Journey to the Heart of Music

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Chapter 9 – Identity and Change

MODULATING OSCILLATORY SYSTEMS

The topic of this chapter counterbalances the formal approach pursued in Chapter 1, where harmony, the core structural feature in tonal music, was portrayed in terms of positional mutable base numbers. Here again, an understanding of the role of harmony in western music is the central goal, but in this chapter a dynamical perspective is taken upon the subject, casting tonal music in the role of a physical system viewed in quasi-kinematic terms. These two approaches, formal mutable numbers (abr. MBN) and physical modulating oscillatory systems (abr. MOS) are complementary. Indeed, the 'metrical algebra' of modulation exchanges presented below is entirely equivalent to the computations of formal mutable numbers, given in Chapter 1 and elsewhere. The consideration of western music in dynamical terms forms a longstanding and vibrant thread in music theory, which has on occasions been characterised as *Energetics*.

Perhaps the two most fundamental categories which can be applied to the material world are those of *identity* and *events*: what 'things' are and how 'things' change.[1] In this chapter the outline of a model of pseudo-physical musical 'worlds' (tonal compositions, in principle), constructed in terms of these two primal categories, is viewed from the perspective of the interaction of nested harmonic/arithmetic series. The model focuses on the most basic relationships – ratios of (positive) natural numbers – and how these simplest of entities (expressed as regular periodic behaviour, oscillation) might interact to develop structure, process information and create variety. The model describes a physical process of *oscillatory computation*, notionally driven by the second law of thermodynamics and governed by a simple, yet subtle, process: the algorithm of symmetrical exchange. 'Identity' in modulating oscillatory systems derives from arrangements of harmonic or arithmetic series, while 'events' are changes in these configurations brought about through the mechanism of this *modulation* algorithm. (For the sake of completeness, arithmetic series have been included above, as the MOS model works equally with either. However, as our topic is music, the discussion below is framed for the most part in terms of harmonic series.)

At the heart of the scheme lies a general relational model: *little worlds* of relationships. Essentially, the model consists of relationships and a process by which the relationships can be changed. The relationships are arranged as two or more entangled harmonic (or arithmetic) series, one a fundamental *nesting series*, within which the other *nested series* reside. The mechanism whereby the constituent series interact – *modulation* (used in the broader sense of change in relationships) – denotes a process of exchange or transformation, within and between harmonic or arithmetic series. In computational terms, the nesting and nested series are data, while the algorithm or rule which acts upon them is modulation.

These *little worlds* of relationships – pieces of tonal music viewed as ideal self-organising dynamical systems – do require some level of internal structure. A separation of internal parts is necessary because there is no absolute or fixed background against which a wholly relational system can *compute* its evolution. The system can only *see* its internal relationships and cannot grasp at handles outside itself. Thus it requires at least two related sets of coordinates within the system to negotiate change, allowing for each to anchor the other, while manoeuvring between old and new reference points. These two sets of relationships are the *nesting* and *nested* harmonic or arithmetic series, the manoeuvring is *modulation*.

Before ploughing into the detail it might be helpful to briefly lift our gaze to the broad sweep of landscape. Toy worlds, models and thought experiments are familiar tools, where idealised schemes, often reduced to their barest essentials, are devised in order to gain some understanding of complex real world systems and processes. In regarding a piece of music (or any other oscillatory system) as a self-contained, self-consistent world of relationships, the particular little world under consideration is being viewed, in principle, as a little universe or whole world: its internal relationships are *all that is*, viewed from inside the system. Though, at some different scale, this little world might appear as no more than a tiny unit in some other broader scheme: and of course in the case of music, the ear, aural cognition and the human mind provide a wider territorial context, within which these notionally self-organising musical systems operate.

When viewing music from a dynamical standpoint, as a pseudo or quasi-physical scheme, to begin with the question arises: what, at bottom, is this system? Or even, what is its elemental unit, what is a note? If you take a note out of a piece of music, what have you got? A frequency (plus harmonics of timbre), an amplitude and a duration – say half a second of sound at 440Hz – the note ‘A’ above middle C. Once the note has been removed from the context of the piece as a whole – its own little world – there is no way of knowing much about it. Was it the highest pitched note in a melodic phrase? Was it loud in relation to the other notes in the piece? Was its duration long or short compared to other simultaneously sounding notes? The note has lost its ‘identity’, its meaning. What this poor little note is (or rather was), depended on all the other notes in the composition: the network of close and distant relationships shared amongst the whole.

It is the relative relationships within the piece that carry the musical information. After all, transposing a piece of music to a different pitch and/or tempo, within reasonable limits, doesn’t alter its essential qualities. On average, most of the music of the baroque and classical eras is performed today at a higher pitch and different tempi than in their own time, which rather makes the point; the ‘music’ is the internal relationships between the notes and chords, not any absolute collection of frequencies and durations. The conclusion to be drawn is that what a note 'is' – its identity or meaning in a piece of music – is the sum of its relationships with the other notes and chords within the piece (and vice versa). And, by extension that a piece of music, as a whole, is a scheme of changes to note relationships, a structure of musical events, experienced sequentially in performance.

However, not just any arbitrary set of relationships between notes produces a satisfactory musical experience, only relationships that are governed, in general, by a particular algorithm or rule, have proved to be acceptable to the vast majority of participants in the western tradition. This particular process of computing whole number relationships is known to musician's as modulation; and although modulation is thought of traditionally in terms of changing from one key to another, the basic mechanism, as demonstrated by the dynamical approach pursued below, permeates many other areas of tonally organised music. In particular, the modulation algorithm of symmetrical exchange is shown to operate between individual notes and chords, and thereby govern music's central structural feature, harmonic progression.

TABLE OF HARMONIC SERIES (abr. THS)

As already mentioned, some time ago while attempting to devise a music data format based entirely on harmonic series, I constructed a table or matrix of series to help me think out how it might be done. The seed from which this idea grew, came from studying the output of cellular automata.[2] This table of nested harmonic series (Figure 9.2), which is effectively the *Sieve of Eratosthenes* – the ancient method for deriving prime numbers – contained intriguing patterns of *arrowheads* and *conjunctions* between the various ratios. The start of the sequence of *primary conjunctions* is marked by a dashed line in Figure 9.2.

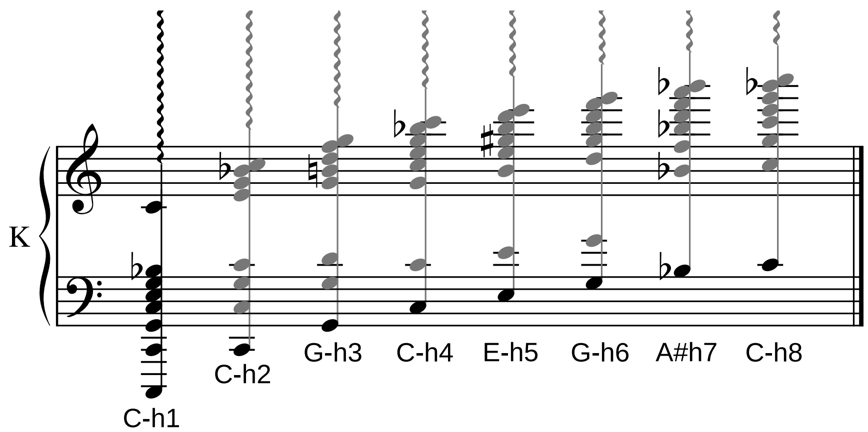

Figure 9.1 The first eight ratios of the harmonic series, labelled C-h1 through C-h8 and illustrated with black-headed notes on the bass staff. Above this 'arpeggio' of the *chord of the harmonic series*, in gray-headed notes, are the harmonic partials of each note of the series. (A wiggly extension of the note stems indicating further harmonics to hn.)

The table, Figure 9.2, could be thought of as a visual plot of *all the harmonics* arising from the notes in the series, that is the harmonic partials of the black notes in Figure 9.1. This *chord of the harmonic series*, is in a sense fundamental, in that no harmonics will be found in the series based on the individual black notes that are not also present in this chord's extrapolation – the fundamental series. (In practice there might well be slight differences, but for this discussion we can enjoy the luxury of an ideal system.)

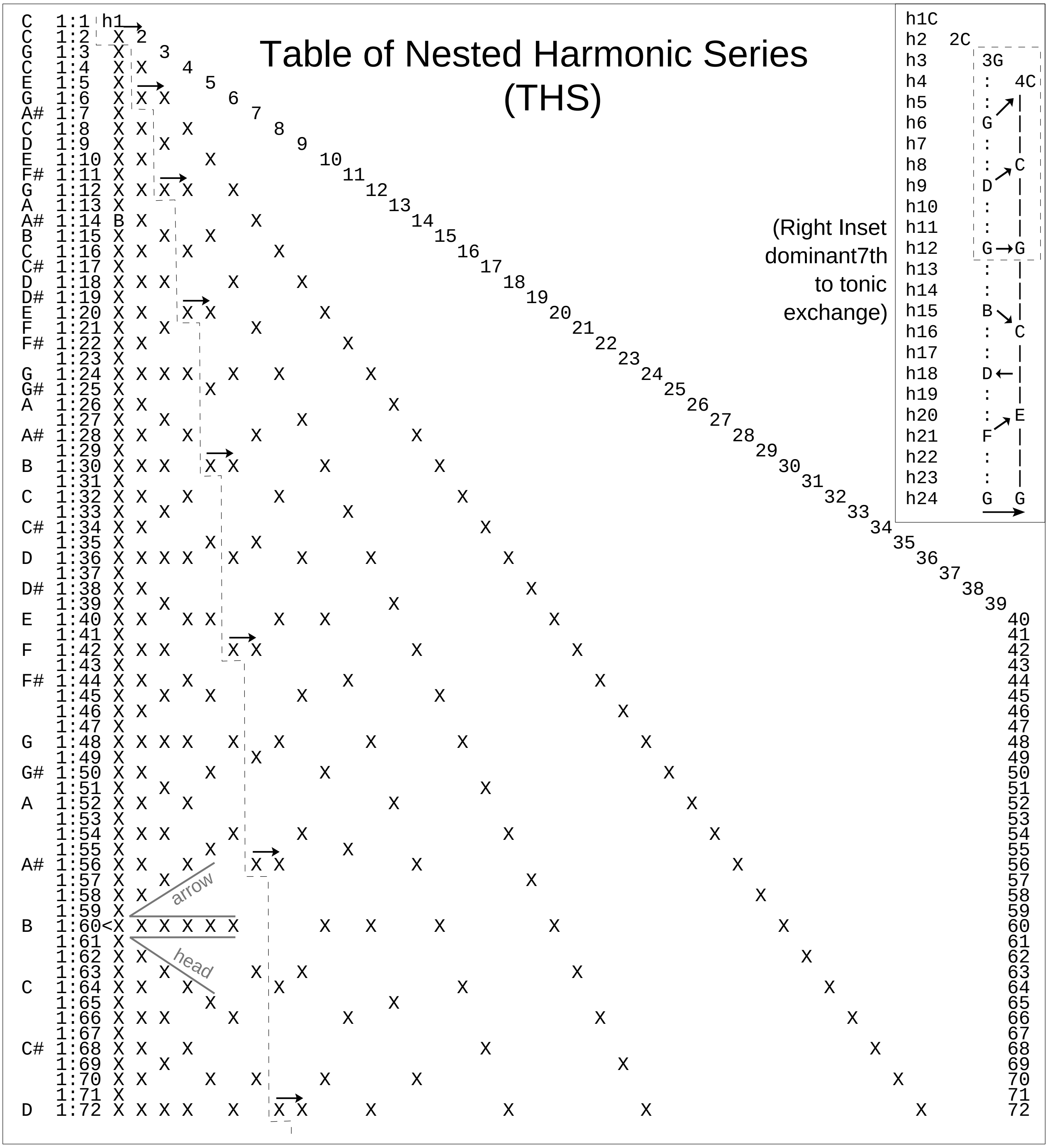

Figure 9.2 A fundamental nesting harmonic series (the 45 degree column numbers) drawn with a matrix of all possible nested harmonic series up to h36 – the actual columns of Xs – with the primary conjunctions connected by the dashed line and mark by small black arrows. (The further extension of the dashed line is charted in Figure 9.14, where the black arrow chord progressions provoke an upward cycle of key shifts.) The arrowhead at h60 is outlined in gray. Right inset: a secondary sesquitertia 3:4 modulation exchange – a perfect cadence – discussed below, with the primary sesquitertia 3:4 modulation exchange enclosed by the broken line box.

Looking at these conjunctions, starting from the top left corner, it can be seen that moving across the columns from left to right, following the dashed line, produces exchanges of two ratios for one, three ratios for two, four ratios for three, etc. From a musical point of view, the exchanges follow the pattern of an octave for a unison (C+C -> C), a twelfth for an octave (C+C+G -> G+G), a fifteenth for a twelfth (G+G+D+G -> C+C+G), etc. However, if the fundamental series (the column numbers) are retained in these exchanges, it becomes apparent that ratios are not being lost by this procedure, but transformed by a process which passes them down from the upper nested harmonic series to the lower fundamental nesting series. The nested series is moving in steps which trace out the ratios of nesting series one by one.

Modulation

This exchange process might be described as a sequence of computations – addition followed by transformation. Below, with the entangled upper nested harmonic series and the lower fundamental nesting series enclosed in curly brackets, i.e. {{nesting series}, nested series}, the computations could be written:

```
                  {h1C} + {h2C} -> {h1C,h2C}
          {{h1C,h2C}, +h4, +h6} -> {{h1C,h2C,h3G}, h6G}
{{h1C,h2C,h3G}, h6G, +h9D, +h12G} -> {{h1C,h2,h3G,h4C}, h8C, h12G}
```

In this last exchange above, one can see the outline of what is in musical terms a full or perfect cadence in the key of C-major – the chord progression G-major -> C-major (broken line box in Figure 9.2). Or we could say chord V resolves to chord I in the musician’s vocabulary, or ‘set G’ is mapped to ‘set C’ in mathematical terminology; and, if the upper nested section where to be extended to h24G we could obtain the more urgent sound of the dominant-seventh chord resolving to the tonic (below and Figure 9.2 inset).

```
{{h1C,h2C,h3G}, h6G, h9D, h12G, h15B, h18D, h21F, h24G} --->
                        ---> {{h1C,h2C,h3G,h4C}, h8C, h12G, h16C, h20E, h24G}
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The ‘primary’ full cadence (on the dashed line) moves between the fundamentals h3G and h4C, by exchanging four ratios for three ratios while this latter and rather more elaborate ‘secondary’ full cadence exchanges eight ratios for six ratios, again on the same foundations. Continuing this upward extension finds unlimited numbers of conjunctions yielding ever more complex ‘tertiary’, ‘quaternary’, etc. exchanges in the THS. (Beyond a quaternary exchange the term *‘n’-fold* exchange is introduced, e.g. *five-fold* exchange.)

Computing Internal Arrangements

However, returning to the primary exchanges or modulations (following the dashed line in the THS) it is apparent that for as long as ratios are added to the top of the nested harmonic series, this sequence would compute the set of positive whole number ratios: h1C, h2C, h3G, h4C, h5E... hn – the fundamental series. Thus the upper nested harmonic series is extending the lower nesting harmonic series of the system by one ratio per modulation/transformation, with each step in the sequence achieved by an exchange of n -> (n – 1) ratios of the *nested* harmonic series based on the *nesting* fundamental frequencies h(n – 1) and hn. These primary exchanges could be viewed as the *primitive operations* of the model’s programming language. In Figure 9.3 the first eight of these primary modulations are illustrated in musical format (with each one adjusted so as to begin on a C-major chord).

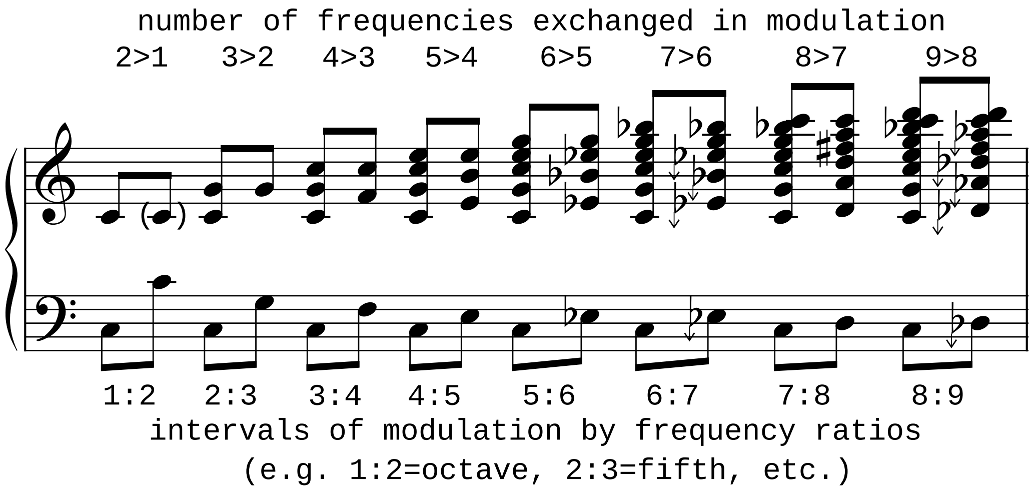

Figure 9.3 Primary modulations expressed in musical (harmonic) terms. Symmetrically, the interval of modulation (in the bass clef) is mirrored by the reverse number of notes exchanged, i.e. two notes exchanged for one results in an octave modulation, three notes for two a fifth, etc. The modulations 6:7 and 8:9 fall outside the fractions of the musical scale and the (just intonation) semitone modulation 15:16 is not reached in this illustration. While equally tempered modulation has no place in a 'natural' whole number based system, the modulation 17:18 gives a close approximation to the tempered semitone i.e. 18/17 = 1.0588 against 1.0594 for the tempered interval (as noted by Vincenzo Galilei).

In principle, these *modulation* exchanges might be interpreted as a process whereby a piece of music (or any oscillatory system) changes from one arrangement of internal relationships (constituent frequencies) to another. In tonal music, the rearrangements appear to operate at a number of levels: the levels of key/tonal center, chord/harmonic progressions, rhythm/meter, timbre/tone color. Most often the reconfiguration will be from a more complicated way of generating a given number of fluctuations per period to a less complicated internal arrangement which achieves the same goal with greater economy – as the Law of Increasing Entropy demands. A musical phrase typically launches from rest with a rapid increase in complexity, followed by a gentle release of information as it drifts back down to a closing cadence. However, under particular conditions an oscillatory system might move in the opposite direction, absorbing energy by moving to a more complex internal configuration – reversing the small black arrows in the THS. Modulation is a two-way thoroughfare. In general terms the modulation algorithm can be expressed in the form:

$$n_{f(\frac{m}{a})} \leftrightarrow m_{f(\frac{n}{a})}$$

e.g. secondary sesquitertia modulation exchange
n = 8, m = 6, a = 2
dominant-seventh to tonic progression

for series of 'm' and 'n' harmonics, with integer values of 'a' representing primary, secondary, tertiary, etc. exchanges and 'f()' signifying the relative fundamental frequency of the respective series.

Historical Perspectives

Remarkably, medieval and renaissance scholars and composers from Franco of Cologne in the 1250s onward,[3] gradually worked out these modulation exchanges, in the form, principally, of rhythmic proportions in the *mensural system of notation,*[4] providing a thoroughly systematic scheme of classification. For example, proportio dupla 2/1, sesquialtera 3/2, sesquitertia 4/3, sesquiquarta 5/4, etc., using the Latin prefix to denote a numerator larger by one than its denominator – which is exactly what is needed to classify the conjunctions and exchanges in the THS. So, adopting their scheme, the primary exchange of four ratios based on h3 for three ratios based on h4 could be termed a *primary sesquitertia 3:4 modulation* and the fancy dominant-seventhth exchange of eight ratios for six, also on h3 and h4, would be a *secondary sesquitertia 3:4 modulation* – the Figure 9.2 inset.

Name	Proportion	Ratios/Harmonics	Notes/Chords Exchanged
Dupla	1:2 exchange	h1, 2 --> h2.	2 notes C+C --> 1 note C
Sesquialtera	2:3 exchange	h2, 4, 6 --> h3, 6.	3 notes C+C+G --> 2 notes G+G
Sesquitertia	3:4 exchange	h3, 6, 9, 12 --> h4, 8, 12.	4 notes G+G+D+G --> 3 notes C+C+G
Sesquiquarta	4:5 exchange	h4, 8, 12, 16, 20 --> h5, 10, 15, 20.	5 notes C+C+G+C+E --> 4 notes E+E+B+E
Sesquiquinta	5:6 exchange	h5, 10, 15, 20, 25, 30 --> h6, 12, 18, 24, 30.	6 notes E+E+B+E+G#+B --> 5 notes G+G+D+G+B
Sesquisexta	6:7 exchange	h6, 12, 18, 24, 30, 36, 42 --> h7, 14, 21, 28, 35, 42.	7 notes G+G+D+G+B+D+F --> 6 notes A#+A#+F+A#+D+F
Sesquiseptima	7:8 exchange	h7, 14, 21, 28, 35, 42, 49, 56 --> h8, 16, 24, 32, 40, 48, 56.	8 notes A#+A#+F+A#+D+F+G#+A# --> 7 notes C+C+G+C+E+G+A#
Sesquioctava	8:9 exchange	h8, 16, 24, 32, 40, 48, 56, 64, 72 --> h9, 18, 27, 36, 45, 54, 63, 72.	9 notes C+C+G+C+E+G+A#+C+D --> 8 notes D+D+A+D+F#+A+C+D
Secondary Sesquitertia	3:4 exchange	h3, 6, 9, 12, 15, 18, 21, 24 --> h4, 8, 12, 16, 20, 24.	8 notes G+G+D+G+B+D+F+G --> 6 notes C+C+G+C+E+G

Figure 9.4 Table of the first eight primary modulation exchanges plus the secondary sesquitertia 3:4 (V^7- I) exchange.

Like the pedestrians on the Millennium footbridge, once musicians strode forth in harmonic steps they became unknowing players in a metrical game. Trial and error, plus an evolutionary process, seeking out the optimal organizational principle for an oscillatory system – structure through nested modulation. The evolutionary path trod by western music was (is) I think, beyond the overall control of individuals and groups. The surface details no doubt were influenced by what might be termed ‘fashion drift’ and more directly by composers through innovative changes of style. However, the development of the underlying structure of an evolving system would essentially obey the laws of a musical form of natural selection controlled en masse by the musical ‘consumers’, the agents of selection, but acting beyond prescriptive direction. The unseen hand of selection would, I believe, inevitably guide a system of music in harmonic parts (i.e. polyphony) toward the organizing principle of modulation – at every level. The point at which the system of harmonic modulation ‘took off’ is probably marked by the change from stepwise ‘melodic’ motion to movement by interval leaps in the bass part, as it started to track the whole number relationships of modulation from chord to chord (as well as key to key). Which would put the dawn of the tonal era circa 1450 – 1500, halfway between the writing of the two examples in Figure 9.5.

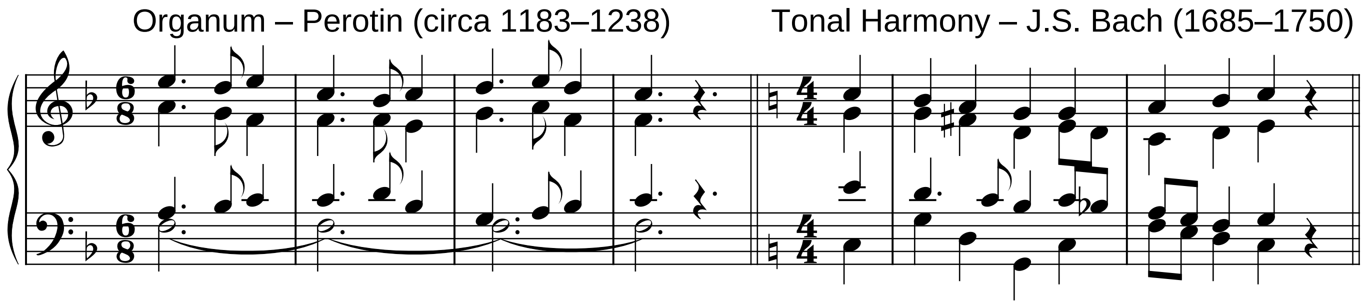

Figure 9.5 Whilst the ecclesiastical music of the middle ages evokes a vision of the unchanging certainities of heaven the tonal music of Bach presents a dynamic system of this world, of motion and computation.

The organum (an early form of music in parts) is taken from *A History of Western Music* by Donald J. Grout and the accompanying text makes the point: “The sustained note sections, which sometimes involve hundreds of measures [of 6/8] over one unchanging bass note, form great blocks of fundamentally static harmony... [and] do not have the quality of harmonic movement to which we are accustomed in music of the eighteenth and nineteenth centuries, organized around clearly related tonal centers and working with dominant-tonic relationships; one cannot speak of chord progressions in Perotin, but only chord successions. The musical shape of a Perotin organum is defined by the design of the Gregorian Chant on which the piece is built...”

Gradually, over the past 500 years or more I suspect, musicians have been discovering and elaborating a system of music which contains its own internal logic – musical structure based on the whole number relationships of nested modulation. The need for ‘external scaffolding’ like cantus firmus chant melodies and the like, to hold the edifice up, fell away, as little by little, composers gained the confidence to build with the new materials of tonal relationships – which they found, miraculously, were self-supporting. It is, I think, the defining discovery of western music: *that tonal compositions – oscillatory little worlds – can stand on their own as self-coherent entities supported solely by their internal relationships,* given that the relationships are (broadly) those generated by the process of nested modulation.

What also fell away over the years was the elaborate framework of medieval theory, much of it relevant, in terms of understanding ratio and proportion, but no longer explicitly required by composers for the new, viscerally intuitive, tonally organised music. The mathematics – i.e. the structural scaffolding – was still there holding up the building, but now internalised as chord progressions and key relationships, computed by the modulation algorithm, to produce self-sustaining musical structures – *little worlds*. However, a side effect of this shift from elaborate theorising about ratio and proportion to the actual *living out of the theory in sound*, in the form of tonal compositions, was that music became detached from its medieval sister disciplines of mathematics and astronomy, drifting over time into the arms of ‘the arts’. Just as musicians were discovering through practical usage, what I believe is a partial implementation of a more basic system of nature – the generation of structure by the mechanism of nested modulation in oscillatory systems, or in the parlance of the computer, input converted to output by the algorithm of modulation – the gap between the artists and the men of the new science proved too wide for this intuitive discovery to leap or leak across.

Of course the development was nowhere near clear cut. Music’s little worlds of tonal relationships are by no means rigorous applications of the principle of modulation. Also medieval music contains a fair degree of tonal reference, and J.S. Bach, arguably the greatest tonalist, was extraordinarily fond of and adept at, adding extra (non-harmonic) contrapuntal elaboration. However, the fact that his music can be understood and enjoyed without the slightest awareness of these additional layers underlines the pre-eminence of tonal relationships as the primary source of form and structure. The change in external style, which came after Bach, exemplified in the *idealised simplicity* of Haydn and Mozart, was to mark a classical phase of pure, internalised, tonal architecture. Musical structure and expression, form and function, as one:

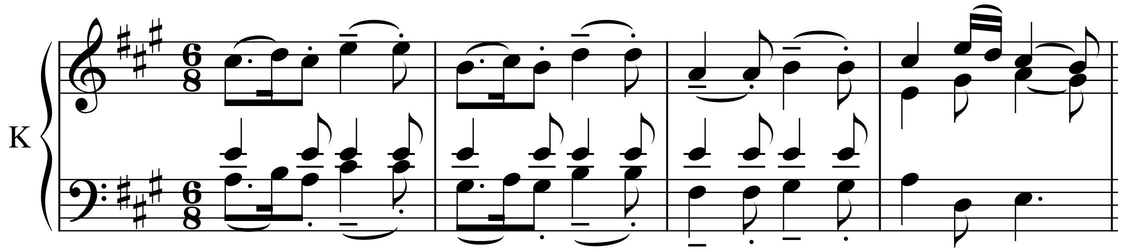

Figure 9.6 Theme from the Piano Sonata in A Major (K331/300) by W.A. Mozart.

Nesting and Nested Harmonic Series

Illustrated below is a harmonic series, and to keep things simple only the first six harmonics of the series are illustrated. So in effect we are looking at columns 1, 2, 3 and rows 1:1 to 1:6 in the top left corner of the Table of Harmonic Series, Figure 9.2. (An expanded version of Figure 9.7 can be found in the CHPT19 folder, Quick Start Outline Figure 5.)

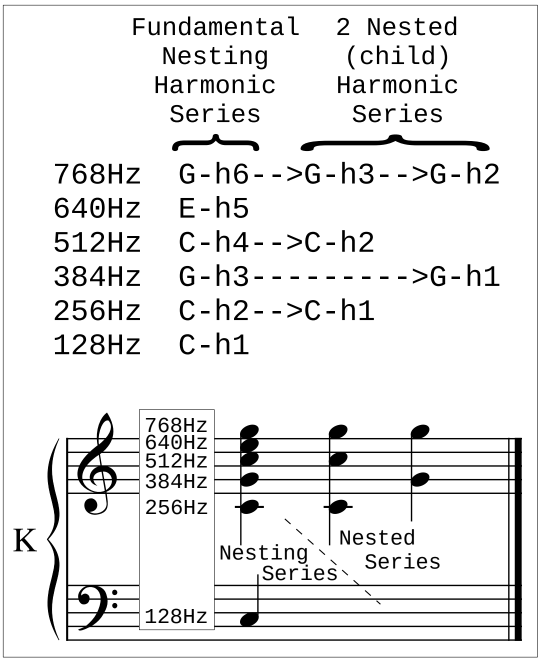

Figure 9.7 Extract from the opening of the Table of Harmonic Series expressed in ratios and chords.

Within a harmonic series there are potentially an unlimited number of nested harmonic series. On the left of Figure 9.7 is a fundamental nesting series consisting of the first six ratios of the harmonic series built on frequency C-128Hz. These six ratios form the common C-major chord expressed as a complete harmonic series, most often in music, chords appear in a less full and organized arrangement – though the overtones deriving from the lower notes in a chord will plug many of the gaps.[5]

To the right of the six ratios are two *child* or nested series built on the frequencies C-256Hz and G-384Hz. The nested series are subsets of the nesting series, that is to say, there are no new ratios in the child series that are not also to be found in the *parent* series; and though we are considering here only six ratios, however far the parent and child series are extended, no ratios will be found in the child series which are not in the parent series. You can also see that C-256Hz, the h1 foundation of the first child series, is the second harmonic of the fundamental parent series and that G-384Hz, the h1 foundation of the second child series, is the third harmonic of the parent series. The THS above, Figure 9.2, illustrates the further expansion up to the start of the 36th child series. One might also draw a computational analogy between the parent/nesting series as hardware and the child/nested series as software.

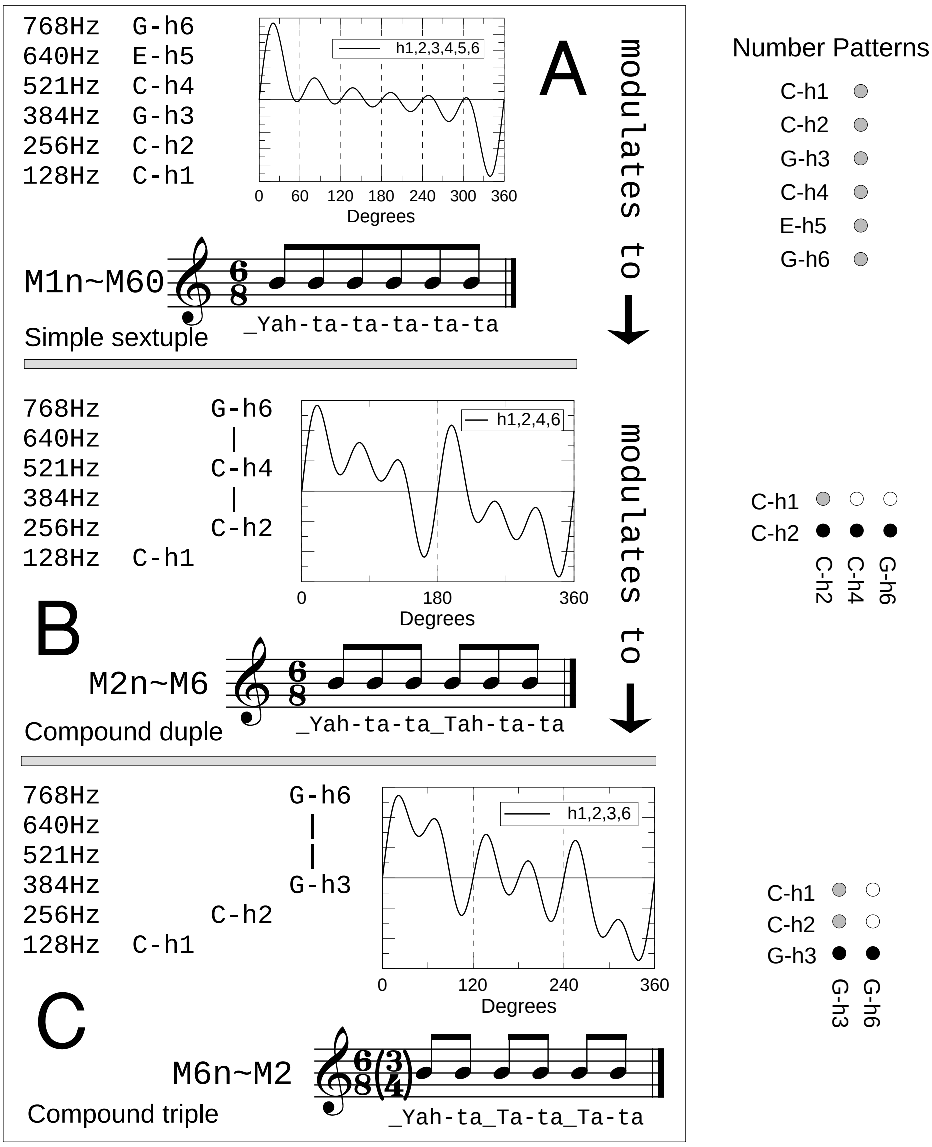

Figure 9.8 Four ways of describing internal arrangements: (top left) ratios in a table of harmonic series, (top right) as a graph, (bottom left) as nested Meters and (bottom right) as time signatures and meters/rhythmic patterns.

As a vibrational system, the fundamental parent series is a rather labored and complex way of producing an interference pattern of period 128Hz containing six fluctuations – this is illustration A/A1 in Figures 9.8/9.9 (assuming roughly equal amplitudes and uniform phase). There are two less energetic alternatives. Illustrations B/B1 and C/C1 both produce a period of 128Hz and 6 fluctuations per period but in a more economical way. The A, B and C boxes in Figure 9.8 illustrate four different but equivalent ways of describing and thinking about the alternative arrangements of internal relationships, which a system can adopt to generate six fluctuations period.

In these four views of a system of nested harmonic relationships, the graphs of the interference patterns generated by the harmonics are aligned with the relevant time-signature/meter. Interpreting interference patterns, the *sum* of an oscillatory system's parts, in metrical terms, is helpful in understanding how a small and gradual change at the level of the whole system, a slight leaking of *metrical accent*, can be linked with a *step change* within the individual parts.

Triangulation

The existence of two entangled harmonic series is a crucial element in the process of modulation, as it allows a form of *triangulation* to occur. In the illustrations A1, B1 and C1 in Figure 9.9, two vertices of a triangle of relationships, the period and the number of fluctuations per period, are held constant at h1 and h6 respectively in the interference pattern generated by the combined nesting and nested series. The third vertex is the point where the nested series/sub-system joins onto the fundamental nesting series. This joining point has some freedom to manoeuvre between 128Hz in illustration A1, 256Hz in B1 and 384Hz in C1 – which are the harmonics h1, h2 and h3 of the nesting series. No other frequencies (joins) are compatible with a period of 128Hz and six fluctuations per period (except the inverse of A – not illustrated).

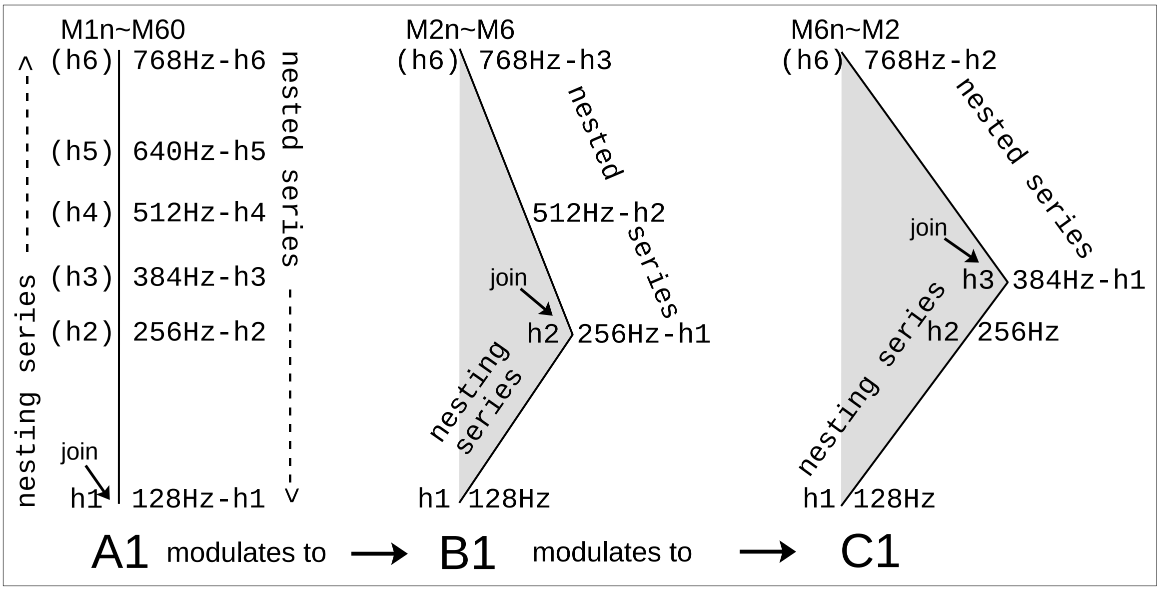

Figure 9.9 From left to right, the system relaxes, under the influence of the second law of thermodynamics, to the most economical internal arrangement, which it finds in the form: [h1C + h2C + (h3G] + h6G).

After Modulation

Once the nested subsystem has exchanged three ratios based on 256Hz (illustrations B/B1) for two ratios based on 384Hz (illustrations C/C1), the upper vertex of the system is ready to absorb two more ratio/oscillators, in the build-up to the next conjunction at row 1:12 in the Table of Harmonic Series. The sequence is illustrated in boxes D, E and F. This process is continuing the system's evolution as described by the dashed line in the THS, Figure 9.2, the process is moving the system forward through another energy cycle of gain, gain, modulation and loss.

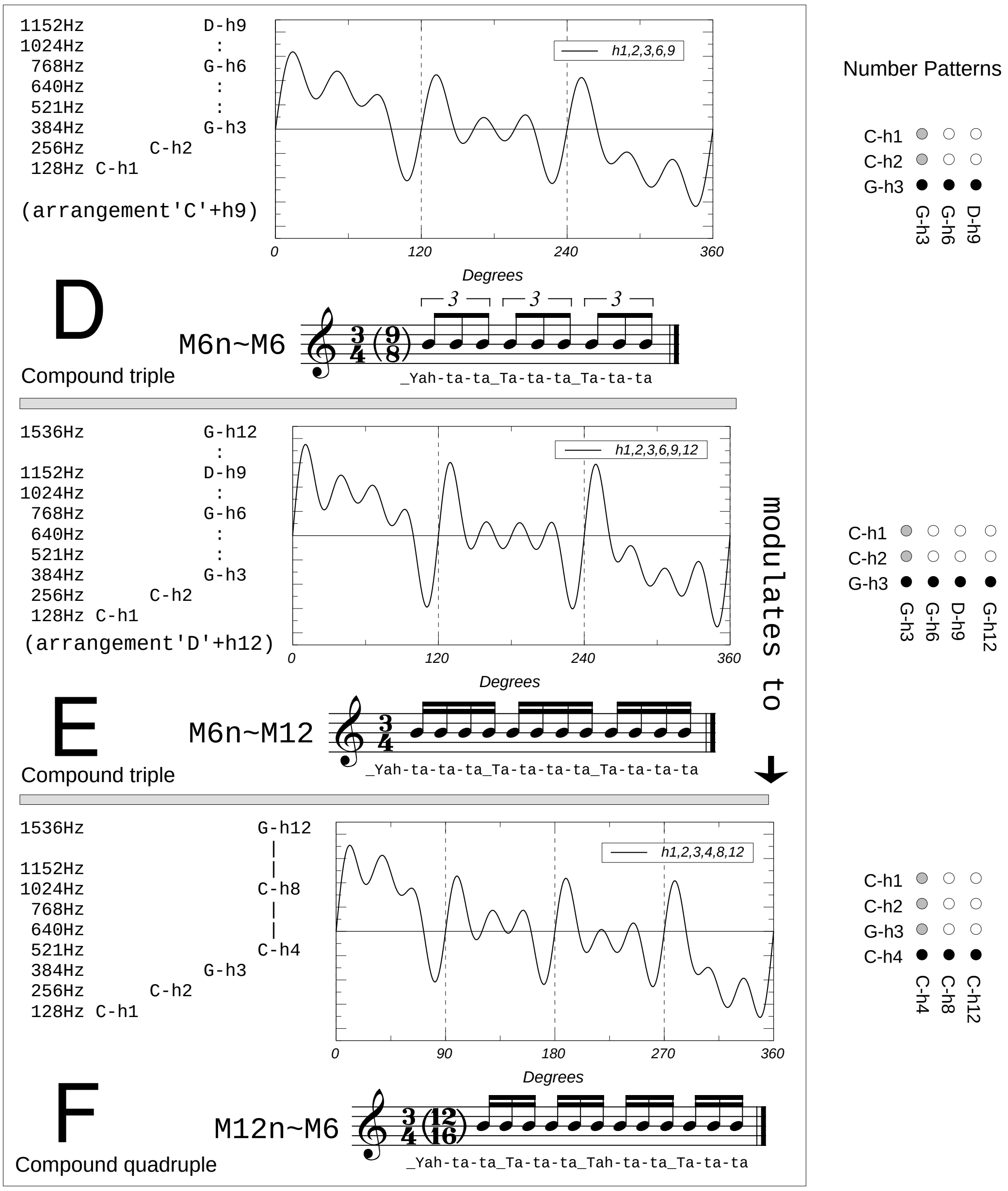

Figure 9.10 Four ways of describing internal arrangements, as in Figure 9.8. Illustrations D, E and F follow on from A, B and C, showing the further development of the system up to the next conjunction (h12) in the Table of Harmonic Series, where another modulation/exchange can occur. In both figures the relevant number pattern is appended on the right-hand side.

Number Patterns

As noted in Chapter 1, probably the oldest forms of abstract mathematics involved the use of physical tokens to represent quantities and magnitudes, with pebbles, seeds, sticks, etc... among the most likely counters beyond the ten finger-digits; and in such presumably purely additive number systems, the arrangements of tokens can fall into patterns. The ancient Greek mathematicians classifying some arrangements as square and oblong, plus other shapes.

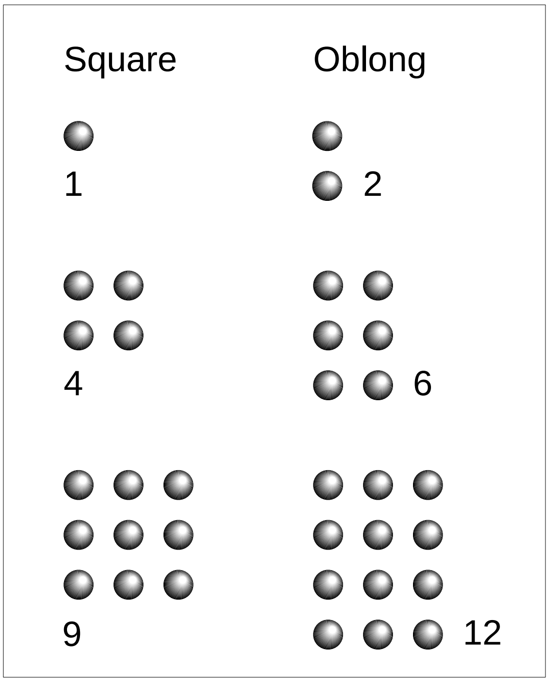

Figure 9.11 The first few square and oblong numbers.

The square and oblong numbers, viewed in the two coordinate dimensions of a vertical and horizontal array, have a particular connection with the modulation algorithm. Interestingly, the square and oblong numbers form an alternating sequence, which, beginning from unity, runs: 1, 2, 4, 6, 9, 12, 16, 20, 25, 30, ...; with each square number adding one column to form the next oblong number and each oblong number redefining its axis through modulation, before adding another column to form the next square – illustrated in Figure 9.12. It is precisely this same sequence of numbers which is mapped out by the path of primary modulations in the Table of Nested Harmonic Series (THS Figure 9.2 dashed line).

Visually, the *rectilinear number patterns* – fully filled rectangular arrangements in all dimensions occupied – of which squares and oblongs are a subset, mimic all the possible nested configurations of whole number oscillatory combinations and mutable numbers. In the discussion here, there are just two levels, the fundamental vertical dimension and a nested horizontal layer, and while up to three levels of nesting can be represented on the Cartesian ‘xyz’ axes graphically, beyond the cubic (Chapter 1, Figure 1.9), rectilinear number patterns lose their direct physical representation and so also their illustrative usefulness.

In Figure 9.12, the sequence of the first four primary modulations, beginning from unity (i.e. the first natural number) is charted in these visually illustrative rectilinear number patterns. Each step involves either addition or modulation. Addition occurs when the system attaches the next commensurable ratio/oscillator in the harmonic series, to its ‘outer edge’ – its highest frequency. (Subtraction would involve the release of one or more ratios.) Addition continues until a rectilinear pattern is encountered, which has an equivalent but less energetic arrangement. At such points, a modulation or exchange may occur – mediated by the algorithm of symmetrical exchange, acting in accordance with the second law of thermodynamics. Basically, the system seeks out the configuration that yields its lowest viable level of energy and complexity – its ground state – that is, the most efficient internal arrangement capable of preserving the systems identity or integrity.

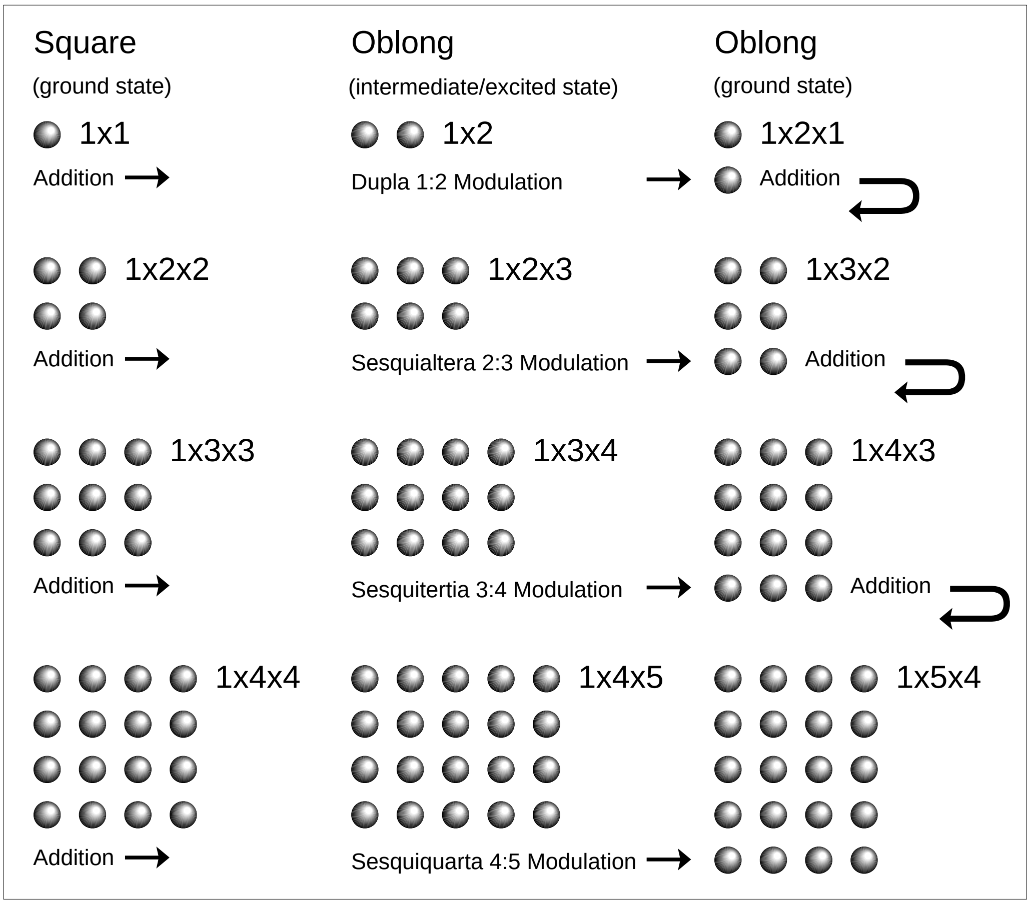

Figure 9.12 The first four primary modulations in the Table of Harmonic Series covering the numbers 1, 2, 4, 6, 9, 12, 16 and 20. The vertical axis represents the fundamental nesting series and the horizontal axis the nested series. Expressed as mutable numbers, the sequence of additions and modulations can be written:
MBN: 1_1 , 2_1 --> $1_2\,0_1$, $2_2\,0_1$, $3_2\,0_1$ --> $2_3\,0_1$, $3_3\,0_1$, $4_3\,0_1$ --> $3_4\,0_1$, $4_4\,0_1$, $5_4\,0_1$ --> $4_5\,0_1$ ...

The first step is that of addition, H1 + H2, a single, one-dimensional harmonic series. Rather wonderfully, although this series defines an axis (which here is illustrated vertically) it doesn’t preclude the existence of other axes, and with these other possibilities in play, the system has options. The question is what next to add, H3 or h4? If the system acquires H3 it continues its one-dimensional growth from MBN: 2_1 to form the prime state mutable number MBN: 3_1. Alternatively, the system could redefine itself (modulate) from MBN: 2_1 to MBN: 1_2 0_1, in other words become a two-dimensional entity, and then be open to the possibility of adding h4 rather than H3. The system doesn’t have to decide which form to take, one or two dimensions, until the second step of addition looms, but, once the addition happens the die is cast, and the system follows one or the other route: single axis H1, 2, 3 or dual axes H1, 2 nesting h4. After this choice is made, the two-dimensional system's next step of addition would be to attach h6 which leads to another modulation exchange, from H1, 2 nesting h4, 6 to H1, 2, 3 nesting h6 (second row Figure 9.12).

The alternative route of the one-dimensional harmonic series H1, 2, 3 next leads to the addition H4. However, the formation of the relatively complex prime state series, H1, 2, 3, 4 would provide another opportunity for the system to assume a two-dimensional form – this time via a secondary dupla 1:2 modulation from: H1, 2, 3, 4 to H1, 2 nesting h4. This exchange would involve the loss of h3, a large relaxation of energy and complexity and therefore highly attractive under the auspices of the Second Law. If the system again stubbornly resisted relaxation to a lower energy configuration, and continued its one-dimensional growth through H5, it would again find a route to relaxation proffered when it acquired the ratio/oscillator H6:

H1, 2, 3, 4, 5, 6 ------- tripla 1:3 modulation --------> H1, 2, 3 nesting h6 (MBN: 6_1 --> $2_3\,0_1$)
H1, 2, 3, 4, 5, 6 – tertiary dupla 1:2 modulation --> H1, 2 nesting h4, 6 (MBN: 6_1 --> $3_2\,0_1$)

The point illustrated here is that no matter how stubbornly a system resists a multi-dimensional structure, the modulation algorithm will incrementally offer an ever-increasing number of escape routes to a lower energy existence, while simultaneously, the Second Law will make the probability of the system taking one of these routes, increasingly large. In a low energy environment, only the prime number oscillatory patterns are safe from the siren call of modulation – as they already occupy their ground state configurations.

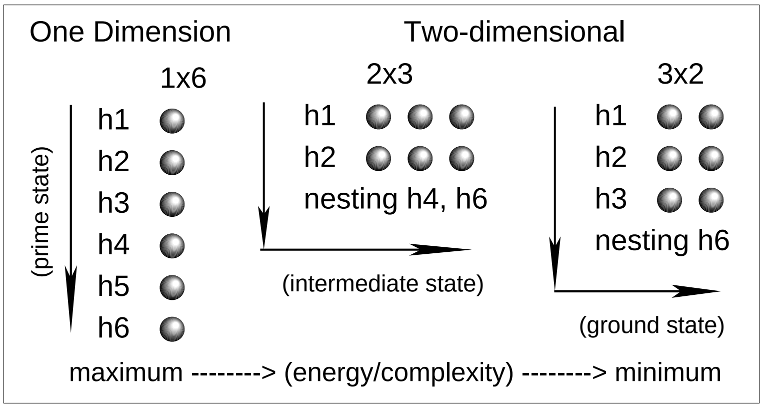

Figure 9.13 The number six has three possible rectilinear configurations: 1x6, 2x3 and 3x2, MBN: 6_1 , $3_2\,0_1$, $2_3\,0_1$.

All these scenarios lead to the addition of h6 to the system by one means or another. Six, as one of the set of fecund numbers and formed of the factors two and three, has two two-dimensional arrangements: 3×2 and 2×3. These two arrangements translate to the compound triple meter of time signature 3/4 and compound duple meter of 6/8 time – Figure 9.8B/C. Once a growing system has adopted a multi-dimensional structure, it will repeatably encounter points – mutable numbers like six – where it can release energy and complexity by rearranging its internal structure. In each of these modulatory steps between oblong number patterns (i.e. adjacent columns in the Table of Harmonic Series), the transformation is driven by the exchange of a higher energy oblong configuration for its corresponding lower energy equivalent, as illustrated in Figure 9.13 for six. The process fostered by the modulation algorithm (in principle, chord progression in tonal music) is powered by these transformations, as the Second Law relentlessly drives systems toward equilibrium.

The first step of modulation, the dupla exchange is perhaps special, somewhat subtle; in that the system is moving from a one-dimensional structure to an arrangement, which while remaining apparently one-dimensional, has become open to two dimensions by pairing ratios and growing by twos. Ultimately, the system still has a one-dimensional existence, there are still four tokens in the square number pattern 2×2 and there are still four fluctuations in the interference pattern of the combined frequencies H1, 2 nesting h4 (assuming uniform amplitudes). The difference between H1, 2, 3, 4 and H1, 2 nesting h4 (number patterns 4 and 2×2) is one of energy and complexity. A multi-dimensional structure is more efficient. Figure 9.14 shows this simplification visually. By grouping the underlying ratios of the fundamental series into twos, and then threes, a less complex arrangement of the whole system is obtained, but one which still maintains its identity – the value of the mutable number. From the perspective of aural cognition, such a process would allow a relatively simple (and therefore more intelligible) nested harmonic series, to be extracted from a perhaps complex and extended underlying series implied by a particular objective chord or progression.

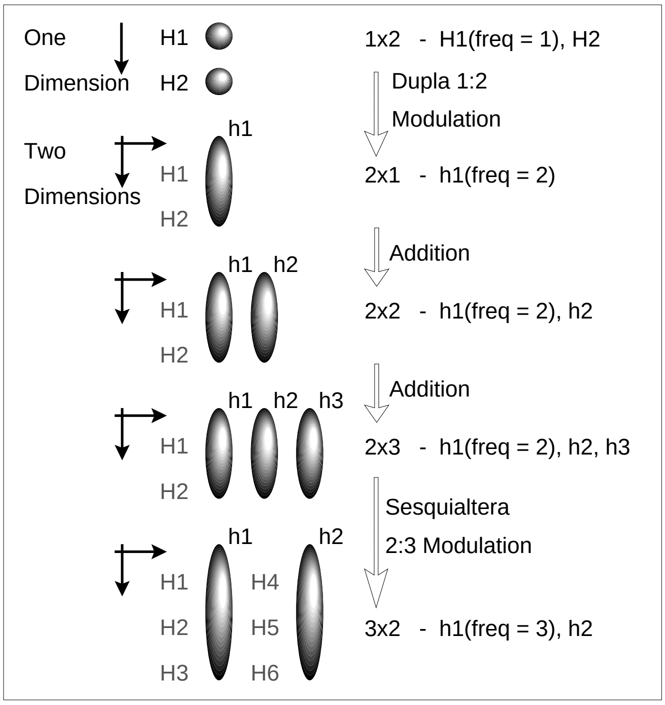

Figure 9.14 The simplified view of the complex fundamental series H1 through H6. (The color-scheme of gray and black for background and foreground ratios, is carried over into the examples in Chapter 12.)

Computing Structure

Briefly moving now from the details of the modulation mechanism to the broader canvas again. In applying this oscillatory model to our appreciation of music, the experience of tonal music can be likened to a journey *through the structure* of the piece, with the passage from chord to chord and key to key understood (more or less rigorously) as a sequence of *computational steps*, taken by means of the *modulation algorithm*. In a modulating oscillatory system analysis, the chord progressions of the piece represent (parts of) the upper nested series in the model, and our internal sense of key/tonal center (generated from listening to the chordal exchanges) are represented by the nesting fundamental series; all of which, combined with other sources of information, such as directional and visual inputs, is processed to recover the full musical experience.

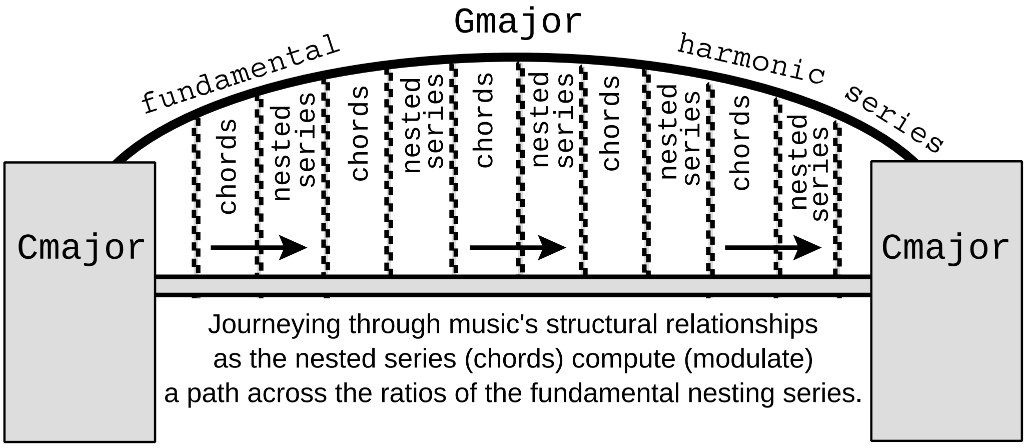

Figure 9.15 A musical performance is a journey through the structural relationships of a 'little world'.

For musical compositions, we travel from one end of the structure to the other, experiencing the relationships sequentially, with each performance a rebuilding of the relationships anew. However, if the model were applied to self-organising physical systems, the structure would be built once, with the modulation algorithm charting a course through a (perhaps complex) set of 'cadences', until eventually the system ran out of ratio/oscillators to compute. Thereafter, like the Sydney Harbour Bridge, the system would maintain a permanent and stable existence.

MODULATION, METER AND TIME SIGNATURES

Concepts of meter, pulse and time signatures, can be usefully applied to oscillatory systems containing nested series because they follow the same logic of organising vibrational/rhythmic patterns into groups and hierarchies, that is to say, whole number patterns of strong and weak beats. In particular, compound meters are able to encapsulate a separation of rhythmical levels, which match the separation of nesting and nested harmonic series.

In illustration Figure 9.8A above – the 'labored' method of generating six fluctuations per period – the metrical expression is shown as a measure of 6/8, with the six eighth-notes all joined together by the 'beam'. This is indicating that the meter is simple sextuple. That is, one strong beat followed by five weaker pulses. In performance this is very difficult to maintain. It's highly unstable. Almost any contour of melody or harmony would unbalance the music, tempting the players into adding secondary accents. (Like a pencil balanced upright on its end, any slight disturbance results in it falling to the more stable state of lying on its side.) The effect of adding one or more secondary accents, is to transform the 6/8 measure from simple sextuple into a compound form, with a secondary accent on the fourth eighth-note producing a 6/8 compound duple meter, or with accents on the third and fifth, a 3/4 compound triple meter – as shown in Figure 9.8B and 9.8C respectively. This is a practical example of the all-pervasive power of entropy increase, the second law of thermodynamics. Keeping to a simple sextuple meter takes a player a great deal effort or stress, relax for a moment and it's gone. More generally, in the durational domain this principle of relaxation – entropy increase – is felt in delightful hemiola effects, while in the domain of harmony it lurks behind the various cadences and many satisfying chord progressions – all brokered by the modulation algorithm.

When an oscillatory system alters its internal arrangement, by the process of modulation, the external effect is no more than a subtle change of accent within the interference pattern – which I have termed the system's *Meter*: for example, the slight relaxation from 6/8's _Yah-ta-ta_Tah-ta-ta to 3/4's _Yah-ta_Ta-ta_Ta-ta, in stepping from configurations B/B1 to C/C1 in Figures 9.8 and 9.9.

In a way a relational oscillatory system has two existences, one as a collection of separate parts and one as a whole unit. At the level of the whole system – the interference pattern – it only takes a small leakage of energy away from fourth fluctuation in the pattern of a 6/8 compound duple meter toward the third and (principally) fifth fluctuations, for the system to slip – that is modulate – into a 3/4 compound triple meter. However, this slight change of *accent* at the level of the whole system, a mere fluctuation, results in a *step change* in the arrangement of the internal parts, as h4 'evaporates' from the system, to be replaced by h3.

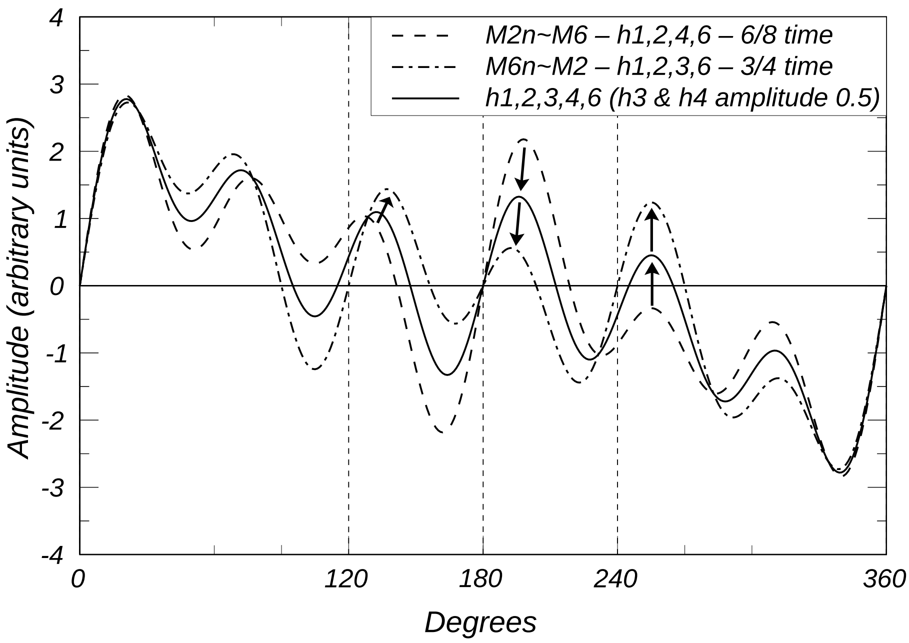

Figure 9.16 The full line marks the intermediate stage between a 6/8 and 3/4 meter interference pattern, with half of h4's amplitude/energy syphoned away into h3 and h5.

Such a step change within a system, a re-calibration of the internal arrangements, might be viewed as a form of *oscillatory computation*: a mechanism whereby an oscillatory system could, perhaps, compute its internal relational evolution – its structural development.

'M' numbers – Meter/Metre

Meter with a capital 'M', first introduced in Chapter 4, is essentially a concept borrowed from Leonhard Euler's theory of consonance and dissonance, and can be defined as: *the lowest common multiple of the constituent frequencies in a system, expressed in simple whole numbers*. Meter/Metre, with either American and English spelling, is written with a capital 'M' to distinguish it from the more general meaning of 'meter/ metre' and is most often abbreviated to a simple capital 'M' plus the LCM of the frequencies in the system, e.g. M6 `(h1+h2+h3)`, M2[f=3] `(h3+h6)`, M12 `(h1,2,3,4)` or M2520 `(h1,2,3,4,5,6,7,8,9)`.

Originally these 'M numbers' were developed as a useful and compact way of handling many levels of nesting in oscillatory systems, and expressing some other aspects of nested rhythmic/durational patterns; later came the realisation that this metrical description was equivalent to a variable base number system – mutable numbers. However, when thinking in terms of physical systems, as we are in this chapter, this 'metrical algebra' remains an appropriate method. So, for example, 4/4 time might be simple quadruple or compound duple meter, one cannot tell; but the former would have a Meter 12 `(h1,2,3,4)` ascribed to it, and the latter Meter2-nesting-Meter2 `(h1,2,4)` – M12 and M2n~M2 for short. Below is a chart of 'M syntax'.

M	Meter followed by the lowest common multple (LCM) of the constituent frequencies of the system or sub-system, expressed in relative terms. e.g. M12 (h1,2,3,4 - LCM=12)
n~	abbreviation for 'nesting' e.g. M6n~M6 (9/8 time)
->	arrow abreviation for 'modulating to' e.g. M2n~M6->M6n~M2 (6/8 -> 3/4 time)
f'n'	frequency of h1 of Meter e.g. M2[f1.5]n~M6 thus:h1[freq=1.5],h2[f=3]n~h1[f=3],h2[f=6],h3[f=9]
( )	indicate the division and/or grouping of nesting and nested portions of a system e.g. M6n~(M2n~M2)
[ ]	enclose any other information relating to that system or sub-system.

Figure 9.17 A table of symbols and examples of 'M number' usage in expressions of Meter/Metre.

The expressions are read in order from left to right, proceeding from the most fundamental level of nesting outward, through each succeeding layer of nesting, in turn; and are in all essentials mutable numbers in factor format. There is a notional, usually unwritten, M1 at the beginning of each expression. Although it might look like algebra at first sight, it is just a straightforward shorthand for nested patterns – thus the expression: M2n~M6 -> M6n~M2 translates to Figure 9.18

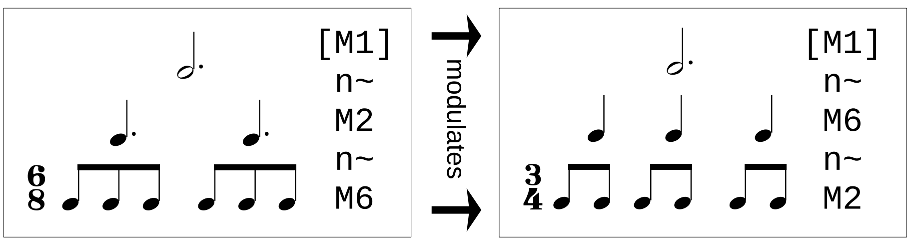

Figure 9.18 A diagrammatic/rhythmic representation of the transformation of nested patterns in the primary sesquialtera 2:3 modulation – M2n~M6 -> M6n~M2.

Overall the metrical patterns formed by modulating oscillatory systems, when acting strictly in accordance with the second law of thermodynamics, appear to produce a factorisation of the total number of fluctuations per period, arranged in order, so that the rhythmic grouping of largest magnitude occupy the (first) lowest frequency and so forth in ascending order, thus minimising the energy embodied in the system.

Nested Meters of Maximum Economy

Fluctuations_per_Metrical_Period

	Nested Meters.	Harmonics.........	Full Meters...	Factors
1	M1............	h1................	M1............	1......
2	M2	h1,2	M2	2
3	M6	h1,2,3	M6	3
4	M2n~M2	h1,2,4	M12	2×2
5	M60	h1,2,3,4,5	M60	5
6	M6n~M2	h1,2,3,6	M60(6)	3×2
7	M420	h1,2,3,4,5,6,7	M420	7
8	M2n~M2n~M2	h1,2,4,8	M840	2×2×2
9	M6n~M6	h1,2,3,6,9	M2520	3×3
10	M60n~M2	h1,2,3,4,5,10	M2520(10)	5×2
11	M27720	h1-11	M27720	11
12	M6n~M2n~M2	h1,2,3,6,12	M27720(12)	3×2×2
13	M360360	h1-13	M360360	13
14	M420n~M2	h1,2,3,4,5,6,7,14	M360360(14)	7×2
15	M60n~M6	h1,2,3,4,5,10,15	M360360(15)	5×3
16	M2n~M2n~M2n~M2	h1,2,4,8,16	M720720	2×2×2×2
17	M12252240	h1-17	M12252240	17
18	M6n~M6n~M2	h1,2,3,6,9,18	M12252240(18)	3×3×2
19	M232792560	h1-19	M232792560	19
20	M60n~M2n~M2	h1,2,3,4,5,10,20	M232792560(20)	5×2×2

Figure 9.19 Nested Meters in their most economical arrangement (left-hand column) with their constituent harmonics. The ‘full’ Meters echo the situation of box A in Figure 9.8 with all harmonics present, e.g. M12 (h1, 2, 3, 4) compared to M2n~M2 (h1, 2, 4).

Finally, for the last conjunction on the dashed line, at column h9 of the THS (Figure 9.2), the metrical expression for the whole system of harmonics – the D-seventh chord in the key of C major – could be written: M2520[f=1]n~M840.

{ { h1C, h2C, h3G, h4C, h5E, h6G, h7A#, h8C, h9D }, h18D, h27A, h36D, h45F#, h54A, h63C, h72D }

Spiral of Fifths

In Figure 9.20, the ‘start’ position represents the last conjunction on the dashed line in the THS (Figure 9.2), at h72. The system, nominally in the key of C major, has reached this point through a sequence of eight primary exchanges (involving sixteen ratios), which has taken the upper tonal structure (nested series) from a solitary h2C to the eight ratios of the D-seventh chord, the dominant-seventh of the key of G major:

h9D + h18D + h27A + h36D + h45F# + h54A + h63C + h72D – Meter840[f=9].

This D-seventh chord marks the point in harmonic terms, where the system might step across a tonal boundary, to a new set of coordinates based on the tonal center of G. By taking such a step, an oscillatory system is beginning to etch out the familiar structure of key relationships that we call the cycle or spiral of fifths: Except that here exclusively whole numbers are being employed which produces strictly, a spiral of twelfths.

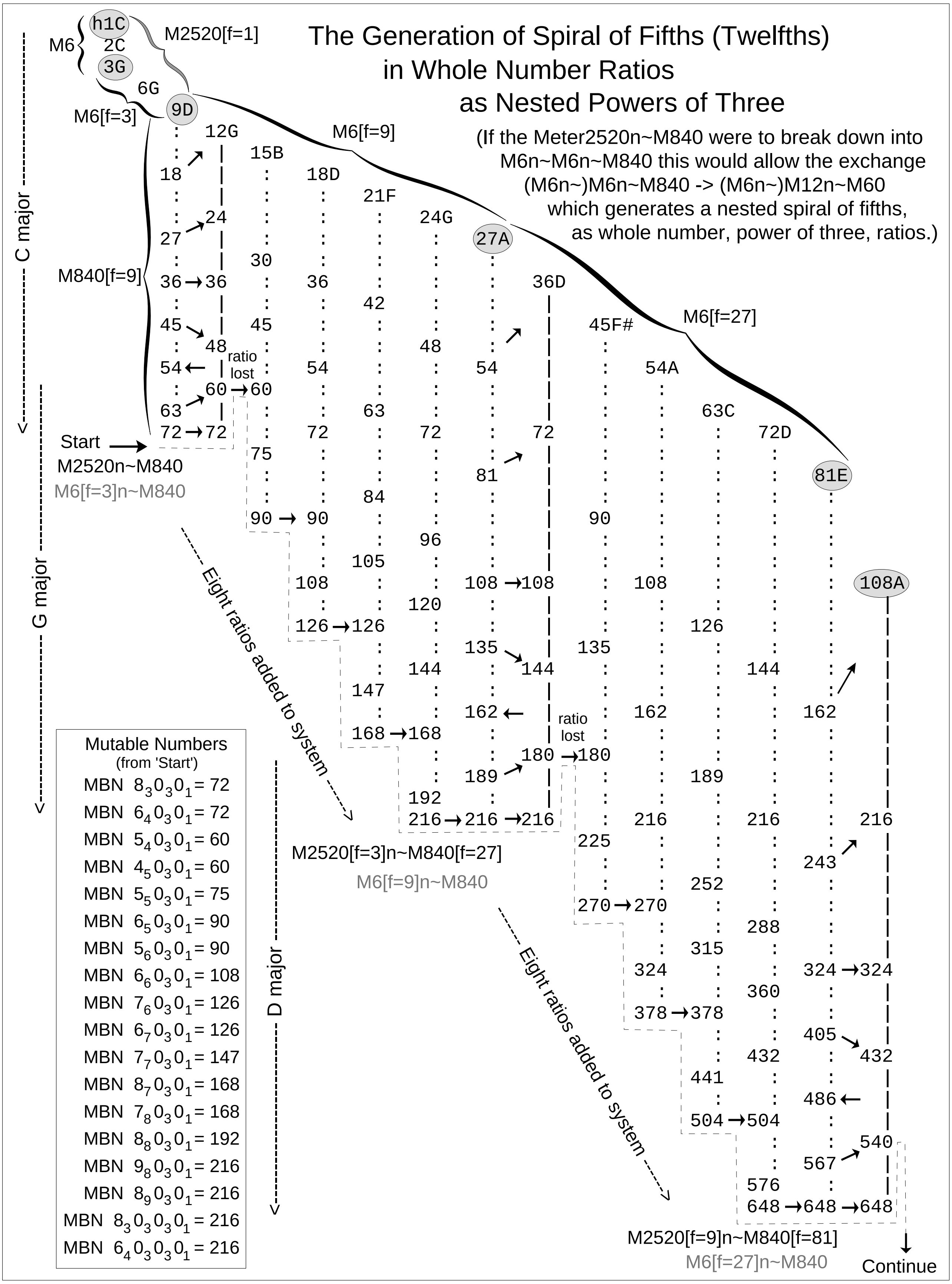

Figure 9.20 The breakdown into nested groups of three, forming key 'areas' and tonal boundaries. Inset box: the equivalent mutable base numbers for the G major tonal center, from 'start' h72 to h216.

In its most elementary form, the change of key from tonic to dominant, from C major to G major, is signalled by one or more conspicuous perfect cadences (D-seventh -> G-major) onto the new tonic chord. Through this progression, music forces a *jump or boundary* upon the model's formerly smooth evolution. By jumping from an upper nested series of eight ratios built on D-h9 to six ratios built on G-h12, the chord progression provokes the 'collapse' of the fundamental nesting series into two levels of nesting, from the relatively complex and unstable Meter2520[f=1] `(h1,2,3,4,5,6,7,8,9+h12)` to the more stable (M6n~)M12[f=3] `(h1,2,h3,6,9,12)`. This recalibration of base units into rhythmic groups of three, under the ever-watchful eye of the law of increasing entropy, is another manifestation of the principle first seen in the primary exchanges, but with a multiplier: 3n -> n ratios exchanged between fundamentals hn and h3n. Continuing the addition of ratios to the top of the nested series reproduces the key cycle jump every eight ratios, and in this way eventually the whole key cycle/spiral can be generated. Figure 9.20 illustrates three cycles of the process. To digress for a moment, interestingly, after the system has 'booted up', that is reached 'start' h72, all further ratio/oscillators in the system are divisible by three, and overall all these ratios take one of the forms: 3^n, $3^n \times 2^n$, $3^n \times 5^n$, $3^n \times 7^n$. This feature lends a four-dimensionality to the relationships. All ratios belong to the family of three. Yet while all ratios are united by their threeness, the occurrence of prime factors two, five and seven, defines a 'relational space' of three dimensions: that is to say no value of the form $3^n \times 2^n$ can equal $3^n \times 5^n$ or $3^n \times 7^n$, or vice versa. Though united by threeness, they maintain a separation.

THE INTEGRATION OF MAJOR AND MINOR

The aggregation of ratios into subgroupings is an important feature of the MOS model, and could be viewed in terms of an additional layer(s) of nesting. Also, the formation of aggregated units appears to play a central role in the integration of major and minor harmonies.

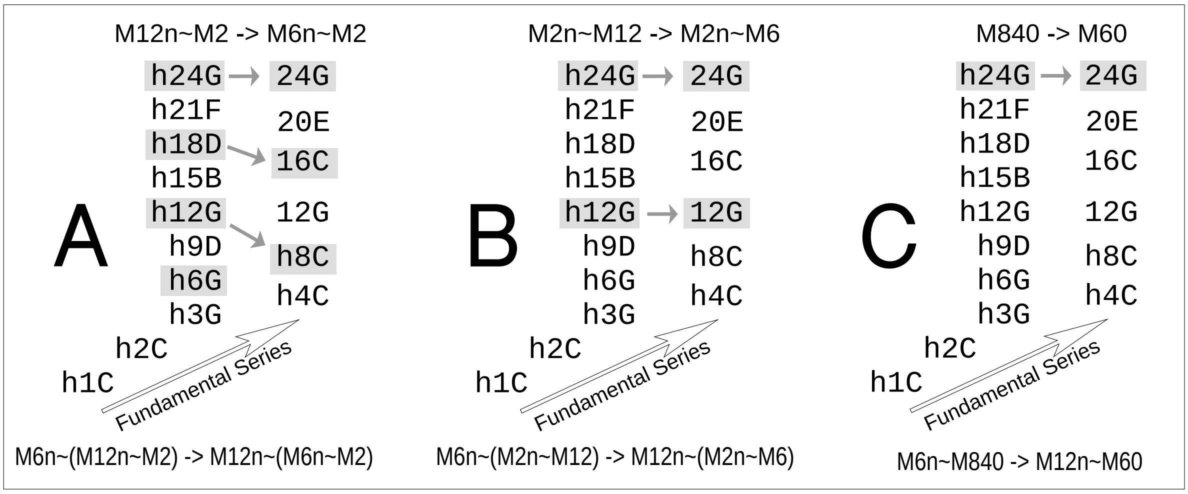

Figure 9.21 Three alternative groupings in the secondary sesquitertia 3:4 exchange's computation of the dominant-seventh to tonic chord progression. A: Four groups of two for three groups of two (4x2 -> 3x2), B: Two groups of four for two groups of three (2x4 -> 2x3) and C: One group of eight for one group of six (1x8 -> 1x6).

Already in Figures 9.20 and 9.21B, the secondary sequitertia modulation gives hints of aggregation within the upper level nested series, into subgroups of two (2×4 -> 2×3). However, this is a rather special case, as the first chord contains, potentially, an additional layer (2×2×2), providing an extra flexibility: the possibility of changing the number of aggregations from four to three (4×2 -> 3×2), or the actual units of aggregation from four to three (2×4 -> 2×3), in addition to the *full* exchange at the level of eight for six (1×8 -> 1×6). Here, it can be helpful to think in terms of the various different ways in which a Meter could be *parsed*.

The second law of thermodynamics implies that more complex Meters should break down into nested groups where possible. However, in Figure 9.21C, the full Meter 840 version of the exchange is prevented from breaking down into the aggregated forms of Figure 9.21A and 9.21B (M2n~M12 or M12n~M2) by the presence of the major-third B-h15, and particularly the seventh F-h21. The addition of the seventh to a common major chord forces the ear to encompass the relatively complex (and unstable) full Meter 840, which relaxes by modulation to a Meter 60 common major chord. (Though it would be possible for one or other of the configurations Figures 9.21A/B to be ‘contained’ within the more energetic right-hand arrangement, but not both simultaneously.)

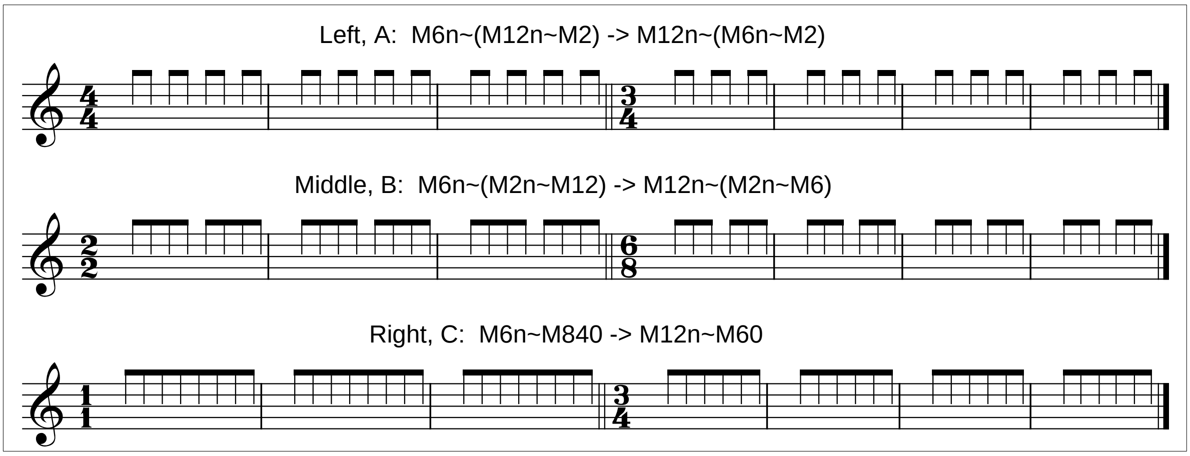

Figure 9.22 The modulation exchanges shown in Figure 9.21A/B/C, expressed rhythmically (Meters written above).

The Instability of the Minor Mode

A question seldom asked is: Why did western tonal music dispense, in the main, with the rich diversity of the medieval church modes? All but two of these modes or scales, each with its own individual character, progressively dropped out of use at the dawn of the tonal era. Only the Ionian and Aeolian modes lived on to become the major and minor scales of the music so familiar today. Also the Aeolian (minor) mode is not particularly stable; music in the minor key often slips back and forth between major and minor scales, and in particular, the dominant chord in the minor key (e.g. the chord of E in the key A minor) is very prone to altering its minor-third into a major-third, that is, mutating from an E-minor to an E-major chord. Why does this sound so right, so satisfying? In his insightful book *Harmony in Western Music*, R.F. Goldman[6] puts forward the view that there is only one ‘mixed scale’ or mode and quotes Busoni: “Strange, that one should feel major and minor as opposites ... a mere touch of the brush suffices to turn one into the other. The passage from either to the other is easy and imperceptible; when it occurs frequently and swiftly, the two begin to shimmer and coalesce indistinguishably.”

The underlying cause of this instability, I suspect, lies in the minor chord – the notes EGB in this example – being h10, h12 and h15 of the Meter 60 major chord/series – i.e. CCGCEG+EGB+EBB – h1-6, h10, h12, h15, h20, h30, h60. Indeed, this instability appears to hint at a general relationship between the minor and major chords. Perhaps our ears and processes of aural cognition, with the aid of partials and difference tones, are to a greater or lesser degree able to construct the broader context of an underlying harmonic series from the 'objective' E-minor chord, and so in effect, *compute* the E-minor chord as an *extended* C-major chord/series. (This would apply to all minor-major pairs of chord/series.) Remarkably, the first difference combination tone also very neatly makes an octave transposition, delivering the same fundamental for the chord of E-minor as it would for a chord of C-major!

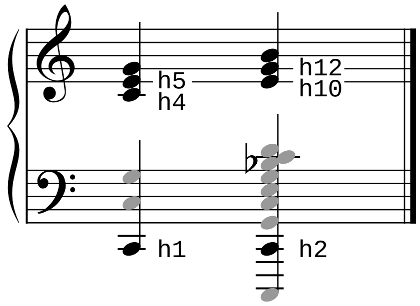

Figure 9.23 The first difference combination tone of the C-major chord: h5E – h4C = h1C (64Hz); while for the E-minor chord: h12G – h10E = h2C (64Hz).

Aggregation

But how can this be when the root note of the E-minor chord is clearly heard as E and not C? An answer can be found in the bundling-up or *aggregation* of fluctuations per period into groups of five, which provides the root note E, derived from h5 (the outer edge of the first 'fundamental' bundle – h1 through h5) in an underlying C-major harmonic series. The crucial note/interval is the minor-third, G in the E-minor chord, which, similar to role of the seventh F-h21 in Figure 9.21C, forces the ear to entertain an extended harmonic series up to and beyond G-h12. However, the period of such a complex extended harmonic series is prone to break down into subdivisions or aggregations, as decreed by the second law of thermodynamics. In this case, the more efficient grouping of five aggregations in the nested series, is interwoven into the fabric of the underlying C-major series (Figures 9.24 and 9.25).

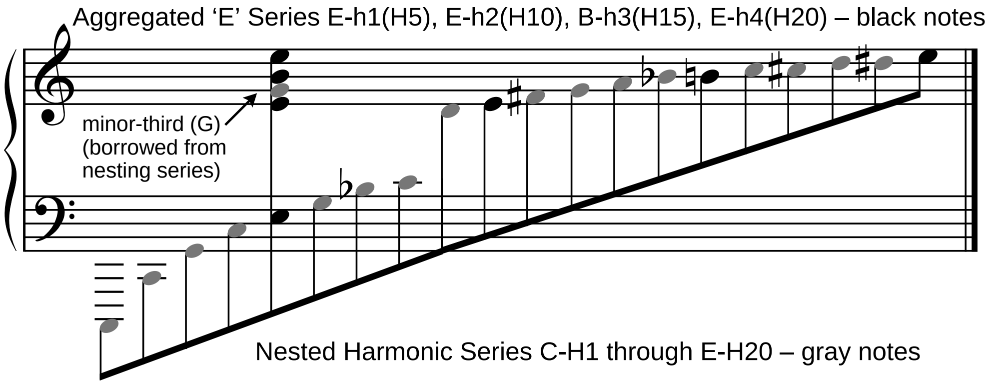

Figure 9.24 An extended series built on C-H1 (gray notes) provides the background harmonic context capable of encompassing all the ratios (H10, H12, H15) of an objective E-minor chord.

The MOS model's view of the overall effect of the ear and aural cognition's struggle to understand and unravel the complexity of the objective E-minor chord – with the apprehended frequency ratios of a minor triad of 10:12:15(:20), implying a fundamental tone C-h1, difference tones of C-h2 and G-h3 and an assorted cluster of other frequencies of timbre – leads to a *dual perception* of a foreground *E-based series*, set within the wider background of an implied *C-based harmonic series*. The rationalisation or reduction of complexity inherent in the face value relationships of the minor triad (i.e. h1 through h15 or h20), into an aggregated arrangement of foreground ratios of 1:2:3(:4) (i.e. E-h5, E-h10, B-h15, E-h20) resting on the foundations of a background series of h1:2:3:4:5 (C-h1 through E-h5), leads to a considerable release of stress without compromising the system's identity – its mutable number value. (A fuller account of the integration of the minor mode within an underlying 'major' tonal center is given in Chapter 11.)

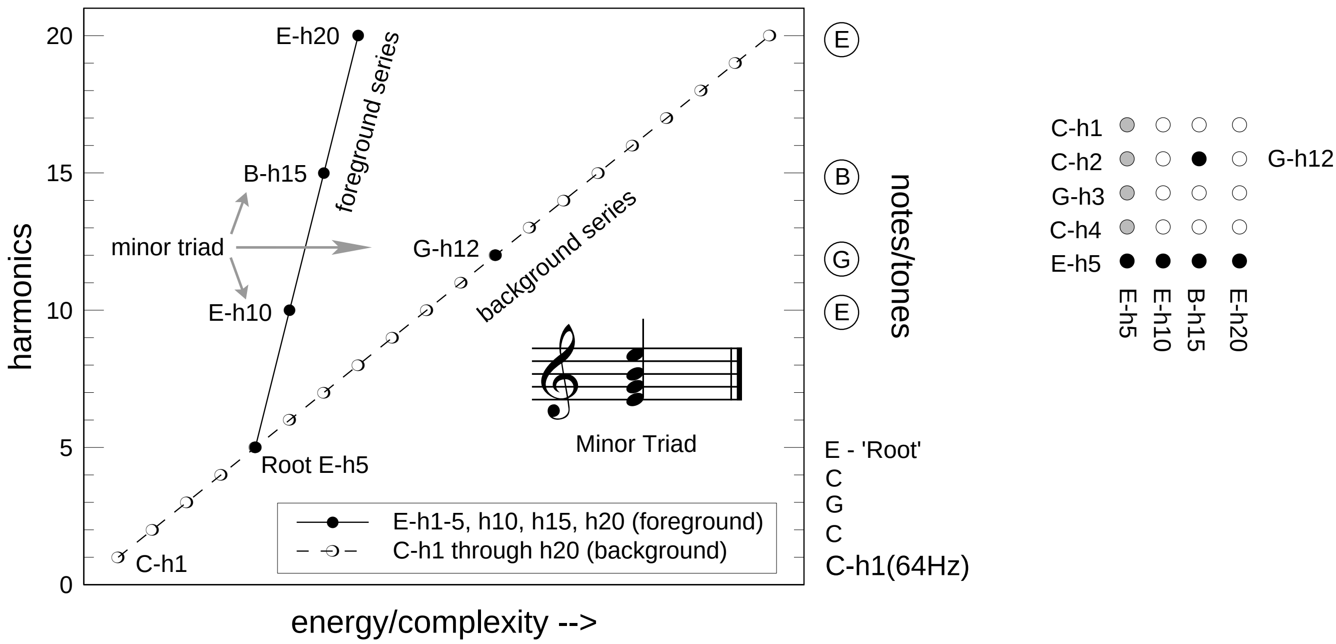

Figure 9.25 A graph of the E-minor chord as foreground and background series. The nested combination of two series reach h15 (and h20) with much greater efficiency: 7 and 8 ratios against 15 and 20 for the single series configuration.

Expressed in the form of mutable base numbers, the difference between the single series background configuration for a minor triad and that of the more efficient, nested, foreground and background arrangement, is that of the prime state and ground state digit sequences of the number fifteen.

MBN Fifteen: prime state 1x15 --> ground state 1x5x3

MBN: 15_1 --> $3_5\,0_1$

Generalising this process of aggregation within nested series, produces a superimposing of the minor key upon the relationships of an underlying major key – the subdominant key of the relative major (i.e. the key of A minor superimposed on F major). Effectively, the minor principle is accommodated in the MOS model by means of nesting the basic MOS concept of a *nested/nesting* pair of intertwined harmonic series, within itself. The outcome of this recursive procedure, when applied to non-trivial tonal compositions, is three levels of structure: an aggregated series, a nested series and a fundamental nesting series. At the top of the system, in the foreground, an aggregated series mirrors the objective chords in a piece of music. Towards the middle of the structure lies the more shadowy nested series, forming a contextual background to the explicit objective relationships above. At the bottom, a notional fundamental nesting series links the dynamic motion (modulation exchanges) of the aggregated/nested pair of harmonic series above, to an absolute, fundamental tone – the embodiment of the tonal center or key.

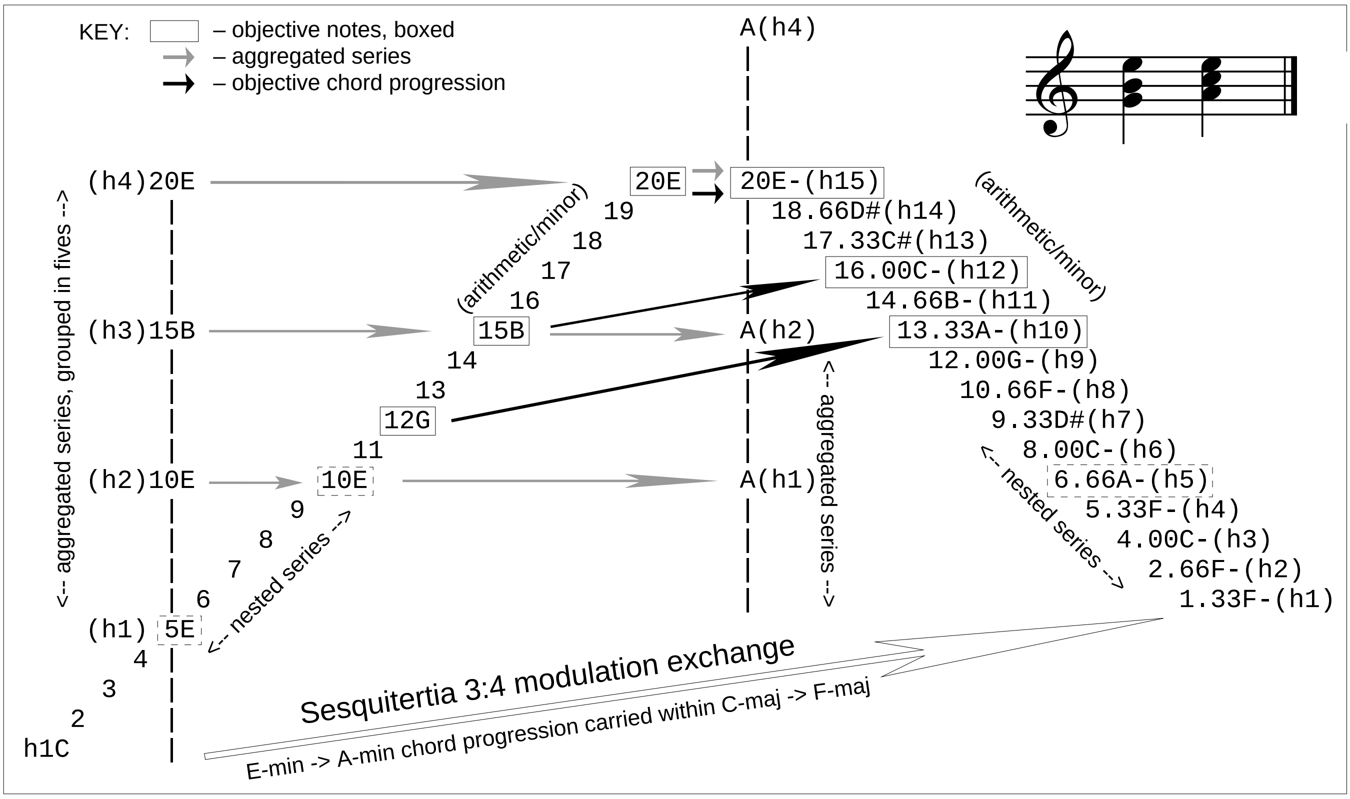

Figure 9.26 The minor chord progression E-minor -> A-minor, carried within the extended harmonic series of C and F (major) and supported by aggregated series grouped in fives.

Figure 9.26 illustrates the effect of aggregation in a minor-minor chord progression. The clue or hint to this relationship is, the instability of the E-minor chord in the key of A minor, that we feel and respond to, could be caused by the aural processing of minor chords as surrogate major chords; and through this superposition of the minor configuration onto the major, we are able to extend the modulation computations of the *normal* 'major' harmonic series (i.e. the low order ratios h1-8), into a parallel mode – the minor key. However, this trick can only be done once and not with complete stability. More layers of aggregation might well be too unstable to be workable at all. Thus only *two modes* are feasible in tonally (harmonically) organised music, the major and minor keys – rather than twelve 'melodic' church modes of the pre-tonal era. And the second 'parallel' minor mode is apt to breakdown to its underlying relationships, as in the dominant minor chord altering its third, the delight we take in this transformation, the sweet modulation of a major third, being the *computational dividend* derived from the resolution of the sesquiquarta 4:5 modulation – Figure 9.27.

As an individual example of computational dividend: the relaxation of the *meta-stable* E-minor chord (superimposed upon a C-major chord/series) to the more stable configuration of E-major, can be charted as a tertiary sesquiquarta 4:5 modulation exchange between columns four and five of the THS (Figure 9.2); where fifteen (5×3) ratios are exchanged for twelve (4×3) at the conjunction 1:60.

C C G C E G A# C D + E (F#) G (A A#) B ----> E E B E G# B D + E (F#) G# (A) B

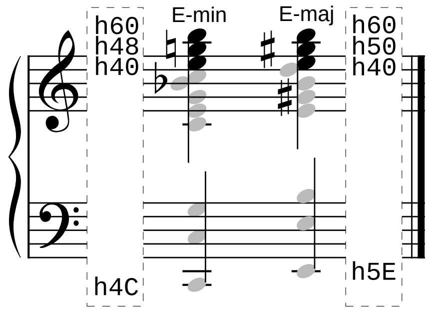

Figure 9.27 In principle, a dividend of energy and information is released by the (tertiary) sesquiquarta 4:5 modulation, through the exchange of fifteen ratios based on h4 for twelve ratios based of h5. (Not all ratios are shown.)

Interestingly, another example of the altered third, the *tierce de picardie*, a telling effect where the final chord of a composition in a minor key turns major originated about 1500 AD, as did the neapolitan-sixth mentioned by Arnold Schlick in *Spiegel der Orgelmacher und Organisten*[7] 1511, both of which coincide pretty well with the dawn of (fully harmonic) tonal organisation in western music. It is perhaps the flexibility afforded by multiple, interwoven, layers of nesting: *fundamental nesting* series, *nested* series and *aggregated* series, which accounts for the high level of mutability within and between chords, and rich vocabulary of progressions, which so characterise the period of common harmonic practice. In reality, as noted above, the major and minor modes are not really separate entities but rather differing inflection of the one tonal center (and its near relations); and the shifting exchanges between major and minor chords (in both major and minor keys) help to support and maintain these multiple layers of nesting, in that while the major chords could be reduced to a single series, minor chords cannot.

Units of Aggregation Other Than Five

Are other units of aggregation possible? Indeed it appears that they are, but perhaps within limits. Aggregating fluctuations per period in groups of two (powers of two) would simply produce octave shifts, which aids flexibility but leads to no meaningful extension of musical experience. However, the prime vibrational groupings of three and seven both look promising, with aggregations of three appearing to have a relationship with the chord of the added sixth and aggregations of seven the neapolitan-sixth chord. (These chords are discussed further in Chapter 10.)

A possible limit, appears to be reached if units of aggregation are taken further, to groups of nine. Here again, like aggregating powers of two, no gain is made (save that of flexibility) as the groups of nine would simply break down into another layer of nesting, as three groups of three (powers of three), though the shift, in this case, would be of a twelfth – effectively producing the spiral of fifths as in Figure 9.20. Although there would appear to be no limit, theoretically, to the number of layers of aggregation or prime number units of aggregation mathematically possible, at some point practical limits would emerge regarding what degree of interwoven detail can be perceived and interpreted by the ear. Though of course, the limits of aural cognitive processes, wherever they may be, would not apply to the scope of mutable base numbers which in the guise of an abstract formal system, is unlimited. And I suspect that it is in the nature of cognitive processes to reach out and sift through, many layers and levels of detail, perhaps unconsciously percieved, in an attempt to make links and conjunctions across the widest possible spans of aural stimuli.

CONCLUSIONS

Though focused on traditional tonally organised music, the model of modulating oscillatory systems might have wider connections to other oscillatory/periodic system via *the unifying perspective of information and computation*. Through the language and techniques of information and computation it is possible to contemplate stepping back from the details of individual systems, to higher levels of abstraction, facilitating a wider inclusive view of music, as one member of a perhaps broad set of oscillatory/periodic phenomena.

The development of this essentially computational approach to tonal music has been influenced by the thoughts and insights of many scholars, from a broad range disciplines, and is most certainly a work in progress, rather than a polished finished product. Therefore no doubt at this early stage in the development of the model there will be many omissions, weaknesses and flaws, which, in the light of further thought and the most welcome counsel and advice from colleagues and friends, will require amendment, together with considerable elaboration and augmentation. It is a model that can only ever loosely describe pieces of music in their entirety (though hopefully capturing their structural essence) as there are so many arbitrary and disparate elements in such complex, externally driven, little worlds. However, when the principles of the model are applied rigorously, as if in a truly self-organising system, the outcome appears to take on the character of *digit sequences* in a *position value counting structure*, i.e. mutable base numbers. Likewise, while not a rigorous example, tonal music should perhaps in principle, at its most ultimate level, be viewed as a form of *positional number system* generated and governed by the *algorithm of modulation*: Though we relish its theorems more for the intrinsic qualities of their computations, than the resultant proofs themselves.

One interesting understanding to emerge from the MOS model is that for each and every tonally organised piece of music, there is another piece of music sharing the same set of computations (modulations) arranged as a sequence of reciprocal series. This arises from the model's view of *the minor superposed upon the major* encompassing the alternate mode in each computation. The effect of this transformation is not unlike that of turning a coat inside out: the end result may be rather odd but it still does work as a coat! By expressing the harmonic information, the ratios between note frequencies, as relationships of wavelength, the effect is that of turning the music inside out. For the most part these *inverse reflections* don't make a great deal of sense to ears attuned to the familiar progressions of traditional music, though some do have a strange charm, and all of them produce ordered sequences of chords driven by the inverted logic of the original. Given the model's apparent ability to integrate these *two dimensions* of tonal music, the major and the minor, into a single stream of computation, a revisiting of the dualistic tradition (from G. Zarlino to H. Riemann) might, perhaps, prove fruitful. (The topic of harmonic reflection is treated at length in Chapter 14.)

[7/10/09]

Notes

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4. Morris R.O., *Contrapuntal Technique*, (OUP, 1922) page 27; Apel W, *Harvard Dictionary of Music*, (Harvard University Press, Cambridge, Massachusetts, 1966) page 439.

5. Sethares, W.A., *Tuning, Timbre, Spectrum, Scale*, (Springer, London, 2005) Chapter 2 – What Is a Spectrum, Fig. 2.5.

6. Goldman, R., *Harmony in Western Music*, (W.W. Norton & Co. Inc, New York, 1965) page 82.

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