

Journey to the Heart of Music

Philip Perry

Copyright © P.J.Perry 2003, 2006, 2009. All rights reserved.

The right of Philip J. Perry to be identified as the author of this work has been asserted by him in accordance with the UK Copyright, Designs and Patents Act, 1988.

Chapter 11 – The Minor Mode

DUALISM

In this chapter the idea of *Harmonic Dualism*, an elegant but largely discredited tonal theory, founded upon a conception of inverse partnership between the major and minor triads, chords, scales and keys, is re-examined and set in the wider context of a system of nested harmonic series. Principally developed in Germany during the second half of the nineteenth century by Moritz Hauptmann, Arthur von Oettingen and Hugo Riemann, dualism has failed to find general long term acceptance amongst music theorists, due in part to its central tenet appearing to be contradicted by natural science (i.e. the existence of *undertones* – fractional standing waves). However, below, one crucial aspect of the theory – the ‘root’ of a minor triad – is explored and developed within the context of mutable base numbers; leading to the conclusion that it might be possible to reconcile some elements of dualism with acoustics. This new approach to dualism is based on the mechanisms of the broader model of *Modulating Oscillatory Systems*.

In mathematics and the physical sciences, I believe it is rare, perhaps even unknown, for a truly elegant idea to be wrong. Although this observation may not be quite so trustworthy for all disciplines, and leaving aside the difficult question of defining true elegance, sometimes, a discipline may be in possession of what appears to be a beautiful idea, but only poorly and imprecisely understand in what manner or context the concept might be rightly applied. The idea of *harmonic dualism* could fall into this category. The most fully developed and satisfactory account of dualism, a tonal theory which places the major and minor triads, scales and keys in an equal inverse relationship, was given by Arthur von Oettingen (1836-1920) in *Das duale Harmoniesystem* published in 1913. However, the origins of dualistic theory stretch back as far as Zarlino’s observation in *Le Istitutioni Harmoniche* (1558): that while the harmonic division of a string or monochord leads to the major triad, an arithmetic partition yields the minor triad.

11.2 - DUALISM

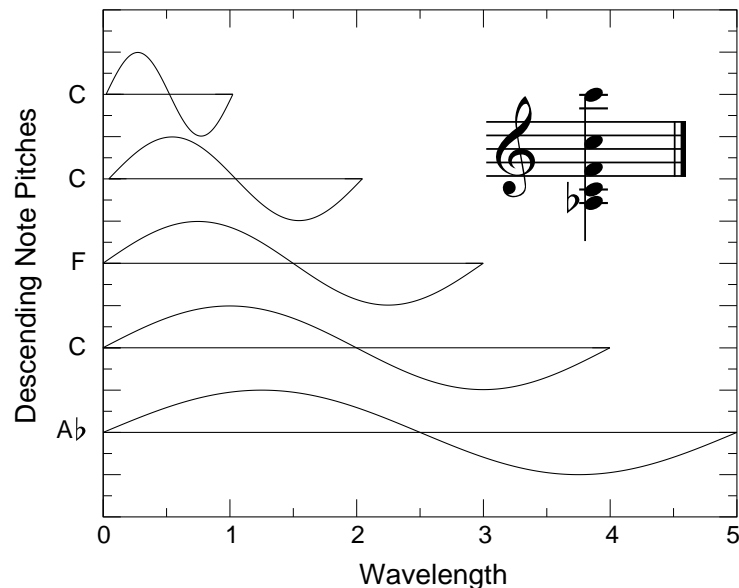


Figure 11.1 The arithmetic division of a string into five equal segments yields the notes of a minor triad, arrayed in intervals of an octave, fifth, fourth and major-third descending. The inverse of the familiar ascending harmonic series.

Since Gioseffo Zarlino's initial foray, there has been sporadic interest in dualistic ideas over the centuries, with contributions from Rameau 1737, G. Tartini 1754, F. Vallotti 1778, M. Hauptmann 1853, A. v. Oettingen 1866/1913, H. Riemann 1905; and more recently V. d'Indy, P. Hindemith and S. Karg-Elert^{1&2}. To be fair, most contributors have approached the topic with differing perspectives, yielding a variety of conclusions; with Hauptmann rather philosophic, Oettingen scientific and Riemann musicological, in focus.

Rameau was the first to seriously develop Zarlino's initial observation, by placing the generative procedure for the minor triad below the fundamental; and there the matter rested until Moritz Hauptmann's book on *The Nature of Harmony and Metre* emerged in 1853. Hauptmann accepted Rameau's top-down generation of the minor triad, but in place of Rameau's rather weak argument of the arithmetic principle being 'suggested' by the predominantly harmonic ordering of music, he substituted a highly abstract dialectic approach. Hauptmann identified the intervals of the octave, fifth and third (major and minor) with the three fundamental categories of thesis, antithesis and synthesis, respectively; and used this association to fashion a wide ranging theory of music, which worked outward from mirroring major and minor triads to mirrored scales and keys, and beyond, to encompass the rhythmic and metrical domain of music. The weakest elements in Hauptmann's work were 1) the theory's philosophical foundations made the concepts remote from musical practice and 2) the theory could not account for the root of the minor triad. Indeed, Hauptmann maintained, rather in the tradition of medieval philosophy, that music theory and musical practice were in some respects distinct and separate endeavours.

The key observation of dualistic theory is that by strict inversion the major triad is transformed into a minor triad: thus major-third plus minor-third reckoned upward produces C-E-G, the major triad; but when applied downward (from the highest note) produces G-Eflat-C, the minor triad. By setting this inverse or mirrored relationship within the wider context of extended harmonic and arithmetic series, Oettingen proposed that the notes of the major triad have a focus in the fundamental tone two octaves below their root (C), which he described as the *tonic* fundamental; while the tones of the minor triad find a mirroring focus in

their first common overtone two octaves above their ‘root’ (the upper note of the C-minor triad, G) – the *phonic* ‘fundamental’. While this beautiful symmetry is most appealing, there is a problem: we do not hear and recognise G as the *root* of a C-minor chord, but rather identify C as the root of both the C-major and C-minor chords.

This flaw at the heart of harmonic dualism is forcefully identified by the entry in the Harvard Dictionary of Music¹: “The greatest shortcoming of the theory [dualism] lies in the fact that in a minor mode the triad is determined, not by its lowest, but its highest tone, ... This forced explanation is in contradiction to the most elementary facts of acoustics and of musical experience.” Theory and observation appear to brutally collide. Although this didn’t appear to worry Moritz Hauptmann, it has been a thorn in the flesh of most promoters of dualism. Arthur von Oettingen’s approach to the problem was to suggest that our musical senses had been led astray by the forceful rootedness of the major triad and that musicians needed to be ‘re-educated’ so as to understand, if not hear, the mirrored logic of the minor principle. Oettingen a highly competent scientist knew he would need to look more to the fields of physiology and psychology for support, that to physics. Some other dualist, for example the great musicologist Hugo Riemann, were at times to go as far as to claim the existence of *undertones*², fractional standing waves, but such contrivances could not withstand scientific scrutiny. Whether dualists ignored the problem of the minor triad’s root, tiptoed around it or rather rashly made unscientific assertions, the clash of theory and practical experience remained an imperturbable and unwelcome elephant in the drawing room.

The Context of Nested Harmonic Series

Here I would like to suggest a context, in which this clash might be resolved. A solution that attempts to incorporate v. Oettingen’s *theoretical* arithmetic/minor principle within the harmonic/major system of *real experience*. The scheme does not challenge the concept of a fundamental bass or the primacy of the major mode over the minor, but rather, accommodates the minor within the major. However, at the outset, it must be admitted that the solution itself is rather theoretical in character, in the sense that although it embeds the minor mode within the reality of the ascending overtone series, whether this scheme bears any relation to how human beings actually hear and process musical sounds, is an open question. Primarily the focus here is on an abstract and mathematical approach to the problem; yet the question of musical cognition cannot wholly be ignored, as the organisational structure of tonal music ultimately depends on human choices, largely made in response to the perception of aural stimuli.

Essentially, *the context* is that of the ‘natural’ major and minor triads (h4, h5, h6 and h10, h12, h15 respectively) as they occur in the harmonic series. And though some small reference will be made to the broader setting of chord progressions in *Modulating Oscillatory Systems* (MOS), for the most part discussion will be limited to the relationships within one single chord/harmonic series.

To construct the desired context requires the expansion of the written chords found in scores, outward, into extended harmonic series, taking the process to even greater lengths than v. Oettingen originally envisaged. For a full dynamical MOS analysis, this expansion of the written chords into harmonic series is taken further than simply placing the individual notes/tones in positions within an harmonic series where all the constituent notes in a chord can be expressed (allowing for a reasonable margin of error arising from

scale temperaments) as harmonics of a single fundamental frequency. Indeed, to achieve a necessary flexibility, the expanded context of a single harmonic series has itself to be *nested* within another, broader harmonic series and this broader harmonic series *nested* within yet another, even more fundamental, harmonic series! To these three levels of nesting the terms *fundamental/nesting* series, *nested* series and *aggregated* series are applied. However, for the purposes of this discussion of natural major and minor triads, only the two upper levels are required – nested series and aggregated series. And although this may appear a great extension and extrapolation of what is only a clutch of written notes, in performance it must be born in mind that a wide spectra of vibration assaults the ear, and these myriad objective frequencies are then further enhanced and supplemented by subjective combination tones of the ear’s own making. Indeed, it is probably a reasonable assumption that the processes of aural cognition operate on the whole frequency range delivered by the ear, even though the listener’s conscious attention may be focused on the notes alone.

Also like v. Oettingen, the notes/tones of music are considered, in principle, to be *just* or natural harmonics in intonation, with the ear’s accepting tolerance being relied on to smooth over any irregularities. The convention of labelling the ratios of the harmonic series from the fundamental tone h1, h2, h3, etc., is adopted; with the addition of the upper case: H1, H2, H3, etc., to distinguish the more fundamental of two or three enmeshed harmonic series.

Nesting and the Harmonic Series

Before moving to the core material concerning the root of the minor triad, it might be helpful to recap a little on the nesting of harmonic series discussed in Chapter 9 and illustrated in Figure 9.7 – where the idea of parent and child series was introduced. Figure 9.7 shows two nested series but just one level of nesting. However, if for example, the nested series built on G-384Hz were extended upward to incorporate six harmonics, the same pattern of nested child series could be repeated to provide a third level of *grandchild* series built on G-768Hz and D-1152Hz.

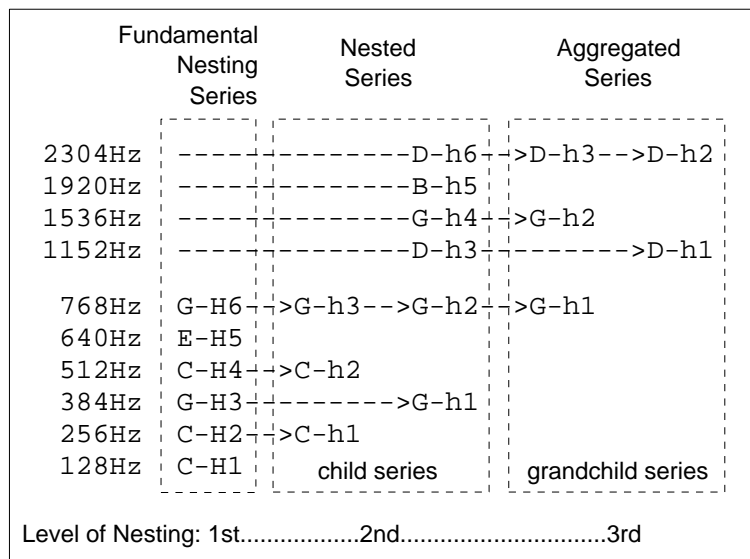


Figure 11.2 Three levels of nesting: Fundamental/Nesting, Nested and Aggregated series.

In Figure 11.2 the series built on G-384Hz (one of the two nested series illustrated in Figure 9.7) has been used as the basis for another level of nesting, but just as easily the nested series built on C-512Hz could have been used, or for that matter, any other higher partial H4, H5, H6, etc., of the fundamental nesting series. The third level series, the grandchildren of the fundamental, are termed *aggregated series*.

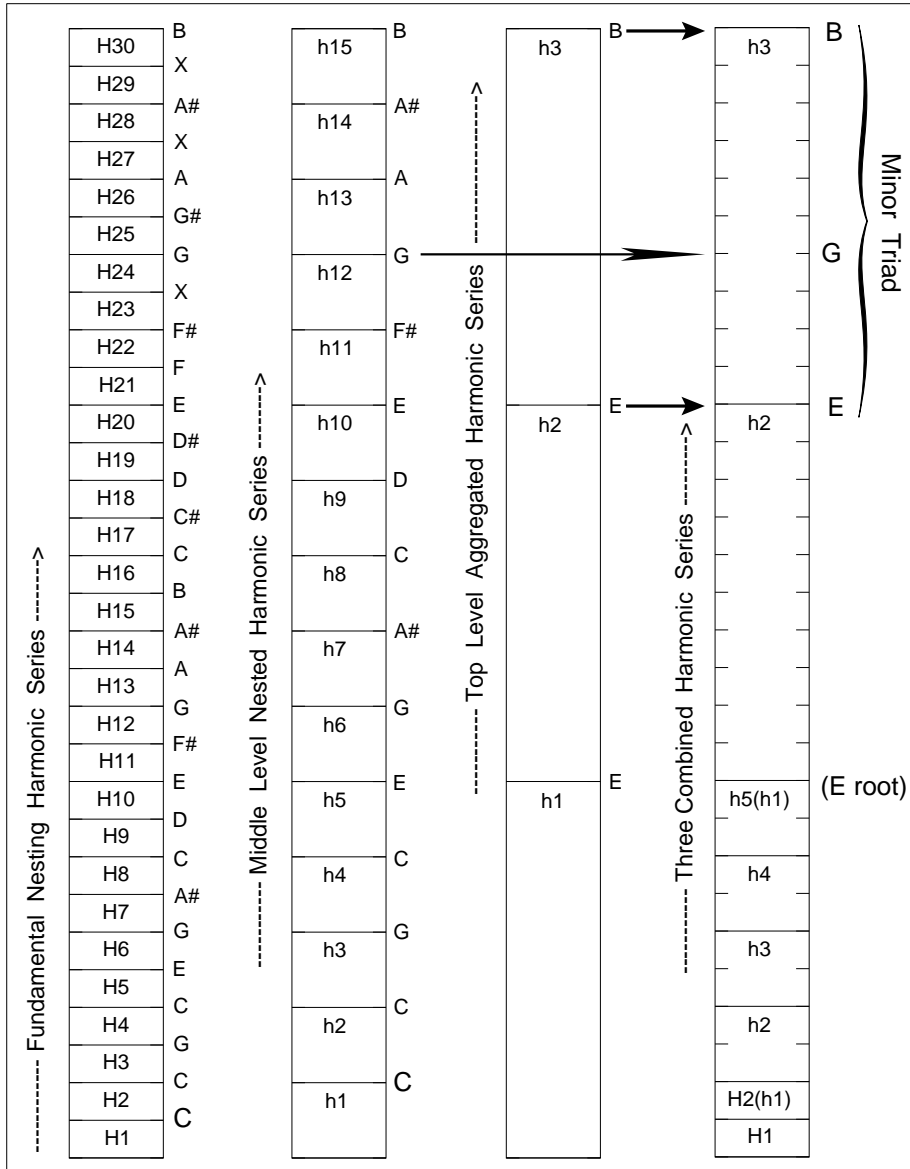


Figure 11.3 Three levels of nesting underlying the minor triad: Fundamental/Nesting, Nested and Aggregated series from left to right, with the whole combined system illustrated on the extreme right. (Value thirty, MBN 3₅ 0₂ 0₁)

The ‘ladders’ of Figure 11.3 illustrates the application of three levels of nesting to the hosting of the minor triad. However, this third level of nesting, the aggregated series as I have termed it, is to some extent a useful fiction in that the grandchild series could equally be viewed as nesting within the parent series and this would be true for any further levels one might care to construct. It is simply convenient to give two different series, both nested within a fundamental series, distinguishing names indicating a clearly defined relationship between them. Ultimately it is difficult to draw absolute distinctions beyond the two levels of *nesting* and

and minor-third ascending from h1 and descending from h60. The descending pattern of frequencies from h60 might be viewed as an inverse harmonic series or arithmetic series of wavelength relationships. These relationships are illustrated in Figure 11.5.

The great stumbling block for dualism has been that whilst the major triad emerges naturally from the real *modes of vibration* of a physical object, the arithmetic/minor relationships have remained stubbornly theoretical. We don't hear a phonic 'root', nor scientific experiment detect 'undertones'. So it is to these real material phenomena, the ratios of the ascending harmonic series, that the elegant ideal of dualism must be adapted.

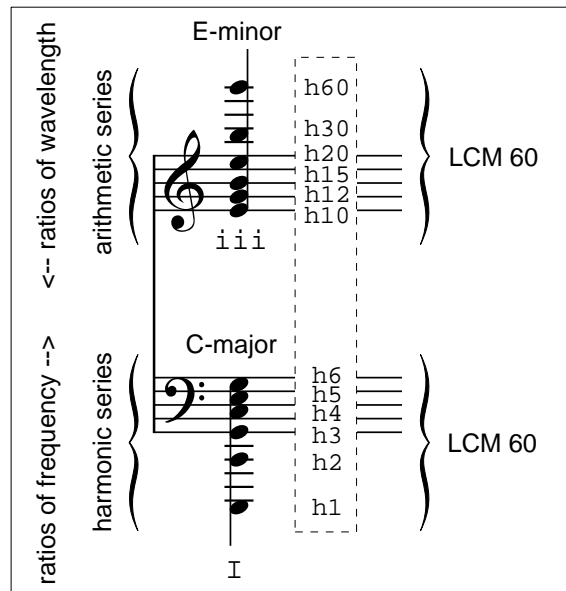


Figure 11.5 The 'natural' C-major and E-minor triads set within the wider context of an extended harmonic series.

THE PERCEPTION OF TONAL MUSIC

To digress for a while and venture upon the difficult terrain of the human perception of music – though being clear that the interpretation of the minor mode, and indeed tonal music in general, presented by the MOS model stands independently upon a mathematical foundation – nevertheless, it might be useful to review a little of what is known about aural cognition and attempt some integration of the two strands. The following overview draws upon James Beament's most readable book: *How We Hear Music* (2005)³, but also differs from it in some regards.

For a single note the ear easily identifies a repeating pattern, the period of the fundamental frequency, which is perceived as pitch. Also the configuration of overtones, their strengths and any significant frequency deviation from that of an ideal series is identified too, and conveyed as the tone color or timbre of the perceived pitch. (Beament gives timbre a particular meaning not adopted here.) The ear apparently resolves the lower overtones as more or less separate entities – about h2 through h10 – however, as their envelopes of resonance overlap on the sound detecting membrane of the inner ear, they are signalled in combination, thus emerging as the generalised tone color component of consciously perceived musical sound. Although there is no clear dividing line, the higher harmonics beginning around h8, falling ever closer together, become increasingly difficult for the ear to detect, due to gross mutual interference. The random

nerve impulses generated by this mutual interference gradually overwhelm the ordered signalling of overtones. Such random patterns are the hallmark of noise. [This feature might also have a bearing on the apprehension of conjunctions between chords for the MOS model: Generally, this mutual interference of overtones in simple chords will still leave clear headroom in the 1000Hz to 4000Hz frequency band where the ear has greatest sensitivity (roughly the top two octaves of the piano keyboard) but in very complex chords it may prove somewhat more restrictive.]

But to what degree is it legitimate to separate pitch from tone at the level of perception? Overtones appear to contribute to the perception of pitch, as evidenced by the lower registers of many instruments, like the bassoon for example, where the fundamental harmonic is only weakly present and most of the energy in the objective sound is shared between h_5 , h_6 and h_7^4 . Nevertheless the listener perceives a strong and resonant fundamental! Equally, a poorly constructed instrument generating strong ill focused overtones does not produce a perception of clean clear pitched notes but rather a ‘pitch-band’ to use Beament’s term; as well as an infelicitous tone. The perception of a single fundamental pitch involves more than h_1 alone – excepting tuning forks and scientifically generated sound – though that said, the fundamental frequency of a single note does resonate a significant part of the ear’s detector membrane unhindered by overtones: Thus signalling one pitch alone in that portion. Perhaps the perception of a single clear note, the perception of pitch with tone, should be thought of as the characteristic signature of one pure harmonic series? And one might ask, what does the ear identify in a chord? Are the notes of a triad acting as three separate *itches* or three powerful harmonics of *timbre*, or something of both⁵?

The ear-brain hearing system might usefully be divided into three stages, the ear mechanism, unconscious processing and conscious processing. Obviously, the system evolved long before the invention of tonal music, or speech for that matter, and its primary function was, and still is, to keep us safe. To warn of danger and give us an automatic omnidirectional awareness of changes in our surroundings. An acoustic early warning system. The ear harvests other information from the sounds which fall upon it beside pitch and tone color. It is very sensitive to the transient sound at the beginning of notes and to the volume of sounds. Plus non-acoustic information from other senses, like sight, and expectations, assumptions, experience, etc., both conscious and unconscious, must all be taken into account. Most of the sound we hear and almost all of the sound our hearing system was evolved to deal with consists of complex chaotically changing frequencies: transient frequencies, noises. With a few exceptions like bird song, most natural sounds are perceived as containing little *fixed* pitch and tone information. A noise might be perceived as high pitched, medium or low, or perhaps changing from one level to another; hollow, brittle, dull, etc., but change is the dominant feature. We have difficulty grasping any settled pitch or tone because noises generally don’t have simple settled frequency relationships of fundamentals or integral overtones; usually they are constantly varying. Obviously there was, and is, an advantage in the ear and unconscious processors delivering a clear signal of possible danger at the conscious level of perception. That clear signal takes the form of a unified perception a sound, with the attributes of direction, distance (volume, reverberation) and character (patterns of frequency change). The ear and unconscious processors’ particular strength is in matching up transients, from which it is able to distinguish between different simultaneous sources of sound and assign direction relative to the axis of the ears. Most natural sounds are composed almost entirely of transient frequencies. In contrast, musical sound generally consists of settled sustained frequencies, musical sound is an atypical

subset of noise which contains a much reduced level of transients. Transients are generated at the start of a note's vibration, which is principally how the ear distinguishes between instruments and follows parts in contrapuntal music. Also mini-transients are created by the random fractional instability in note frequency – a characteristic of sound generation in traditional instruments – this Beament terms 'timbre'. However, overall, musical sound is very different from natural noise and our ears confirm that this is so. Regardless of this difference, the ear performs the same low level processing on musical sound as for noises – as far as the dearth of transients allows – and depending on how much information it has been able to glean, delivers one or more unified perceptions of a musical sound or sounds. Importantly, part of the information influencing the received perception will come from within: memory, experience, expectation, conscious thought.

Depending on the amount of information available to the ear mechanism, the processes of aural cognition probably exhibit some degree of flexibility in the interpretation of aural stimuli. Three simultaneous notes of the triad with different transients, volumes and directions, like for example music played by a violin, clarinet and piano spread across a stage, might well be apprehended primarily as separate *packages of pitch*, three parts, each with an associated tone color. Though the musical listener will tend, unconsciously, to turn these separate sound sources into a perception of harmony – as well as enjoying the independent motion of the parts. The same three simultaneous notes played on the piano alone, thus produced with more or less similar transients, direction, volume and harmonic spectra, would, in contrast, tend to be perceived as one *package of tone color* (the sound quality of a triad) with the pitch element less well defined. Here the notes of the triad, which bear the frequency relationship of the harmonics h4, h5 and h6, in a major chord, are being combined: As like overtones of a single note, their resonances overlap on the ear's detector and thus are signalled as an intermingled group⁶. Like overtones; intervals, triads and chords have their individual pitches smeared out into something not unlike the perception of tone color. Again, equally, the musically experienced listener could unpick this package into parts, identifying the separate notes, though this does require an element of conscious effort. However, the overall impression created by a unified chord perception is one of tone. The strident bare fifth chord C (h2) to G (h3) finds a match in the tone of the artificial 'harmonics' of an organ chorus with mixtures or the Quint registration: C-h1, h2, h4, G-h6 (eight, four, two, one-and-one-third foot); while the sweet major triad finds a parallel in the cornet registration: C-h1, h2, G-h3, h4, E-h5 (eight, four, two-and-two-thirds, two, one-and-three-fifths foot). For the tone quality of the minor triad, no matching flue or reed organ registration can be found, though interestingly, the harmonic spectrum of the bells in a zimbelstern stop, and that of the traditional European bell in general, do match the minor triad. Indeed, by nesting one harmonic series stepping in groups of five, inside a second fundamental series, the principal sequence of bell tones emerges: hum tone h5, fundamental h10, minor-third h12, fifth h15, nominal h20, major-third h25, upper fifth h30, minor-seventh h35 and octave h40. Pitch and tone are somewhat akin to the two faces of a coin, the harmonics of timbre bolster and color a single fundamental harmonic of pitch, multiple simultaneous pitches in chords take on something of the character of harmonics of timbre. But what of the 'pitch' sensation of the whole combined chordal package, the identification of a repeating pattern, a Meter in fact; which at the level of a single separate note is capable of generating a clear perception of pitch?

Given an E-minor triad (E-h10, G-h12, B-h15) just above middle C at 256Hz, the pattern of repetition, the interference pattern, would have a frequency of 32Hz, that is, the C-h1 fundamental of the triad's

encapsulating series would repeat thirty-two times per second. However, when we hear a triad, minor or major, we do not apprehend a clear low pitch of 32Hz (or 64Hz for the major triad C-h4, E-h5, G-h6) analogous to the pitch of a single note – *but we do identify a root*. And the rootedness of chords, a real perception widely experienced, does possess a generalised pitch quality, here E for the minor triad and C for the major. (The nature and role of difference tones is discussed separately below.) What might be happening with the perception of chords, could be somewhat analogous to our perception of noises, where there is only a rough sense of the pitch, no clear repeating pattern, but a strong awareness of the sound's character, its tone. However, in the subset of noises we call chords, the frequencies are unnaturally stable and this does produce a settled repeating pattern, but the pattern is perhaps too long and complex to be perceived as a clear low pitch; or alternatively the unconscious low level processors simply do not pass this information on. The perception instead, as for noises, is of the general quality of the sound, the characteristic tone or signature of the interval or chord. The overall repeating pattern, the fundamental, is there in the Meter of the objective sound, hovering like a shadow behind the scenes; and more explicitly, some of the harmonics of its series are present, in the objective notes of the chord or triad. However this background, this broader awareness, if it does exist at some level of cognition, is only forceful enough, for whatever reason, to lend confirmation to our sense of the rootedness of intervals, triads and the other chords built upon them. This of course is a guess and a guess that would seem to suggest that the root of the E-minor triad should be C and not E! But as we shall see below it is a step in the right direction, a step toward the root of the E-minor chord.

Attempting to produce a model of aural cognition at any level, is necessarily a speculative exercise. To quote James Beament: “looking for a needle in a haystack is much easier than putting one in a cortex and finding useful evidence”. Notwithstanding these words of caution, as the MOS model provides a blueprint for the processing of musical sound – in the form of mutable digit sequence exchanges – it is perhaps worthwhile attempting a brief sketch. Beament describes in some detail the nature of the coding of nerve pulses carrying information from the ear mechanism to the low level unconscious processors. The salient point of which being, generally, that a range of individual nerves are involved in signalling a range frequency information. Thus where the resonances of two or three or more frequencies overlap on the ear's detector membrane, nerves in that area will generate pulses, on a rather irregular basis, but linked to the various periods of resonance, which taken en masse equate to a metrical encoding of the sound's components. And though this metrical information is unlikely to be presented ‘in phase’, the repetition patterns of the integer components of harmonic series will be periodic.

This given, one might draw a rough analogy more than a model, of the processing of musical sound, along the lines of an auditorium where each seat is provided with a small light connected to one of the nerves from the auditory pathway leading from the ear. Seating is arranged such that nerves which lie close together beside the detecting basilar membrane will be close together in the auditorium. The auditorium is full, and each individual is instructed to clap their hands once, each time their seat light flashes. The role of auditory perception, is play by one individual standing on the stage listening to the aggregate of all clapping but also capable of focusing his or her attention on particular areas within the auditorium. I shall term this function, the sensor. The effect of this ‘fuzzy focus’ is to extract the lowest common multiple from areas of clapping.

As the signals arrive for a single note heard, particular areas in the auditorium will, taken overall, produce the clap rates of the individual harmonics detected (LCMs), though no one contiguous group will be

producing a single clap rate, save the portion of seats connected to the region of the detector vibrated by the fundamental alone. Thus the perception of tone color, the overtones, is melded together; however, in addition to the production of the tone color perception which could be thought of as the predominating subdivision of the period (i.e. a meter) the overall repeating pattern of overtone claps will match with the clapping from the contiguous region signalling the fundamental only. This generates the perception of pitch. Even if the fundamental is weak or non-existent, sufficiently strong overtone claps may be able to produce the perception of a pitch in the sensor, by their combined period. For the perception of intervals, triads and chords, the same effects are at work but without any significant contiguous group of seats signalling one clap rate alone. Thus an overall perception of tone color pervades the sensing of intervals and chords: With the exception of the octave interval which is more like that of a single pitch. Also there will be a period generated by intervals and chords (though often a rather long period) analogous to the pitch period, delineated by the repeating pattern of the sum of all claps, a period not directly sensed beyond 'rootedness'.

For chord progressions, in this application of the MOS model, the crucial factor perceived between commensurable chord steps, like for example the dominant-seventh to tonic exchange, is that *in one area in the auditorium*, the seats receiving signals which overall delineate the clap rate of the conjunction frequency, *remain constant*. The clap rate does not change in this oasis of stability, common to both harmonies. Thus the chordal steps in tonal music are connected by a simple logic. The sensor has learned to scan all the seats searching for these continuities, for by navigation via these beacons, the flux and flow of otherwise apparently unrelated aural perceptions can be tamed, classified and interpreted. Harmonic progression is rendered intelligible by the relationships of the mutable base number system.

The above overly simple sketch is but a brief and speculative excursion into largely unknown territory. There are two strands of thought being pursued in this chapter: with the principal strand operating at the mathematical level of the interaction of waves in the abstract; and a secondary strand, working by implication from the first, concerning the much more intractable processes of the human ear and mind. The second strand is purely conjectural. And to conjecture a little further, perhaps one could argue for the site of the above described auditorium to be at the level of automatic unconscious processing. Such a location might then account for the rather direct emotional response which music can undoubtedly illicit, acting along similar lines to the automatic responses to sounds indicating danger, surprise or contentment.

Neuroscience is in its infancy, with progress to date finding complex, distributed, adaptive processes, particularly at the higher levels of cognition, where the course of development taken by mental structures may differ between individuals. However, hopefully, at some stage in the future a general understanding of the brain's processing of musical stimuli will become available: "Collectively, studies of patients with brain injuries and imaging of healthy individuals have unexpectedly uncovered no special brain 'center' for music. Rather music engages many areas distributed throughout the brain, including those that are normally involved in other kinds of cognition".⁷

AGGREGATION AND THE MINOR TRIAD

An interesting feature of any complete group of harmonic frequencies h_1 to h_n , of equal amplitude and uniform phase, in other words, a collection of waves of equal peak-to-trough displacement which all begin

and end together; when brought together in an interference pattern, is that they describe exactly 'n' equal subdivisions of the fundamental period. For example, the rare chord of the first fifteen tones of the harmonic series would, under these conditions, subdivide its fundamental period into fifteen equal units – illustrated in Figure 11.6.

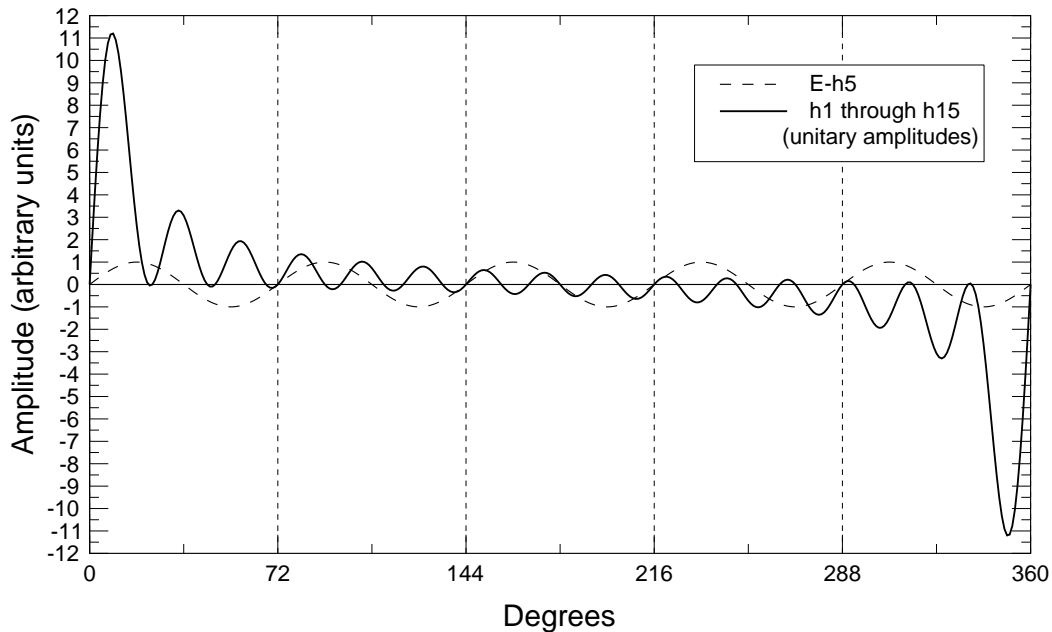


Figure 11.6 The interference pattern of all the harmonics from h1 through h15 (with equal amplitudes and uniform phase). The rare and complex chord which encompasses the natural minor triad (Figure 11.4).

These fifteen subdivisions could be further grouped or aggregated into five groups of three oscillations (MBN $3_5 0_1$) or three groups of five oscillations (MBN $5_3 0_1$), the groupings corresponding to the harmonics E-h5 and G-h3, respectively. While this is a special case, it is useful because it highlights relationships; and, as the processes of aural cognition do not register phase differences for the most part (Ohm's Acoustic Law⁸); together with there being generally some level of equality in note intensities; any real situation, though less uniform, might well possess something of the regularity and proportionality exhibited in Figure 11.6 and the following graphs. As James Beament points out, irrespective of phase differences in the objective frequencies, the mechanisms of the ear-brain system identify underlying patterns, indeed his bar diagrams on pages 68 and 77 of *How We Hear Music*, chart relationships similar to the graphs presented below. (Figure 11.9 illustrates that even with phase shifts, underlying relationships still emerge.)

The Natural Minor Triad

Interestingly, if just the frequencies of the natural minor triad are considered in isolation (E-h10, G-h12 and B-h15 in this example) they likewise support subdivisions of five and three – within a given tolerance. However, this produces only twelve oscillations, aggregated as alternate groups of two and three cycles – three groups of two cycles plus two groups of three – in one full period for E-h5 (Figure 11.7). Or a regular grouping of four cycles for G-h3 (Figure 11.8). A necessary feature of aggregated groupings, is that they are constituted of integral numbers of whole oscillations – complete cycles – so as to be supportive of resonances. Though the aggregations themselves may contain uneven numbers of complete oscillations.

11.13 - DUALISM

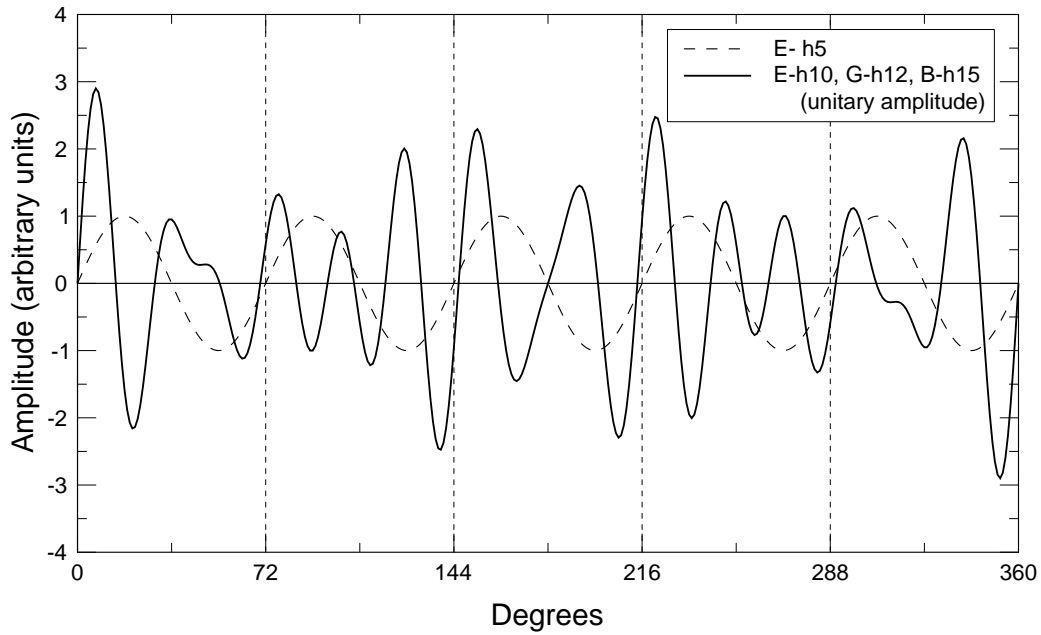


Figure 11.7 The interference pattern of an E-minor triad (notes E, G, B) supports internal subdivisions of five – E-h5.

Where more than one subdivision of the overall period is possible – as here with both subdivisions of five and three, plus two and a questionable four – the highest number of aggregations, E-h5 (i.e. highest frequency/energy), would probably emerge as the most prominent: As this arrangement, set at the bottom of a nested structure, would represent the the lowest energy or ground state configuration for the system considered overall. Though, perhaps, the rather veiled tone of the minor triad stems from a shadowy perception of the period’s subdivision into three aggregations of four complete cycles – G-h3. Also, the uneven numbers of oscillations within the period of E-h5 might, perhaps, contribute to its prominence, as well as ‘locking’ it into the period of C-h1, with the repeated sequence of 2-3-2-3-2 cycles.

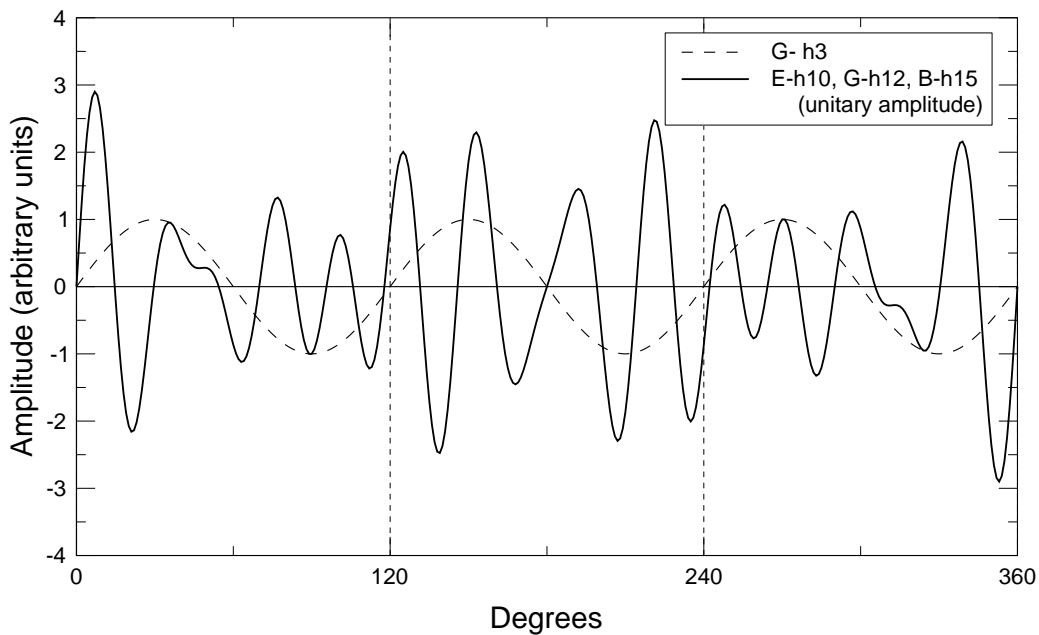


Figure 11.8 The interference pattern of an E-minor triad (notes E, G, B) supports internal subdivisions of three – G-h3.

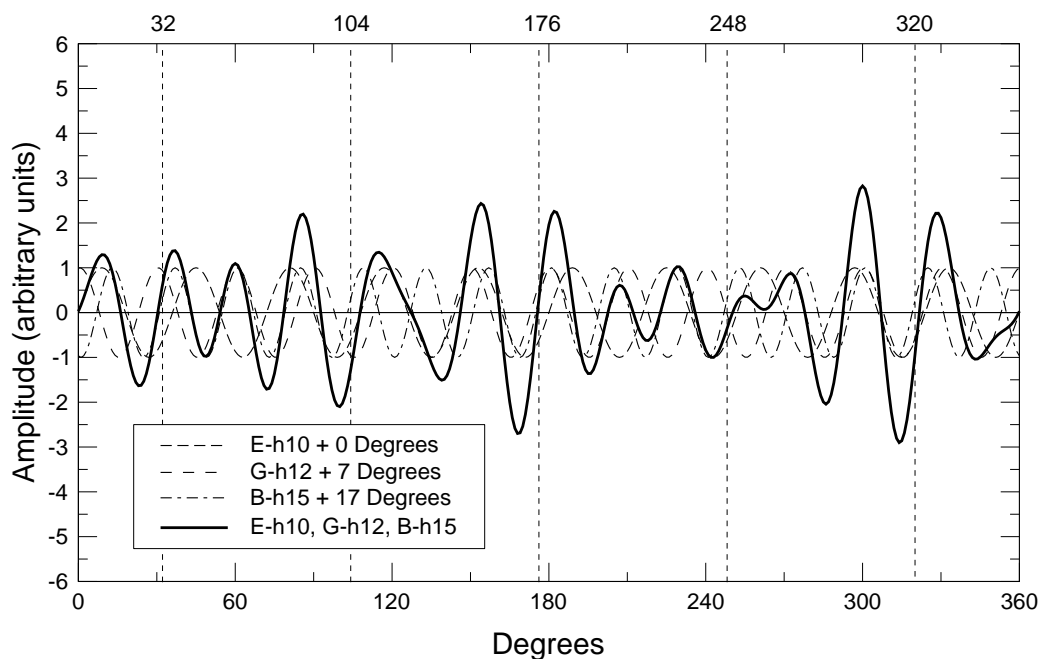


Figure 11.9 The interference pattern of the minor triad with randomly chosen phase shifts for frequencies E, G and B, maintains aggregations spaced at approximately 72 degrees, equivalent to the period of E-h5. (Unit note amplitudes.)

Aggregations and Difference Tones

The combination tones and other similar physiological-cum-psychoacoustic phenomena are fraught with controversy. Some scholars doubt the existence of Helmholtz's summation tones, others classify them as distortion effects. However out of all these phenomena, the difference tones, the easiest to hear, are generally accepted: though their precise cause may be disputed. Crucially perhaps, the lowest frequency difference combination tone formed from the objective notes E-h10 and G-h12, is C (h2), not E (h5); though if B-h15 is included, the additional difference tones G-h3 and E-h5 are also generated. These three difference tones potentially define three modes of aggregation for the overall period of the harmonic series carrying within its higher ratios the objective minor triad: A subdivision of two oscillations – C-h2, a subdivision of three oscillations – G-h3 and a subdivision of five oscillations – E-h5. (The latter two within a given tolerance.) As well as the full period or repetition pattern of the combined notes, of course, exactly matching C-h1 an octave below the lowest difference tone: C-h2 (Figure 11.11). The subdivision of two oscillations (C-h2) is illustrated in Figure 11.10; and in Figure 11.8, subdivisions of three oscillations (G-h3) is graphed.

Though difference tone often 'point to' the perceived roots of chords – any two adjacent overtones create a difference tone equal to the fundamental of the series – however, for reasons associated with the difference tone generated by the interval of a minor sixth (eg. C to Aflat), I believe difference tones are a separate phenomena from the creation of rootedness in chords⁹. Nevertheless they may be considered a contributing factor; and, in the E-minor triad under discussion here, as difference tones are found to be pointing to C, G and E – three possible candidates – they clearly do not isolate a unique root. Yet still this is some advance on the situation above, in that now as well as C-h1, E-h5 has entered the field of 'rootedness'.

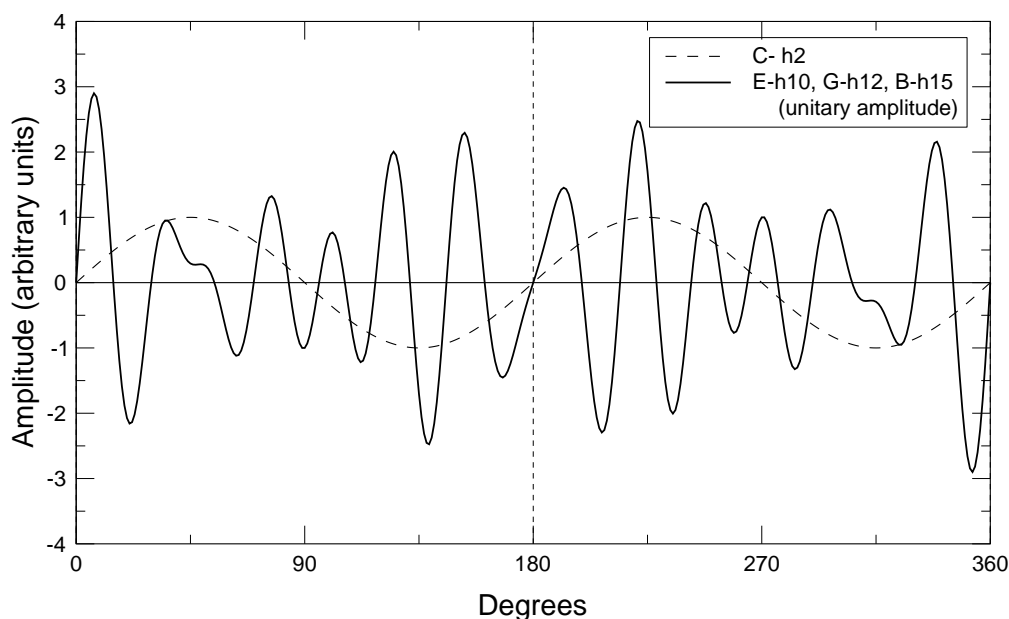


Figure 11.10 The interference pattern of an E-minor triad (notes E, G, B) supports internal subdivisions of two – C-h2 – the period of the lowest and most prominent difference combination tone.

The crux of the matter is that our ears and mental processes, searching for intelligibility within the complex pattern of sound generated by the objective E-minor triad, with the particular aid of the period of repetition or Meter (C-h1), the difference tones and similar consistent periods of aggregated oscillations within the interference pattern (C-h2, G-h3, E-h5); might seek out an accommodation between the complex extended series implied by the 10:12:15 frequency ratios found in the minor triad, and the very attractive low energy configuration of the ratio 2:3 (10:15) presented by the outer pair of notes – the fifth E to B. The accommodation consists of nesting a short low energy/complexity series based on E-h5 within the broader context of an harmonic series based on C-h1. (The slightly more complex nesting based on G-h3 would imply a G-major chord with added sixth, discussed in Chapter 10.) Essentially this approach hints at a recursive application of the hearing system's procedure of sifting out relationships which fit into harmonic series and then combining them into a single perception. An extended interpretation of the minor triad's pitch relationships --E-h10, G-h12 and B-h15 – is perhaps rather too distant for the processes of aural cognition to readily grasp, especially when such a forceful alternative is to hand in the first three ratios of a series built on E-h5. Our hearing system chooses the *simplest series*, based on a *short period* of repetition, the series founded on E-h5 *and from this choice we divine the root of the minor triad*. Nevertheless, the ambiguity inherent in the ratios does not go unnoticed, our perception of the minor triad is less stable and less satisfactory than the major triad, as demonstrated by the early adoption of the tierce de picardie cadential transformation¹⁰, early in the tonal era. This instability might perhaps be the hallmark of two series in contention. The extended context of a 'C-major' harmonic series could operate in a similar manner to our sense of key. For example, in the key of C major the dominant chord, G-major, for the most part, is clearly heard as possessing the root note G, without this necessarily diminishing our sense of being in the key of C major. Equally, the broader context of an underlying harmonic series built on C-h1, could form a background against which the objective E-minor triad's relationships are 'computed' – and if not within the mind itself, at least in terms of the abstract mathematical scheme of mutable base numbers. Indeed, though it is another

story, in the MOS model, the movements of the underlying harmonic series which form this background, might themselves define a sense of key.

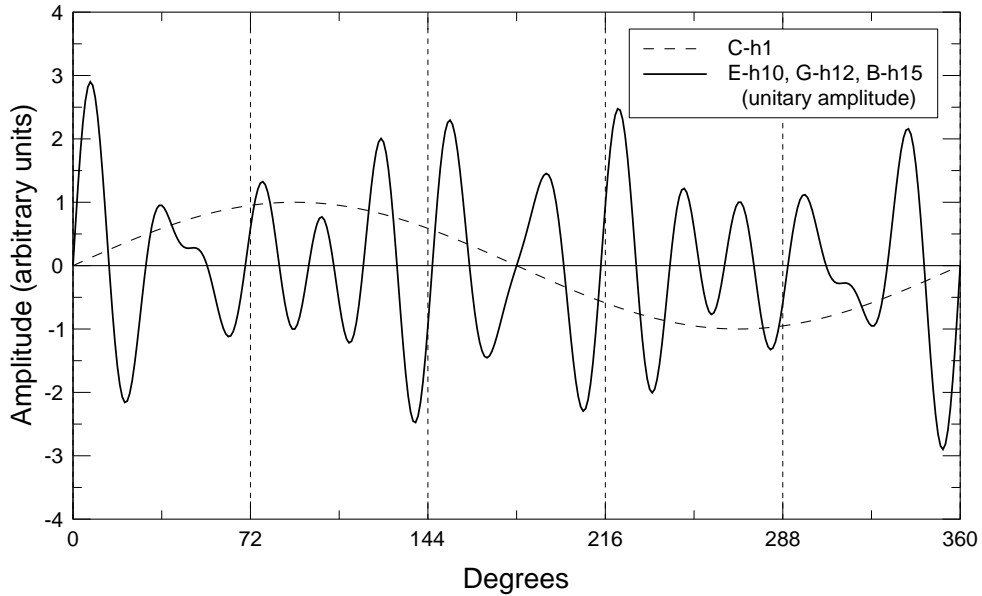


Figure 11.11 The overall period of the interference pattern of the natural minor triad is C-h1, the period of repetition.

But can this extended harmonic context truly exist in the mind, as a real part of the processes of aural cognition, given the strength of the perception of an E, and not C rooted triad? Perhaps our processing of the minor triad penetrates no further than the shortest period of repetition? A definitive answer to this conundrum must probably await further developments in the field of neuroscience. Nevertheless, the simplicity and consistency of the MOS model and mutable base numbers, may perhaps give some grounds to believe that an approach to the mysteries of aural cognition, along these or similar lines, might prove worthy pursuit.

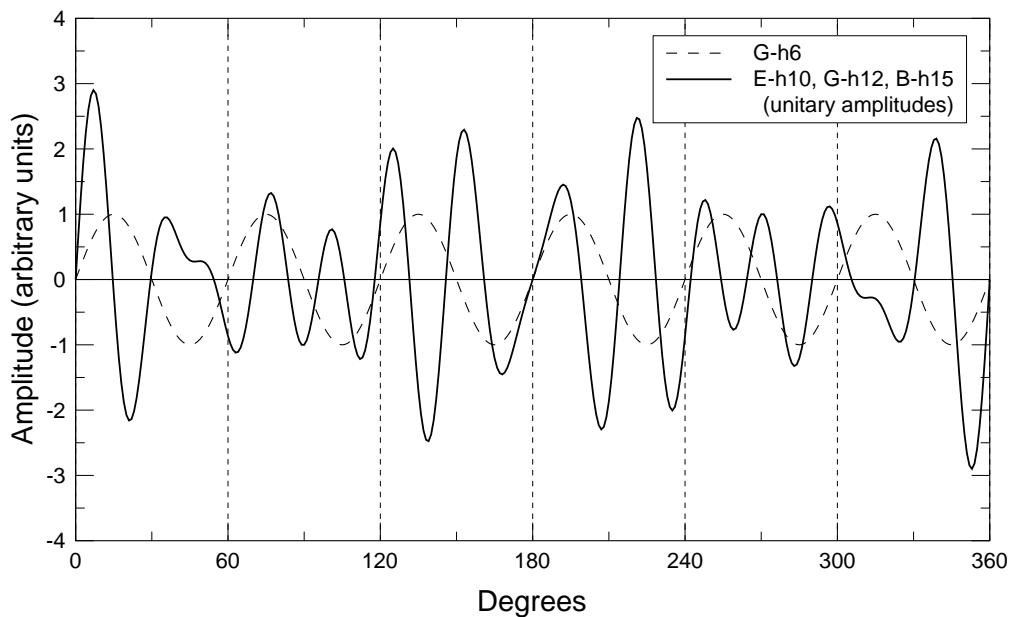


Figure 11.12 The interference pattern of a natural E-minor triad and G-h6 are in almost complete anti-phase at 60 and 300 degrees, destroying any possibility of matching complete oscillations to the pattern to G-h6.

It is interesting and perhaps significant that the run of consistent periods of aggregated oscillations, within the E-minor triad's interference pattern, finishes at E-h5: G-h6, A#-h7, C-h8, etc., are not well supported. For example in Figure 11.12, G-h6 doesn't meld, in complete cycles, with the interference pattern around 60 and 300 degrees; though matching well on either side of 180 degrees.

However, C-h4 with a five degree discrepancy between the interference pattern at 90 and 270 degrees (Figure 11.13); is somewhat less wayward than the nine degree discrepancy of G-h6. Notwithstanding the possibility of other scenarios, the main point is that the E-h5 resonance makes the uppermost clear match with the interference pattern generated by the objective tones of the E-minor triad. And though under normal conditions of transmission to the ear, the waves will form a complex pattern through the processes of reverberation, absorption, etc., (see Figure 11.9), the ear sifts and sorts this jumble and in combination with the processes of aural cognition appears able to reconstitute something akin to the simple relationships illustrated in the graphs. Whatever the detailed mechanisms of musical cognition are eventually revealed to be, it is likely that they will involve processing acting over the whole frequency range accessible to the ear, a range spanning up to ten octaves, and, no doubt, will include both innate and learned elements. Current ideas and models are necessarily provisional.

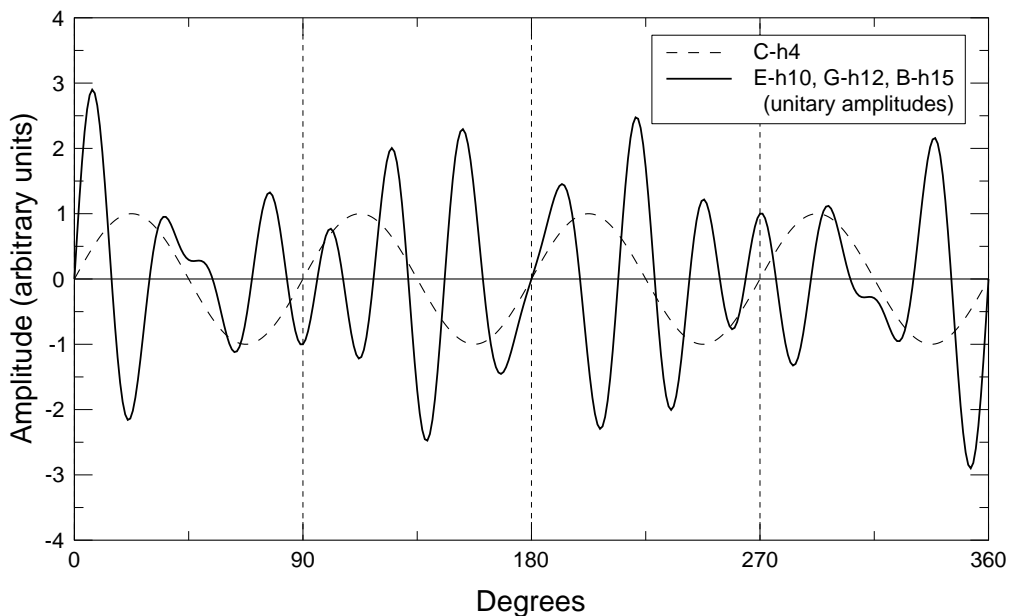


Figure 11.13 The questionable match of C-h4, with a discrepancy of five degrees at at 90 and 270 degrees.

The Minor-third

The most crucial note/interval in all this is the minor-third, G-h12 in the E-minor chord, which, working in conjunction with the period of repetition C-h1, the lowest difference tone C-h2 and the twelve oscillations of the overall period, possibly encourages the ear and mind to entertain the existence of an extended 'background' harmonic series from C-h1 up to, and beyond, h12. However, the period of such an extended harmonic series is prone to break down into subdivisions or aggregations – as decreed by the law of entropy increase, the all pervasive second law of thermodynamics. In the case of the minor triad, groups of five become interwoven into the fabric of the underlying series built on C-h1 – Figure 11.14. (A similar

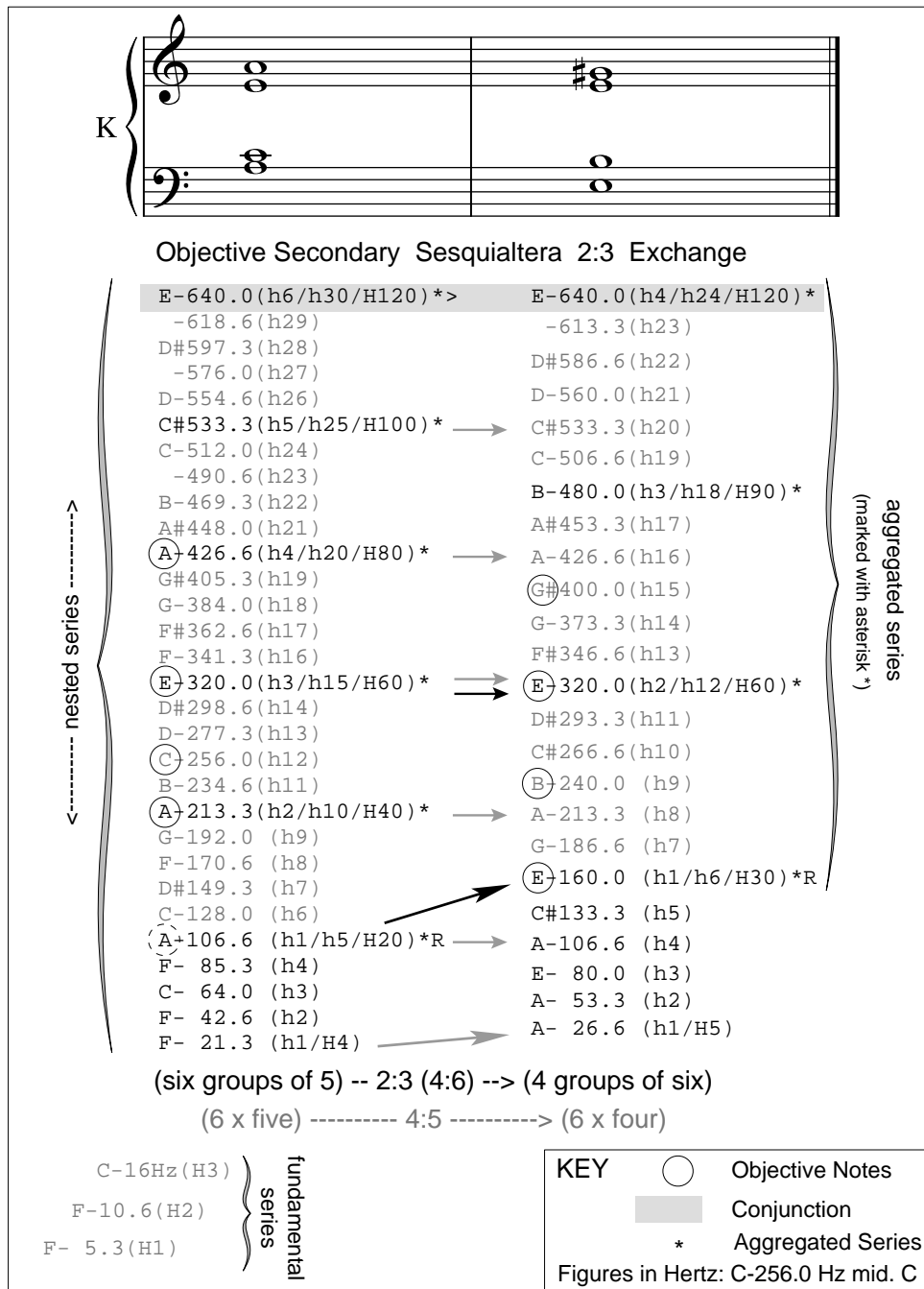


Figure 11.15 On the surface level of objective tones, a 4:3 (2:3) tonic-dominant progression with a fourth/fifth root movement, sets in motion an underlying 4:5 step of a major-third at the lower level of the nested harmonic series – from F-22.3(h1/H4) to A-26.6(h1/H5). In mutable numbers this chord progression of A-minor to E-major would be represented by the two digits sequences: MBN $6_5 0_4 0_1 \rightarrow 4_6 0_5 0_1$ both of which yield the value one hundred and twenty – Decimal $1_{10} 2_{10} 0_1$ in the generalised format.

A hint or clue to this duality of nested relationships, lies in the instability of the dominant chord in the minor key: the chord of E in the key of A-minor. The magical effect of the E-major chord replacing the normal E-minor harmony, that we feel and respond to, could be explained by the processing of minor chords within surrogate ‘major’ series. The delight we take in the sweet transformation to a major-third above the

11.20 - DUALISM

root representing a reduction in stress and complexity, further enhanced by the underlying (nested) 'major' series shifting its ground from H4 to H5 of the fundamental series. As an example of this relational dividend, the replacement of the 'meta-stable' E-minor chord (in the key of A-minor) with the more stable and less complex configuration of an E-major chord is illustrated in Figures 11.15. Where the 4:3 (2:3) relationship of the objective tonic-dominant chord progression of A-minor to E-major, at the level of aggregated series, provokes the underlying nested harmonic series to make a 4:5 (major-third) adjustment to accommodate the note E-160.0Hz; which the series based on F-21.33...Hz does not possess. This example is taking us into the arena of dynamic *Modulating Oscillatory Systems*. A fuller description of the terms and concepts of the MOS model will have been encountered in Chapter 9 and in Chapter 12 these are applied to a complete example composition: the first Prelude from the Well-tempered Clavier by J.S. Bach.

[13/06/09]

Notes

1. Apel, W., *Harvard Dictionary of Music* (Harvard University Press, Cambridge, Mass, 1966)
2. Jorgenson, D.A., *Resume of Harmonic Dualism*, (Music and Letters, XLIV, 1963) p31.
3. Beament, J., *How We Hear Music*, (The Boydell Press, Woodbridge, Suffolk, UK, 2005) p66.
4. Taylor, C., *The Science of Musical Sound*, in *Music and Mathematics*, Eds. J. Fauvel, R. Flood, & R. Wilson, (Oxford University Press, 2003) p59.
5. Beament (2005, *ibid.*) gives an interesting account of notes being apprehended as either pitch or tone in what he calls the 'Three-tone Paradox', section 7.6, p83.
6. This raises the question of what happens when the notes in a chord are sufficiently widely spaced so that their fundamental resonances do not overlap, do we hear a chord or three separate pitches? Given the flexibility of aural cognition, we generally hear what we expect to hear – usually perceiving the intermixed tone of the chord – especially as the overtones of the lower notes will overlap the upper notes, in a similar way to the fundamentals of a triad. Learning and experience, to a great extent, influence perception; even to the point of sweeping up melodic elements into chords, as for example in the unaccompanied suites by J.S.Bach.
7. Weinberger, N.M., *Music and the Brain*, (Scientific American, Nov. 2004) p68.
8. Jeans, J., *Science and Music*, p86 (Cambridge University Press, 1937; Dover, New York, 1968)
9. The interval of a minor sixth will generate a difference tone a major sixth below the lower of the two notes (i.e interval C to Aflat generates Eflat below). The relationship between these notes is that of h3, h5 and h8 of the harmonic series unlike the other difference tone which have the relationship of h1 (to a power of two), hn, hm. That is, apart from the minor sixth, difference tones do otherwise generate the fundamental note letter, of their series and so would seem at least to run parallel to the rootedness of chords phenomena. For example, the interval of a major sixth will generate a difference tone a fifth below the lower of the two notes (i.e interval C to A generates F below) which yields the series F-h2, C-h3, A-h5. However, in the counter example of the minor sixth, the generation of the difference tone Eflat-h3 along side the objective tones of C-h5 and Aflat-h8, produces the perception (at least for me) of an Aflat major chord with a root note of Aflat lying below the difference tone Eflat. Therefore, in this case the rootedness of the objective notes is not directly produced by the process which generates difference tones, wherever that process resides in the ear-brain hearing system.

Objective tones, treble staff.

Difference tones, bass staff in black, enfolding harmonic series in gray.

Figure 11.16 The most prominent difference tones set within the context their enfolding harmonic series. Third from the left is the 'counter example' of the minor-sixth which doesn't define a root note. Also on the right, the crucial interval of a minor-third, C to Eflat, 'points to' the root of a series founded on Aflat, not its perceived root note C.

10. The 'picardy third' replacement of the expected tonic minor chord, at the final cadence of a composition in a minor key, by the tonic major chord. The practice has existed since 1500, and the name suggests, perhaps, an association with the Franco-Flemish polyphonic school.