build the 2p-subshell with the electrons contributed by boron, a metalliod element, followed by the non-metals carbon, nitrogen, oxygen, fluorine and the unreactive noble gas, neon – the upper sections of Figure 15.4. Electrons have a quality termed ‘spin’ which can have the discrete value of plus or minus one-half, also denoted *spin-up* and *spin-down* – as indicated by the arrows in Figure 15.5. Spin one-half particles move through 720 degrees of rotation before returning to their identity symmetry – a rather wave-like attribute also shared by protons and neutrons.

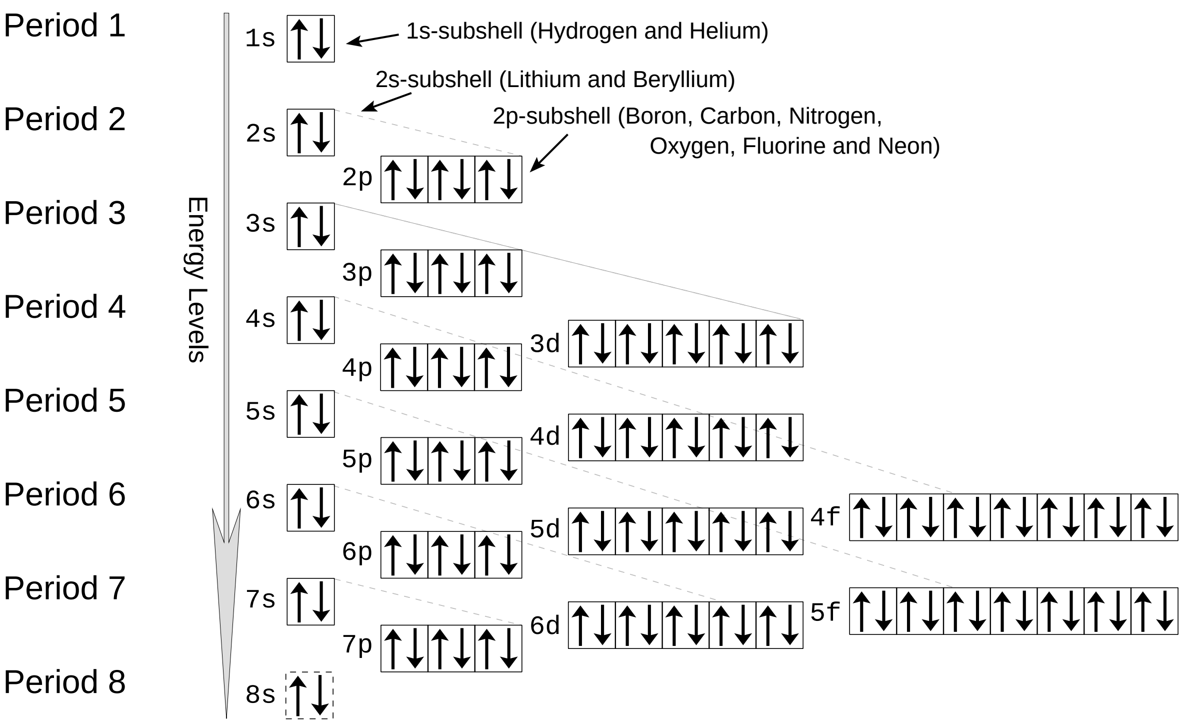


Figure 15.5 The overlapping energy levels of the electrons in their shells (periods), subshells and orbitals. Individual electrons are illustrated as arrows, and orbitals as boxes.

The Full Cadence and Period Two

The justification for applying principles of tonal organisation to atomic structure is based on the recognition of a likeness or parallel between the oscillatory patterns formed by the eight electrons added to the table in period two and the building up of a dominant-seventh chord consisting of eight notes. Both are in effect viewed as *systems of eight ratios* of the harmonic series. The likeness is continued through to the *cadence* at the end of the period where the system of eight ratios/electrons finds stability in the configuration of a family VIIIA noble gas: through the process of resolving (by means of a secondary sesquitertia 3:4 modulation) to a lower energy *system of six ratio*s – the filled p-subshell of a noble gas. Essentially the progress through each period is viewed as a gradual approach to a closing dominant-seventh to tonic *perfect or full cadence* (V^7–I), executed by the atom's outer grouping of eight electrons.

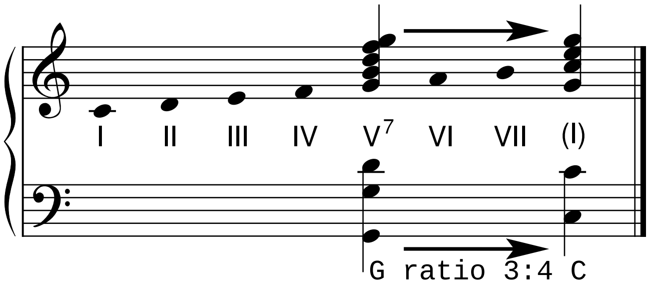


Figure 15.6 Dominant-seventh chord on fifth degree of the C-major scale resolving to the common major chord on the first degree (tonic), a perfect or full cadence mediated by the modulation algorithm of symmetrical exchange.

In this chord progression, eight ratios of the harmonic series built on the note G are exchanged for six ratios built on the note C. The accumulation of eight ratios – musically the dominant-seventh chord: ascending G-G-D-G-B-D-F-G – represents a relatively highly ordered, low-entropy state which is given an opportunity to release energy and order (once it reaches maturity) by a conjunction in the Table of Harmonic Series (Sieve of Eratosthenes) at row 1:24. The system, through the secondary sesquitertia 3:4 modulation, is able to exchange eight ratios of the harmonic series based on G, notionally *frequency 3*, for six ratios built on the note C, *frequency 4*, thus finding increased stability by rearranging its internal relationships into a lower energy pattern with the same overall 'identity' –i.e. interference pattern. This new configuration of the electrons is that of a stable, unreactive noble gas, for example neon.

'Twoness' – the Tonic

Period 1, discounting hydrogen's 'unique' character, exhibits solely 'twoness' – in the form of the stable and almost totally unreactive noble gas helium. Hydrogen with one electron really stands separate from the table, like a system of 'Metric One', it has in a sense, no Meter – a single uniform pulse, metrically undivided. The characteristics with which twoness, or the tonic, imbue an atom (in the outer layer of electron structure), are derived from the system's metrical self-sufficiency; systems based on h1-2 and its doublings are self-sustaining, finished units. Thus the family VIII elements – nested systems of electrons built on the twoness of the sesquitertia 3:4 modulation – form gases: the atoms have no incentive to band together as liquids or solids, or to form molecules. Helium only becomes a liquid at minus 269 degrees celsius and a solid, under considerable pressure, at a fraction above absolute zero (–273 C).

Period 2 contains much 'twoness' also, particularly from carbon onward where there is the possibility of primary sesquitertia 3:4 modulation exchange based on the conjunction at h12, allowing the arrangement of six electrons to relax to a system based on h1-4. Similarly, nitrogen, oxygen and fluorine find twoness in molecular form, all illustrated in Figure 15.7. Other characteristics derived from metrical self-sufficiency are both high and low melting points, and toughness and brittleness! This apparent contradiction flows from the strong and exclusive molecular bonds which twoness engenders. Where the twoness is achieved via molecular association – two or more atoms joining together to make up the number of electrons in their combined outer layers to form a family VIII noble gas configuration – sometimes this will take the form of giant interlocking structures such as diamond, while in others, discrete molecules will be the outcome. Substances formed of giant interlocking bonds will generally be tough and have high melting points as a result of their interwoven type of structure. While substances formed of discrete molecules (again with strong covalent bonds) will generally be brittle (if solid) and have low melting points – as each individual molecule's twoness bonds it together strongly, but only weakly binds it to its molecular neighbours (if at all, as in a gas). Nitrogen, oxygen, fluorine and carbon dioxide molecules are examples of this behaviour.

'Threeness' – the Dominant

'Threeness' first enters the table in Period 2 where the 2s-subshell modulates to h3 of the fundamental nesting series[3] by means of a tripla 1:3 exchange (see lithium and beryllium in Figure 15.7), and similarly each succeeding period follows this pattern to produce s-subshells on h6, h12, etc. The characteristics with which threeness in the outer layer of electrons imbue an atom are derived from an inherent instability. In systems founded on the ratios h1 and h2, metrically, h3 and its multiples produce disruptive ‘unfulfilled’ structures – the unfinished business of the dominant chord. Thus structures of threeness are destined to break down, relaxing back to twoness – the tonic – or where threeness finds a permanent existence, to eventually, by accumulation, break the structure and in so doing, bring the whole system back to a new twoness based on a different frequency level. Though not directly related to the electronic configuration of the atom, perhaps this phenomenon might also be ascribed to a structural change among the protons and neutrons in the nucleus, leading to the radioactive decay found in Period 6/7 of the table?

Between periods 2 and 7 the forward march of threeness is marked by the progressive extension of metallic qualities, such as malleability, ductility and conductivity, gaining ground across the table with each successive period. For example, in Period 2, family VA starts with the tonic-twoness of the gas nitrogen, but by Period 6, toward the bottom of the table, the family exhibits the threeness of a heavy metallic solid, lead.

Hydrogen	h1	MBN	1	value = one
Helium	h1+2	MBN	1×2	value = two
Helium	h1+2	MBN	2×1	(pri. dupla 1:2 modulation)
Lithium	h1+2+3	MBN	1×3	value = three
Lithium	h1+2+3	MBN	3×1	(pri. tripla 1:3 modulation)
Beryllium	h1+2+3+6	MBN	3×2	value = six
Boron	h1+2+3+6+9	MBN	3×3	value = nine
Carbon	h1+2+3+6+9+12	MBN	3×4	value = twelve
Carbon	h1+2+3+4+8+12	MBN	4x3	(pri. sesquitertia 3:4 modulation)
Nitrogen	h1+2+3+6+9+12+15	MBN	3x5	value = fifteen
Oxygen	h1+2+3+6+9+12+15+18	MBN	3x6	value = eighteen
Fluorine	h1+2+3+6+9+12+15+18+21	MBN	3×7	value = twenty-one
Neon	h1+2+3+6+9+12+15+18+21+24	MBN	3×8	value = twenty-four
Neon	h1+2+3+4+8+12+16+20+24	MBN	4×6	(sec. sesquitertia 3:4 modulation)

Molecules--

Nitrogen	h1+2+3+6	MBN	3×2	
(N_2)	4+8+12+16+20+24	&	4×6	value = twenty-four
Nitrogen	h1+2+3+6	&	3×2	
Oxygen	h1+2+3+6+12	MBN	3×2×2	
(O_2)	4+8+12+16+20+24	&	4×6	value = twenty-four
Oxygen	h1+2+3+6+12	&	3×2×2	
Fluorine	h1+2+3+6+12+18	MBN	3×2×3	
(F_2)	4+8+12+16+20+24	&	4×6	value = twenty-four
Fluorine	h1+2+3+6+12+18	&	3×2×3	

Figure 15.7 A table of mutable base numbers in factor format representing the structure(s) of the electrons in periods 1 and 2 in terms of a modulating oscillatory systems analysis. Some elements have more than one structural form (e.g. beryllium). Bottom of table: nitrogen, oxygen and fluorine form molecules in isolation.

Fecund Mutable Numbers

In Chapter 1 it was noted that there was a set of particularly fertile mutable numbers, the numbers constructed from small prime factors (e.g. 3×2×2 MBN twelve). The set of numbers begins:

1, 2, 4, 6, 8, 12, 24, 36, 48, 72, 96, 120, 144, 192...

These are the numbers with the largest range of alternative digit sequences relative to their value. A little later in Chapter 1, the value twenty-four was noted as having twenty alternative mutable digit sequences and two of these twenty digit sequences (3×8 and 4×6) lay behind the full (V^7-I) cadence. Looking at Figure 15.7, it can be seen that the structures possibly created by the electron cloud, when represented as mutable numbers, are actually outlining more or less this set of fertile numbers. And once the evolution of the electronic configuration of the elements reaches the mutable number twenty-four at neon in period 2, it sticks there with the molecular form of the period 2 gases (nitrogen to fluorine) all taking up various different digit sequences of this most fecund number. These four gaseous elements in families V to VIII form a distinct subset and share many characteristics, underlining there common mutable number value. As the periods continue, this common Metric Four heritage in the outer electron layer (gray shading on the right of Figure 15.4) comes under increasing pressure from the underlying Metric Six at the core of all elements – the noble gases excepted. The trend can be seen clearly in the ‘halogens’, the column of family VII, where chlorine retains a gaseous character, but bromine is a liquid and iodine a solid at room temperature. Subsets of elements in a period, with broadly similar characteristics linked to a fecund mutable number, become increasingly influential as the table unfolds. This could be viewed as a reflection of the exponential growth in digit sequences exhibited by mutable numbers and the fecund set of numbers in particular. The foremost examples of this phenomenon being the transition metals and the rare earths of the diffuse and fundamental subshells – two large sets of elements with broadly similar characteristics.

The Chemical Bond

An extension of the metrical approach to the electronic structure of atoms leads naturally on to a similar metrical view of the chemical bond, a little of which can be glimpsed at the bottom of Figure 15.7. The electronic arrangements of the atom exhibit the features of a structure delicately balanced and interwoven, likely to readjust its internal configuration in response to any new event. Even small changes at the extremities may percolate down through many levels of nested structure, in a domino effect, as each level re-computes its relationships and thus creates a new situation for the next lower level. Occasionally, one tiny change at the exterior might cause the entire system to flip to a new configuration (e.g. at the close of a period or when atoms bond to form a molecule), and at others, only a small local responses occur.

The configuration of the nitrogen atom/molecule, N_2, contains much *twoness*, in that nitrogen’s three 2p-subshell electrons could slip away from the *threeness* of the 2s-subshell (h3, 6) to form a not entirely stable nested series built on (h4)+h8+h16+h24 – as illustrated in Figure 15.7. This half-step towards the tonic – the family VIII configuration of neon – is reflected in nitrogen’s natural, non-associative, discrete molecular state as a gas (as also oxygen and fluorine). With each nitrogen atom having three outer layer electrons ($2p^1$, $2p^2$ and $2p^3$) this means that two nitrogen atoms can form a metrical alliance by sharing these outer layer electrons so that together they have the numbers to make up the full 2p-subshell – the noble gas configuration of neon. Put in musical terms, enough ratios/notes to form a dominant-seventh structure of eight ascending tones G-G-D-G-B-D-F-G which reaches up to the conjunction at h24. The resolution of this combined system of eight electrons/ratios to a configuration of six, yields an electronic structure close to that of neon. However, buried below the twoness of this outer grouping of Metric Sixty, lie two atomic nuclei with their close bound electronic structures, each of which harbours the threeness of Metric of Six – the dominant. Illustrated in Figure 15.8 as ratios of the harmonic series on the left and electron configurations on the right.

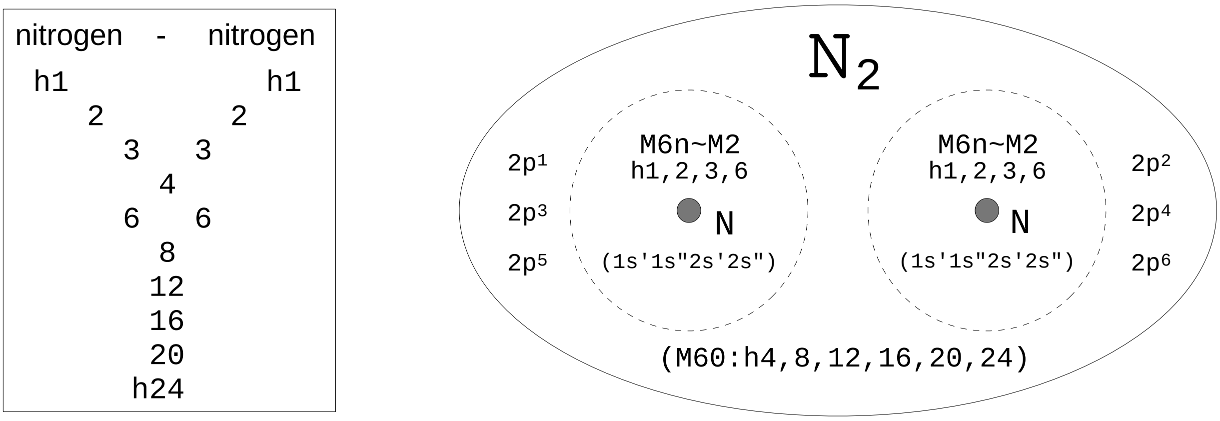


Figure 15.8 Two schematic views of the oscillatory patterns of the electron cloud in a nitrogen molecule.

It is also possible to represent the MOS model and mutable number approach to the vibrational configuration of the nitrogen molecule as chords.

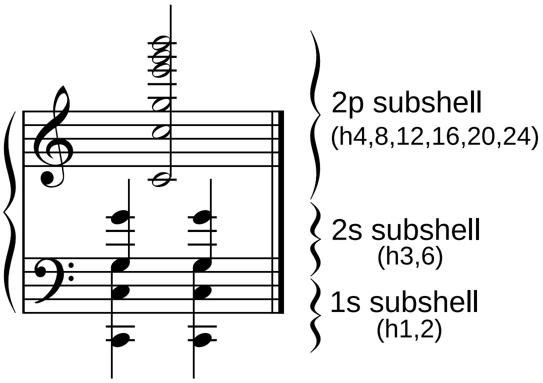


Figure 15.9 The MOS view of the oscillatory patterns of the electronic configuration of a nitrogen molecule (N_2), expressed in musical notation.

Other Self-organising Physical Systems

Though the periodic table of elements is by far the most detailed and useful *reference structure* against which the rather murky and less certain structures of tonal music could be analysed and assessed (and not forgetting the very helpful information available on isotopes and allotropes as well) two other physical systems have also provided some inspiration at times. They are, on the macroscopic level, the celestial mechanics of the *solar system* and at the other end of the scale, the shell structures of the atomic nucleus.

Viewed in terms of orbital periods or resonances (and using the harmonic series rather than arithmetic ratios for consistency), the planets of the solar system, when plotted against each other, appear to echo the telltale pattern of a modulating oscillatory system: A structure through which flows a chain of 2:3 relationships leading to a nested harmonic series built on the 2:3×3 relationship – the dominant of the dominant (V^7 of V^7 in music theory). In tonal music this structure would be described in terms of an underlying tonic key or tonal center (nominally C major), the dominant chord (G^7) and the dominant-of-the-dominant chord (D^7). All these features are presented in the diagram, Figure 15.10. Indeed, the structure is somewhat reminiscent of the electronic arrangement of the atoms toward the end of the 5d-subshell described in the articles *Elements of Music?* (http://www.archivemag.co.uk/). However, any similarities or parallels would be due to the application of the underlying principle of the modulation algorithm of symmetrical exchange lending a ‘family resemblance’ to any dynamic system of simple whole number relationships, upon which it operates. The actual mechanics of the modulatory process bring about such resonant alignments of the planets shown below, and recorded in Bode’s Law, presumably being accomplished by the gravitational effects of scattering, absorption and deflection of large amounts of planetesimal debris during the early formative period of the solar system.[4] Ultimately in this MOS and mutable base number approach, planetary systems are reduced to physical position-value counting schemes, though most often of a rather awkward and incomplete form, which grind to a halt when the process of modulation runs out of material to fuel further computation.

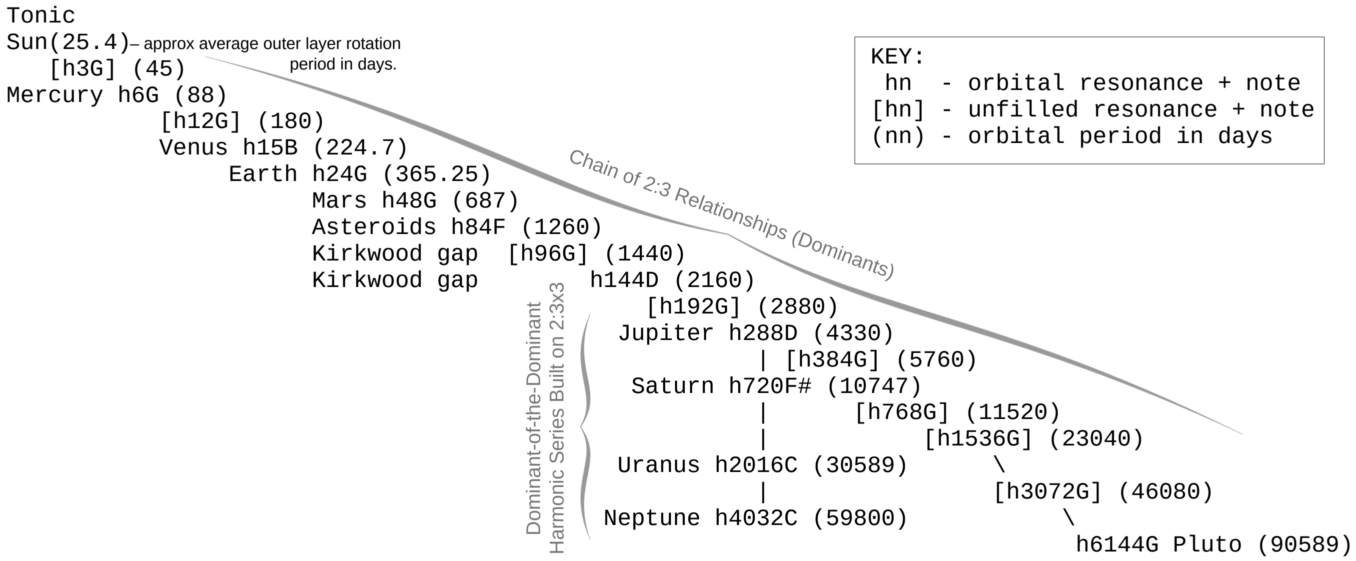


Figure 15.10 The orbital periods of the planets of the solar system (illustrated as harmonic rather than arithmetic ratios for consistency). The inner planets together with the Asteroid Belt, Jupiter and Pluto form a G dominant-seventh harmony, while the gas giants (including Jupiter) outline an incomplete D dominant-seventh harmony.

These essentially tonal relationships, expressed in terms of planetary distance from the Sun rather than orbital period, were noticed by Johann Titus (1729–96) and published by J.E. Bode in 1772 (Titus–Bode Law). In this law the base sequence: 0, 3, 6, 12, 24, 48, 96 and 192 (i.e. the dominant relationship in music and also fecund numbers) is manipulated by adding four to each term to produce a set of relationships which matches well for all the planetary distances, except for Neptune. The confirmation provided by the discovery of Uranus by William Herschel at roughly the right distance (term 192) prompted a search for a ‘missing planet’ corresponding to the base term 24. The search led to the discovery of Ceres and also in time many other asteroids. Gauss was able to calculate the orbit of Ceres and this feat established his mathematical reputation throughout Europe. In the twentieth century the tilted orbit of Pluto, probably a late captured ‘planet like object’, was found to match the term 384 quite closely.

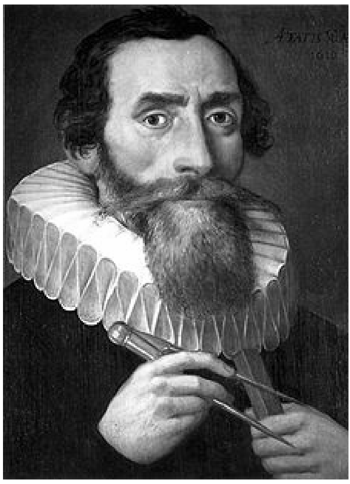
While of course the exact orbital periods are not perfectly aligned with the harmonies of dominant and dominant by one remove (V^7 of V^7), they are near enough to motivate curiosity, or at least justify raising the question of why they are thus and not otherwise: Perhaps a different configuration of gravitational resonances yielding whole-tone harmonies or merely some random arrangement? For example, the ratio of the Earth’s orbital period to that of Jupiter’s is the very musical relationship of 1:12 (with an comma of minus 1.22%) or the gravitational resonance between the Kirkwood gaps in the asteroid belt and Jupiter's orbit the dominant ratio of 1:3. However in contrast, it should be mentioned here that there does not appear to be clear evidence for a fundamental 'tonic' period associated with the Sun. The Sun's outer layer rotates in a rather fluid manner (average 25.4 days), not as a single solid unit, with the equatorial region moving somewhat more quickly than the poles. Near-pole rotation is in the range 33 to 36 days and the Sun's polar axis of rotation is 7.5 degrees off perpendicular to the planetary disc. Though the precise rotation of the core is unknown it is generally thought to be between three and five times that of the outer layer, moving as a rigid body.

To round up this extremely adventurous and exploratory view of the orbital relationships of the solar system, it should be possible to make an interpretation of the structure as a mutable number. Taking only the planets and other objects lying in the plane of the disc that number is MBN $28_6\ 0_8\ 0_3\ 0_1$ or in decimal notation 4032 (excluding Pluto's tilted orbit). This number, expressed as nested ratios with the planet's positions marked by upper case letters is:

h1, 2, 3 nesting 6, 9, 12, 15, 18, 21, 24 nesting 48, 72, 96, 120, 144 nesting 288, 432, 576, 720,... 2016,... 4032
Sun................M...........V................E...............M.........Asteroids...................J.....................S..........U..........N

Neptune is the one planet Bode's Law fails to predict, with a distance from the Sun roughly halfway between term 192 and Pluto at term 384 of the series. In contrast the MOS model of mutable numbers is able to accommodate all the planets including Neptune. Interestingly, recent observations have found evidence for a similarity of distribution in other planetary systems[5] (e.g. pulsar PSR B1257+12), which has apparently bolstered the search for an underlying organisational principle in celestial mechanics.

An interesting feature of this interpretation is the possibility of viewing the asteroid belt as an arena of modulation, an area where the algorithm of symmetrical exchange is at work bringing about a realignment of what was perhaps a planet broken up by gravitational stresses or original planetesimals long corralled. From such a perspective, the arrangement (or changes in the arrangement) of the outer group of planets from Jupiter to Neptune could be seen as percolating down into the next layer of nesting, stirring and reconfiguring these relationships. The process proceeding at the pace of a glacial minuet.



Picture courtesy Wikipedia

Johannes Kepler (1571–1630), the son of a soldier, was born at Weil der Statt near Stuttgart. Though Kepler's grandfather had been wealthy, and Mayor of Weil, his father Heinrich Kepler had not prospered and died early in Johannes' life, leaving his mother Katharina (nee Guldenmann) an innkeeper's daughter and healer to see to his upbringing. Johannes Kepler was not favored with a strong physical constitution, but from an early age showed remarkable natural mathematical prowess. His interest in astronomy grew out of childhood events where he was much struck by seeing comets and an eclipse, an interest perhaps also encouraged by his mother's involvement with the supernatural. In later life Katharina Kepler was to be tried for witchcraft, and luckily, after three tense years, acquitted. Johannes Kepler's education in local protestant schools led on to studies in philosophy, mathematics and astronomy at the University of Tubingen, where he received a Masters degree in 1591. Although Kepler had originally intended to become a Lutheran minister, and began theological studies, in 1594 the offer of a post teaching mathematics at the protestant seminary in Graz tempted him away. Kepler had long accepted the Copernican cosmological system which placed the Sun at the center of the solar system, and while teaching astronomy at Graz he found a way of inscribing the platonic solids in a particular order – octahedron, icosahedron, dodecahedron, tetrahedron, cube – within spheres carrying the then believed circular orbits of the known planets. The results of his speculations were published in the treatise *Mysterium Cosmographicum*, 1596; and this book established his reputation and paved the way for his later career as Imperial astronomer/astrologer. In this work, and throughout the rest of his life, Kepler united a deeply religious streak of mysticism (perhaps stemming from his mother's influence) with a rigorous mathematical mind and attention to detail. The latter trait was over the years to cause Kepler much disquiet, and eventually also to be the source of his greatest achievement, in that the accuracy of his observations (he invented an improved astronomical telescope) forced him to abandon the metaphysical perfection of circular planetary orbits and substitute ellipses. The three planetary laws of motion buried within his writings were a significant step toward an empirical astronomy, and the foundation for Issac Newton's discovery of the inverse square law of gravitational attraction. Johannes Kepler married Barbara Muller, a well-to-do widow of 23 years in 1597; they had five children together, the first two dying in infancy. The times were turbulent, and as the divisions between Catholics and Protestants became increasingly bitter, Kepler's ambitions to develop the geometric ideas presented in *Mysterium* where frustrated. However, by 1601 his contacts with Tycho Brahe, the Imperial mathematician and astronomer based in Prague, brought the offer of a post working as his assistant, and, on Tycho Brahe's unexpected death in the same year, Kepler was appointed as the new Imperial mathematician with the commission to complete the Rudolphine astronomical tables. In the years following 1601, Kepler used and improved upon Tycho's large body of observations and also produced new works: on optics, *Astronomiae Pars Optica,* and a number of astronomical treatises furthering his ideas and speculations. In one of these, *Harmonices Mundi (Harmony of the Worlds)* published in 1619, Kepler attempted to extend his geometric interpretation of divine order amongst the planets to a broader system of proportion, modeled in part on musical theory and practice, but also involving astrology. Again using the best data, coupled with a prodigious mathematical talent, Kepler eventually found that by determining the maximum and minimum orbital velocities of the planets he could interpret their motion, as seen from the Sun, in terms of musical scales. In this scheme the Earth moves through the interval of a semitone and Mercury a minor-tenth. He believed he had uncovered the real basis of the "music of spheres", postulated by ancient philosophers. Essentially, in *Harmonices Mundi* Kepler provided the raw material which, in combination with Christian Huygens' developments in dynamics, would underpin Newton's physically based empirical mechanics. In later life, owing to war and religious turmoil, Kepler moved from Prague to Linz and other temporary havens, married again, happily, to Susanna (nee Reuttinger) and continued writing treatises, the most influential being the *Epitome of Copernican Astronomy*. He died in Regensberg Barvaria, of a fever, aged 58 years.

The Nucleus

Some years after developing the MOS (Modulating Oscillatory Systems) model and applying the idea to the analysis of tonal music, I became aware of a striking parallel between the sequence of primary modulation exchanges (tabulated in Figure 9.4 and traced out by the dotted line in Figure 9.2) and the arrangement of the basic 'shell' structure observed in the atomic nucleus. In a manner similar to the arrangement of electrons, the constituent protons and neutrons in the atomic nucleus form two separate 'shell' structures built on a succession of states and energy levels. For historical reasons connected with earlier work on the electronic configuration of the atom these nuclear subshells are also denoted: s, p, d, f, g, h, i, etc. The s subshell contains two nucleons, the p subshell six nucleons, the d ten, the f fourteen, etc., but unlike in the electron structure, all subshell types are represented at all levels. Thus, there is a 1p nuclear subshell, a 1d, a 1f and so on. Notwithstanding this difference, as in the atomic configuration, the effects of spin and orbital interaction between particles contributes to the fine detail of subshell structure. By convention the letter 'j' is used to represent the total amount of these interactions and 'n' for the level.

Level	**Parity**	**2 States**	**4 States**	**6 States**	**8 States**	**10 States**	**12 States**	**Total**
Spin:		(j = 1/2)	(j = 3/2)	(j = 5/2)	(j = 7/2)	(j = 9/2)	(j = 11/2)	
n=0	Even	1s						2
n=1	Odd	1p	1p					6
n=2	Even	2s	1d	1d				12
n=3	Odd	2p	2p	1f	1f			20
n=4	Even	3s	2d	2d	1g	1g		30
n=5	Odd	3p	3p	2f	2f	1h	1h	42

Figure 15.11 Table of nuclear shells. Notice that most subshells are split allowing nucleons to occupy a variety of energy levels and thereby produce the complex overall configuration apparent from observation – Figure 15.13.

The nuclear shell structure exhibits something of a nested, cellular automata, character (e.g. triangles within triangles in Figure 5.3). Indeed, immediately upon examining Figure 9.4 the sequence of cardinal harmonics (h2, 6, 12, 20, 30, 42, 56, 72, ...) generated by the primary modulation exchanges (dupla, sesquialtera, etc.) yields results in line with that of an ideal three-dimensional harmonic oscillator which provides one starting point for nuclear modelling. To generate the underlying pattern of nuclear shells from the Table of Harmonic Series (Figure 9.2) would only require a simple rule: *Recursively compute primary modulation exchanges for each level, beginning with the largest* – illustrated in Figure 15.12. The AWK script ‘`ns_awk`’ contained in the Chapter 19 archive implements this simple recursive rule, ‘reprocessing the expelled ratio/oscillators’ from ‘higher’ primary modulation exchanges, recursively, to produce a nested subshell structure – an analog of the proton/neutron nucleus. Thus reading the rows of Figure 15.12 from right to left mimics the recursive algorithm employed by the AWK script.

Level	**Parity**	**2 States**	**4 States**	**6 States**	**8 States**	**10 States**	**12 States**	**Total**
Modulation:		(2:1)	(2:1, 3:2)	(2:1, 3:2, 4:3)	(2:1, thro’ 5:4)	(2:1 thro’ 6:5)	(2:1 thro’ 7:6)	
n=0	Even	1s Dupla						2
n=1	Odd	1p Dupla	1p Sesquialtera					6
n=2	Even	2s Dupla	1d - ditto	1d Sesquitertia				12
n=3	Odd	2p Dupla	2p - ditto	1f - ditto	1f Sesquiquarta			20
n=4	Even	3s Dupla	2d - ditto	2d - ditto	1g - ditto	1g Sesquiquinta		30
n=5	Odd	3p Dupla	3p - ditto	2f - ditto	2f - ditto	1h - ditto	1h Sesquisexta	42

Figure 15.12 Table of nuclear shells generated by a cellular automata like rule applied to sequences of primary modulation exchanges (i.e. repeatedly following the dotted line in Figure 9.2).

Although the physical reality of the nucleus is rather more complex than any such ideal model, due to the many interactions between the constituent particles, it is rather remarkable that the central organising principle in tonal music – modulation – should exhibit such a close correlation with what is one of the most basic structures of the material world. This shell structure clearly evident at both the atomic and nuclear levels requires a satisfactory explanation. Although Quantum Mechanics yields result in startling agreement with observation and is undoubtedly a triumph of predictive science, it cannot be considered complete as a theory until it also explains why these particular shell patterns emerge.

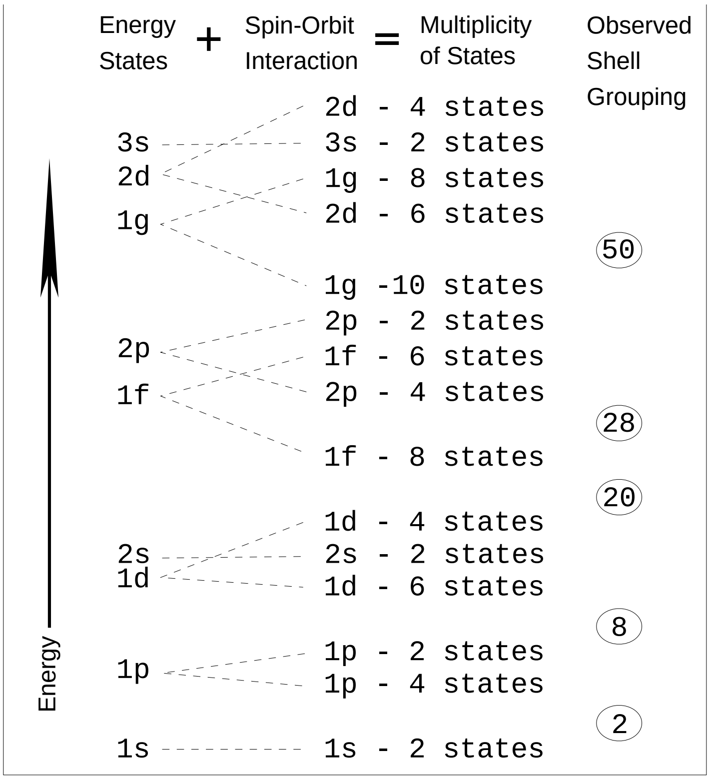


Figure 15.13 The observed energy levels of the nucleus disguise the structure's underlying harmonic origins.

By writing out the sequence of atomic shell structures as sets it can be seen that each structure is reiterated, or perhaps better, *passed through*, in subsequent structures. In the table below a hyphen between figures, e.g. h4-3, indicates the change induced by a modulation exchange.

Dupla	{h1, 2} = 2 states
Sesquialtera	{{h1, 2}, h4-3, h6} = 4 states
Sesquitertia	{{{h1, 2}, h4-3, h6-4}, h9-8, h12} = 6 states
Sesquiquarta	{{{{h1, 2}, h4-3, h6-4}, h9-8-5, h12-10}, h16-15, h20} = 8 states
Sesquiquinta	{{{{{h1, 2}, h4-3, h6-4}, h9-8-5, h12-10-6}, h16-15-12, h20-18}, h25-24, h30} = 10 states

Finally, the number twelve, the point where the spiral/cycle of tonal centers 'returns to itself', as well as featuring in the examples from the periodic table and the solar system, not to mention the scales and keys of tonal music, also crops up in the table of fundamental particles. In all the experiments, observations and analyses available today, using the highest energies and finest measurements, there have been found to be just *twelve* basic matter particles. The twelve particles (or twenty-four if their anti-matter partners are considered separately as in the twenty-four major and minor keys) form family groupings in threes, with a similarity of character running horizontally across the table.

Particle Family One	Particle Family Two	Particle Family Three
Electron	Muon	Tau
Electron-neutrino	Muon-neutrino	Tau-neutrino
Up-quark	Charm Quark	Top Quark
Down-quark	Strange Quark	Bottom Quark

Figure 15.14 The table of fundamental particles.

CONCLUSION

The question mark against the title *Elements of Music?* is more than decoration. Whether the exploratory ideas hinted at above, in regard to underlying processes and structures in the material world, have any chance of seriously linking up with rigorous scientific method, lie far beyond my expertise. Though the general trend in thinking, in the speculative fields attempting to uncover or construct a unified fundamental theory of the physical world, is tipping towards the principles and methodologies of systems formed of discrete units (e.g. quantum mechanics, string theory, quantum gravity). And a certain interconnected logic to schemes founded on simple whole number ratios gives some encouragement that, as well as (hopefully) offering a compelling analysis of tonally organised music, to some degree, mutable numbers and modulating oscillatory systems just might have something constructive to offer other disciplines. So being ever the naive and foolish optimist, I have recorded them here. Yet of course, it must be emphasised that the MOS concept and model is in no way an established method or accepted approach in music theory or any other theory for that matter; it remains entirely possible, probable even, that the whole idea is simply mistaken; fatally and irredeemably wrong.

Overall, this chapter takes a set of ideas hopefully appropriate for understanding the core harmonic structure of western tonal music, and transfers them across to other systems in the material world, in the hope of acquiring a deeper understanding of the systems themselves, music, and the wider world they all inhabit. An understanding of how, at the most basic level, the *simplest of relationships* could interact through the *modulation algorithm of symmetrical exchange* to develop structure, process information and create variety –i.e. the modulating oscillatory systems model. My instinct, from first glimpsing the patterns within the table of (nested) harmonic series, the THS – mathematicians might prefer TSE, The Sieve of Eratosthenes – has been that there might be something here of interest, indeed, something beautiful: to find that Bach and Beethoven, created musical structures which appear to obey essentially the same disciplines that scientists study, seek to understand and attempt to codify, would be yet another wonderful confirmation of the underlying oneness of the 'Great World' in which we live.

I have tried to give some sense of the possibly broad scope and application of *tonalism*, for which there are tantalising hints, at scales both above and below the level of individual atoms found in the periodic table of the elements; that is, in the harmonic 'flavor' of the planetary Bode–Titus Law, the nuclear shell structures of protons/neutrons and the pattern of the *twelve* fundamental matter particles. (And yet, quite frankly, I must admit that this chapter, and probably the whole book, resembles something like the wreckage from a head-on collision between an enthusiasm for music and some rather shaky high school mathematics and science, from which, I fear, the reader perhaps emerges in a dazed and confused state!) However, notwithstanding the many shortcomings, by putting the case, by making the argument, the ideas at least gain an airing, and, while I intend to continue investigating the musical aspects of *tonal computation*, it is my hope that others more knowledgeable in mathematics and the sciences, might find tonalism an interesting enough approach to consider investigating further.

When those first pedestrians set foot on the Millennium Bridge across the Thames, and found to their surprise that the structure began to sway as their random steps were guided into a common metrical pattern, they would perhaps not have realised that this somewhat worrying turn of events was an entirely natural phenomenon. The elasticity of the span provided a means of communication and interrelationship binding the throng together into a whole, a system, a 'little world'. Acting as such a unit, the bridge's ‘elastic field’ induced the pedestrians to convert the disorganised complexity of their individual gaits into an ordered structure. The Millennium Bridge, by providing a means of communication between the pedestrians, drew their randomly distributed steps, gradually, into a commensurable meter. Effectively the bridge was converting the high level of entropy implicit in the individual pedestrian’s varied and uncoordinated paces, into an explicit form of order – a Metric. Metrics are mutable numbers, and, where Metrics are sufficiently complex (i.e. Metrics nested within themselves), mutable number processing via the modulation algorithm is possible, or its equivalent description – structural change in material systems. Likewise in western music, once the ancient Pythagorian inheritance of scale steps consisting of whole-numbered proportions, was bound within a system of multiple melodic threads, the resultant communication across the ‘tonal field’ engendered by polyphony, ineluctably drew musicians towards the discovery of mutable numbers – the metrics of tonality. Similarly perhaps, across the vastly broader arena of the ‘great world’, electromagnetism, gravity and the other forces of nature conspire, at very different scales, to draw together and order the systems of particles and planets. And perhaps aeons later, mathematicians (and musicians) observed and co-opted a little of this material order, to furnish the self-evident foundations of their own endeavours – science and tonal music.

Of course, outward concordance or near concordance doesn’t necessarily lead to a true understanding of inner mechanisms. The Ptolemaic astronomical model of the second century AD, which held sway for so long and so well accounted for the apparent wandering motion of the planets, relative to their starry background (with a scheme of epicycles), was misguided. Copernicus’s simple revolutionary idea that the Sun rather than the Earth might lie at the center of the solar system eventually ousted it – after Copernicus’s circular orbits were replaced by Kepler’s ellipses. The Sun appears to circle the rotating Earth (viewed from the Earth’s inhabited regions) but sometimes appearances are deceptive. Equally, the *appearance* of musical patterns and processes in other systems doesn’t necessarily have to mean or lead on to anything more than blind coincidence, or mistaken observation. And here I should be absolutely clear that I do not have evidence beyond the rather vague ideas outlined above for the proposition that relational processes found in tonal compositions – computation with mutable numbers – might also turn up in other corners of the physical world. However, the existence of these patterns does at least raise suspicions and justify asking questions. Why this particular number of fundamental matter particles? Why this particular structure of planets in the solar system? Why this particular pattern of atomic and nuclear shells and subshells?

It has taken a considerable effort (and much luck) to identify and extract the mutable base number system from the flux of complicated patterns typically comprising tonal compositions of the western tradition. Hints of a numerical underpinning to tonal music abound, almost every area of music reeks of numbers – rhythm, time signatures, harmony, scales, figured bass, etc. – yet to precisely decipher the underlying mechanisms of mutable numbers was neither easy nor straightforward. How much more true might this be of the vastly greater complexity to be found in the processes of the physical world? Our understanding of the material world has progressed in leaps and bounds, particularly from the beginnings of the scientific revolution in the seventeenth century. And it is clearly apparent, to quote Galileo, that nature’s open book “is written in the language of mathematics”. Indeed almost all of science’s successes in unravelling nature’s mysteries, are likewise redolent of mathematics. It is upon this basis that the proposition outlined in the Introduction rests: *that western tonal music is one member of a perhaps broad set of oscillatory systems found in the material world, which are all, ultimately, examples of physical position-value counting structures*. It is no more than the simple guess that, like music, the material world might also be understood, fundamentally, in terms of dynamically evolving whole number relationships; that ultimately these relationships will delineate a form of number system; and that changes in such a number system will amount to number processing – computation.

In essence the idea is far from new, going back at least to the Pythagorean view of the underlying role played by number in the world. And in a later epoch Johannes Kepler similarly constructed, around the orbs of planetary motion, a beautiful mathematical scheme of inscribed and circumscribed three-dimensional platonic solids, which he took to be evidence of the divine ordering of the world. Yet, despite his mystic and metaphysical inclinations, as the years passed and his observations became more accurate, he was forced to yield this static prescriptive model for a more dynamical approach involving harmony and proportion, which, rather than being decreed by perfect, abstract, immaterial forms, emerged naturally from the actual motions of the planetary bodies themselves. Likewise, in contrast to the orthodox Platonistic approach, which situates 'abstract' mathematics in some way beyond material existence – external to physical reality – the mutable base numbers of the MOS model are postulated to arise from processes, and consist of relationships, worked out wholly within the systems they inhabit – whether that be the 'little world' of western tonal music or the 'great world' of the material universe.

Whilst all the mistakes in this book are wholly mine, many of the ideas presented are drawn from a wide range of writers, thinkers and scholars – upon whose footnotes I am not qualified to tread. And I have been acutely aware throughout, of how far beyond any sensible limits of competence the piano tuner has strayed in telling this story. If by some miracle of endurance you have managed to make it to the end of the book, I would like to thank you, the forbearing reader, for your company on the journey.

On that quiet Sunday morning in 2003, when the idea for a new music data format first dawned, in a sense, and totally by accident or even mischance, an unnoticed door opened on to the garden of music, revealing a vista strange, yet not wholly alien. The familiar plants and flowers of tonal music could be decerned amongst the serried ranks of the Table of Harmonic Series, but their usual undisciplined blowsy lushness and natural freedom to grow, entangled by the arbitrary fancies of composers and musicians, was absent. In place of human creativity and invention, the stark parterres I saw appeared to be governed by some strict, unbending mathematical law. That vista glimpsed through the first small crack was entrancing, coldly beautiful in its logic and economy, though not immediately intelligible overall. Intrigued, I pushed again and stepped through the low doorway to begin a long, at times arduous and occasionally exhilarating, *Journey to the Heart of Music.*

PERIODIC TABLE
with the Electronic Configuration of Atoms

	Group VIII
Atomic/Proton Number	2
Chemical Symbol	He
Number of Electrons in Subshell	***2***

K shell

(Dupla 1:2 Exchange, 2–>1 ratios)

<– 1s subshell

	IA	IIA
1s	1 H ***1***	2 He ***2***

L shell – Period 2

(Primary Sesquialtera 2:3 Exchange: 3–>2 ratios)

<– 2s subshell

(Secondary Sesquitertia 3:4 Exchange: 8–>6 ratios)

2p-subshell –>

	IA	IIA	IIIA	IVA	VA	VIA	VIIA	VIII	
2s	3 Li 2 ***1***	4 Be 2 ***2***	5 B 2 ***3***	6 C 2 ***4***	7 N 2 ***5***	8 O 2 ***6***	9 F 2 ***7***	10 Ne 2 ***8***	2p

M shell – Period 3

(Primary Sesquialtera 2:3 Exchange: 3–>2 ratios)

<– 3s subshell

(Secondary Sesquitertia 3:4 Exchange: 8–>6 ratios)

3p subshell –>

3s	11 Na 2 8 ***1***	12 Mg 2 8 ***2***	13 Al 2 8 ***3***	14 Si 2 8 ***4***	15 P 2 8 ***5***	16 S 2 8 ***6***	17 Cl 2 8 ***7***	18 A 2 8 ***8***	3p

N shell – Period 4

(Secondary Sesquiquarta 4:5 Exchange: 10–>8 ratios)

(Primary Sesquialtera 2:3 Exchange: 3–>2 ratios)

<– 4s subshell

3d subshell –>

			IIIB	IVB	VB	VIB	VIIB	V I I I B			IB	IIB							
4s	19 K 2 8 8 ***1***	20 Ca 2 8 8 ***2***	21 Sc 2 8 9 ***2***	22 Ti 2 8 10 ***2***	23 V 2 8 11 ***2***	24 Cr 2 8 13 ***1***	25 Mn 2 8 13 ***2***	26 Fe 2 8 14 ***2***	27 Co 2 8 15 ***2***	28 Ni 2 8 16 ***2***	29 Cu 2 8 18 ***1***	30 Zn 2 8 18 ***2***	31 Ga 2 8 18 ***3***	32 Ge 2 8 18 ***4***	33 As 2 8 18 ***5***	34 Se 2 8 18 ***6***	35 Br 2 8 18 ***7***	36 Kr 2 8 18 ***8***	4p

O shell – Period 5

(Primary Sesquialtera 2:3 Exchange: 3–>2 ratios)

<– 5s subshell

4d subshell –>

5s	37 Rb 2 8 18 8 ***1***	38 Sr 2 8 18 8 ***2***	39 Y 2 8 18 9 ***2***	40 Zr 2 8 18 10 ***2***	41 Nb 2 8 18 12 ***1***	42 Mo 2 8 18 13 ***1***	43 Tc 2 8 18 13 ***2***	44 Ru 2 8 18 15 ***1***	45 Rh 2 8 18 16 ***1***	46 Pd 2 8 18 18 ***0***	47 Ag 2 8 18 18 ***1***	48 Cd 2 8 18 18 ***2***	49 In 2 8 18 18 ***3***	50 Sn 2 8 18 18 ***4***	51 Sb 2 8 18 18 ***5***	52 Te 2 8 18 18 ***6***	53 I 2 8 18 18 ***7***	54 Xe 2 8 18 18 ***8***	5p

4f subshell

6s	55 Cs 2 8 18 18 8 ***1***	56 Ba 2 8 18 18 8 ***2***	57 La 2 8 18 18 9 ***2***	58 Ce 2 8 18 19 9 ***2***	59 Pr 2 8 18 21 8 ***2***	60 Nd 2 8 18 22 8 ***2***	61 Pm 2 8 18 23 8 ***2***	62 Sm 2 8 18 24 8 ***2***	63 Eu 2 8 18 25 8 ***2***	64 Gd 2 8 18 25 9 ***2***	65 Tb 2 8 18 27 8 ***2***	66 Dy 2 8 18 28 8 ***2***	67 Ho 2 8 18 29 8 ***2***	68 Er 2 8 18 30 8 ***2***	69 Tm 2 8 18 31 8 ***2***	70 Yb 2 8 18 32 8 ***2***	71 Lu 2 8 18 32 9 ***2***	72 Hf 2 8 18 32 10 ***2***	73 Ta 2 8 18 32 11 ***2***	74 W 2 8 18 32 12 ***2***	75 Re 2 8 18 32 13 ***2***	76 Os 2 8 18 32 14 ***2***	77 Ir 2 8 18 32 15 ***2***	78 Pt 2 8 18 32 17 ***1***	79 Au 2 8 18 32 18 ***1***	80 Hg 2 8 18 32 18 ***2***	81 Tl 2 8 18 32 18 ***3***	82 Pb 2 8 18 32 18 ***4***	83 Bi 2 8 18 32 18 ***5***	84 Po 2 8 18 32 18 ***6***	85 At 2 8 18 32 18 ***7***	86 Rn 2 8 18 32 18 ***8***	6p
7s	87 Fr 2 8 18 32 18 8 ***1***	88 Ra 2 8 18 32 18 8 ***2***	89 Ac 2 8 18 32 18 9 ***2***	90 Th 2 8 18 32 18 10 ***2***	91 Pa 2 8 18 32 20 9 ***2***	92 U 2 8 18 32 21 9 ***2***	93 Np 2 8 18 32 22 9 ***2***	94 Pu 2 8 18 32 24 8 ***2***	95 Am 2 8 18 32 25 8 ***2***	96 Cm 2 8 18 32 25 9 ***2***	97 Bk 2 8 18 32 27 8 ***2***	98 Cf 2 8 18 32 28 8 ***2***	99 Es 2 8 18 32 29 8 ***2***	100 Fm 2 8 18 32 30 8 ***2***	101 Md 2 8 18 32 31 8 ***2***	102 No 2 8 18 32 32 8 ***2***	103 Lr 2 8 18 32 32 9 ***2***	104 Rf 2 8 18 32 32 10 ***2***	105 Db 2 8 18 32 32 11 ***2***	106 Sg 2 8 18 32 32 12 ***2***	107 Bh 2 8 18 32 32 13 ***2***	108 Hs 2 8 18 32 32 14 ***2***	109 Mt 2 8 18 32 32 15 ***2***	110 Uun 2 8 18 32 32 16 ***2***	111 Uuu 2 8 18 32 32 18 ***1***	112 Uub 2 8 18 32 32 18 ***2***	113 Uut 2 8 18 32 32 18 ***3***	114 Uuq 2 8 18 32 32 18 ***4***	115 Uup 2 8 18 32 32 18 ***5***	116 Uuh 2 8 18 32 32 18 ***6***	117 Uus 2 8 18 32 32 18 ***7***	118 Uuo 2 8 18 32 32 18 ***8***	7p
	7s subshell		5f subshell														6d subshell										7p subshell						

Figure 15.15 The full periodic table of the elements showing the atomic numbers (Z), chemical symbols and electron configurations (shells and subshells as cumulative totals) of the atoms.

Notes

1. Smolin, L., *The Life of the Cosmos*, (Oxford University Press, 1997). Many of the ideas expressed in this section are based on material to be found in Lee Smolin's books and other writings.

2. A recurring pattern of properties is also exhibited in the Transition Metals ('d' block) and Rare Earths ('f' block) sections of the Periodic Table.

3. Hydrogen a gas under normal conditions takes on a metallic character under extreme conditions of temperature and pressure. Emsley, J., *Nature's Building Blocks*, (OUP 2001) page 183.

4. Hahn, J., & Tsiganis K., Gomes R., Morbidelli A., Levison H.F., *When Giants Roamed*, (Nature Vol.435 26/05/2005).

5. Gribbin, J., *Companion to the Cosmos*, (Weidenfeld & Nicolson, London, 1996) Bode's Law, page 66.