9

Identity and Change

MODULATING OSCILLATORY SYSTEMS

The topic of this chapter counterbalances the formal approach pursued in Chapter 1, where harmony and meter, the core structural features of tonal music, are portrayed in terms of positional mutable-base numbers. Here again, an understanding of the role of harmony and meter in western music is the central goal, but in this chapter a dynamical perspective is taken upon the subject, casting tonal music in the role of a physical system viewed in quasi-kinematic terms. These two approaches, formal mutable numbers (abr. MBN) and physical modulating oscillatory systems (abr. MOS) are complementary. Indeed, the 'metrical algebra' of modulation exchanges presented below is entirely equivalent to the computations of formal mutable numbers, given in Chapter 1 and elsewhere. And further, although mutable numbers and their application to tonal music has featured in many chapters up to this point, it was the concept of musical compositions considered as pseudo-physical dynamical systems that occupied my attentions in the early days of the MOS model's development. It was only later that I came upon the formal mathematical interpretation of mutable numbers; notwithstanding this lineage, the consideration of western music in dynamical terms forms a long established and vibrant thread in music theory, an approach which has on occasions been characterised as *Energetics*.

Perhaps the two most fundamental categories which can be applied to the material world are those of *identity* and *events*: what 'things' are and how 'things' change.[1] In this chapter the outline of a model of pseudo-physical musical 'worlds' (tonal compositions, in principle), constructed in terms of these two primal categories, is viewed from the perspective of the interaction of nested harmonic/arithmetic series. The model focuses on the most basic relationships – ratios of (positive) natural numbers – and how these simplest of entities (expressed as regular periodic behaviour, oscillation) might interact to develop structure, process information and create variety. The model describes a physical process of *oscillatory computation*, notionally operating under the aegis of the second law of thermodynamics and governed by a simple, yet subtle, process: the algorithm of symmetrical exchange – the *modulation* algorithm. 'Identity' in modulating oscillatory systems derives from arrangements of harmonic or arithmetic series, while 'events' are changes in these configurations brought about through the mechanism of this algorithm. (For the sake of completeness, arithmetic series have been included above, as the MOS model works equally with either; however, as our topic is music, the discussion below is framed for the most part in terms of harmonic series.)

At the heart of the scheme lies a general relational model: a model that treats tonal compositions, in principle, as being *little worlds* of relationships – autonomous musical universes. Essentially, the model consists of relationships and a process by which the relationships can be changed. The relationships are arranged as two or more entangled harmonic (or arithmetic) series, one a fundamental *nesting series*, within which the other one or more *nested series* reside. The mechanism whereby the constituent series interact – *modulation* (used in the broader sense of change in relationships) – denotes a process of exchange or transformation, within and between harmonic or arithmetic series. In computational terms, the nesting and nested series are data, while the algorithm or rule which acts upon them is *modulation*.

These *little worlds* of relationships – pieces of tonal music viewed as ideal self-organising dynamical systems – do require some level of internal structure. A separation of internal parts is necessary because there is no absolute or fixed background against which a wholly relational system can *compute* its evolution. The system can only *see* its internal relationships and cannot grasp at handles outside itself. Thus it requires at least two related sets of coordinates within the system to negotiate change, allowing for each to anchor the other, while manoeuvring between old and new reference points. These two sets of relationships are the *nesting* and *nested* harmonic or arithmetic series, the manoeuvring is *modulation*.

Before ploughing into the detail it might be helpful to briefly lift our gaze to the broad sweep of landscape. Toy worlds, models and thought experiments are familiar tools, where idealised schemes, often reduced to their barest essentials, are devised in order to gain some understanding of complex real world systems and processes. In regarding a piece of music (or any other oscillatory system) as a self-contained, self-consistent world of relationships, the particular little world under consideration is being viewed, in principle, as a little universe or whole world: its internal relationships are *all that is*, viewed from inside the system. Though, at some different scale, this little world might appear as no more than a tiny unit in some other broader scheme: and of course in the case of music, the ear, aural cognition and the human mind provide a wider territorial context, within which these notionally self-organising musical systems operate.

When viewing music from a dynamical standpoint, as a pseudo or quasi-physical scheme, to begin with the question arises: what, at bottom, is this system? Or even, what is its elemental unit, what is a note? If you take a note out of a piece of music, what have you got? A frequency (plus harmonics of timbre), an amplitude and a duration – say half a second of sound at 440Hz – the note ‘A’ above middle C. Once the note has been removed from the context of the piece as a whole – its own little world – there is no way of knowing much about it. Was it the highest pitched note in a melodic phrase? Was it loud in relation to the other notes in the piece? Was its duration long or short compared to other simultaneously sounding notes? The note has lost its ‘identity’, its meaning. What this poor little note is (or rather was), depended on all the other notes in the composition: the network of close and distant relationships shared amongst the whole.

It is the relative relationships within the piece that carry the musical information. After all, transposing a piece of music to a different pitch and/or tempo, within reasonable limits, doesn’t alter its essential qualities. On average, most of the music of the baroque and classical eras is performed today at a higher pitch and different tempi than in their own time, which rather makes the point; the ‘music’ is the internal relationships between the notes and chords, not any absolute collection of frequencies and durations. The conclusion to be drawn is that what a note ‘is’ – its identity or meaning in a piece of music – is the sum of its relationships with the other notes and chords within the piece (and vice versa). And, by extension that a piece of music, as a whole, is a scheme of changes to note relationships, a structure of musical events, experienced sequentially in performance. Thus, a tonal composition may be viewed as a little universe of musical events, a scheme of connections and relationships unfolding in its own time; and, in principle, a scheme somewhat analogous to the material universe with physical characteristics similarly amenable to rational analysis.

However, not just any arbitrary set of relationships between notes produces a satisfactory musical experience, only relationships that are governed, in general, by a particular algorithm or rule, have proved to be acceptable to the vast majority of participants in the western tradition. This particular process of computing whole number relationships, for the most part applied intuitively, has produced the music of the common practice; and although it is traditionally characterised in terms of standard voice leading, chord progression and key change, the basic mechanism, as demonstrated by the dynamical approach pursued below, permeates many other areas of tonally organised music. In particular, the modulation algorithm of symmetrical exchange is shown to operate between individual notes and chords, and thereby govern western music's central structural feature, harmonic progression.

TABLE OF HARMONIC SERIES (abr. THS)

As previously mentioned, some time ago while attempting to devise a music data format based entirely on harmonic series, I constructed a table or matrix of series to help me think out how it might be done. The seed from which this idea grew, came from studying the output of cellular automata.[2] This table of nested harmonic series (Figure 9.2), which is effectively the *Sieve of Eratosthenes* – the ancient method for deriving prime numbers – contained intriguing patterns of *arrowheads* and *conjunctions* between the various ratios.

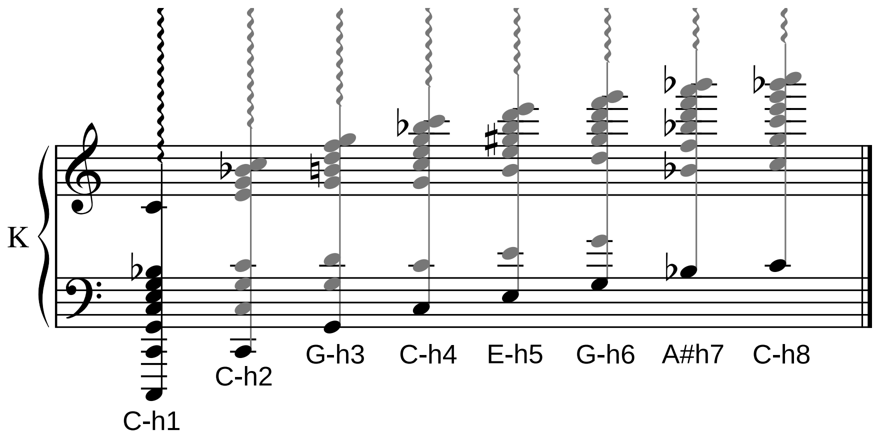

Figure 9.1 Moving from chords to harmonic series: The first eight ratios of the harmonic series, labelled C-h1 through C-h8 and illustrated with black-headed notes on the bass staff. Above this 'arpeggio' of the *chord of the harmonic series*, in gray-headed notes, are the harmonic partials of each note of the series. (The wiggly extension of the note stems indicating further harmonics to hn.)

The table, Figure 9.2, could be thought of as a visual plot of *all the harmonics* arising from the notes in a series, that is the harmonic partials of the roots (black notes) in Figure 9.1. This *chord of the harmonic series*, is in a sense fundamental, in that no harmonics will be found in the series based on the individual black notes that are not also present in the original chord's extrapolation – the fundamental series. (In practice there might well be slight differences, e.g. no material object is perfectly elastic, but for this discussion we can enjoy the luxury of an ideal system.)

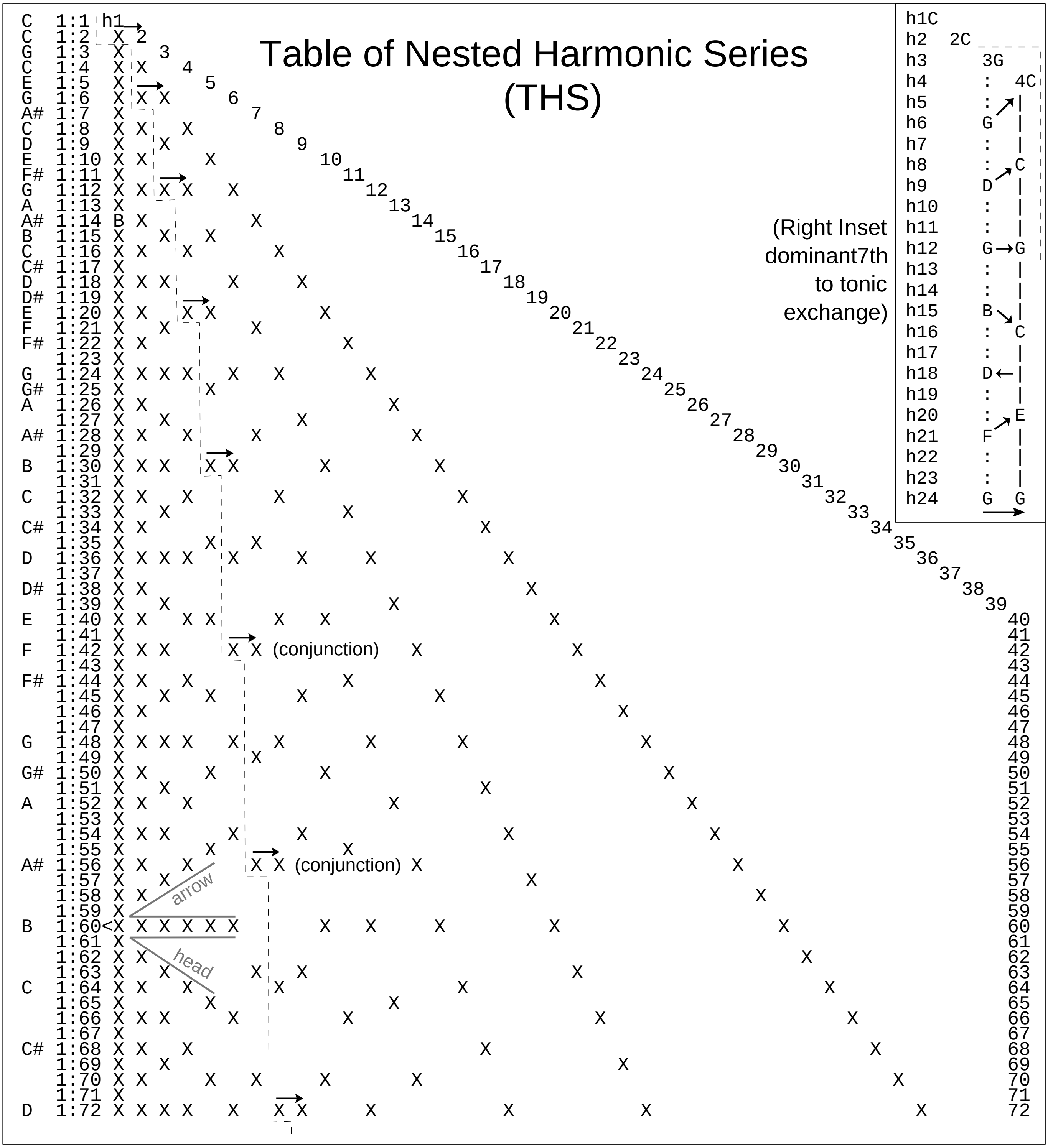

Figure 9.2 A fundamental nesting harmonic series (the 45 degree column numbers) drawn with a matrix of all possible nested harmonic series[3] up to h36 – the actual columns of Xs – with the primary conjunctions connected by the dashed line and mark by small black arrows. (The further extension of the dashed line is charted in Figure 9.14, where the black arrow chord progressions provoke an upward cycle of key shifts.) The arrowhead at h60 is outlined in gray. Right inset: a secondary sesquitertia 3:4 modulation exchange – a perfect cadence – discussed below, with the primary sesquitertia 3:4 modulation exchange enclosed by the broken line box.

Looking at the conjunctions, starting from the top left corner, it can be seen that moving across the columns from left to right and through the rows from top to bottom, following the dashed line, produces exchanges of two ratios for one, three ratios for two, four ratios for three, etc. These exchanges form the mobile structure that caught my eye when first investigating matrices of harmonic series as a possible music data format. From a musical point of view, the exchanges follow the pattern of an octave for a unison (C+C –> C), a twelfth for an octave (C+C+G –> G+G), a fifteenth for a twelfth (G+G+D+G –> C+C+G), etc. However, if the fundamental series (the column numbers) are retained in these exchanges, it becomes apparent that ratios are not being lost by this procedure, but transformed by a process which passes them down from the upper nested harmonic series to the lower fundamental nesting series. The nested series is moving in steps which trace out the ratios of nesting series one by one.

Modulation

This exchange process might be described as a sequence of computations – addition followed by transformation. Below, with the entangled upper nested harmonic series and the lower fundamental nesting series enclosed in curly brackets, i.e. {{nesting series}, nested series}, the computations could be written:

```
                        {h1C} + {h2C} -> {h1C,h2C}
             {{h1C,h2C}, +h4, +h6} -> {{h1C,h2C,h3G}, h6G}
{{h1C,h2C,h3G}, h6G, +h9D, +h12G} -> {{h1C,h2,h3G,h4C}, h8C, h12G}
```

In this last exchange above, one can see the outline of what is in musical terms a full or perfect cadence in the key of C-major – the chord progression G-major –> C-major (broken line box in Figure 9.2). Or we could say chord V resolves to chord I in the musician’s vocabulary, or ‘set G’ is mapped to ‘set C’ in mathematical terminology; and, if the upper nested section where to be extended to h24G we could obtain the more urgent sound of the dominant-seventh chord resolving to the tonic (below and Figure 9.2 inset).

```
{{h1C,h2C,h3G}, h6G, h9D, h12G, h15B, h18D, h21F, h24G} --->
                    ---> {{h1C,h2C,h3G,h4C}, h8C, h12G, h16C, h20E, h24G}
```

The ‘primary’ full cadence (on the dashed line) moves between the fundamentals h3G and h4C, by exchanging four ratios for three ratios while this latter and rather more elaborate ‘secondary’ full cadence exchanges eight ratios for six ratios, again on the same foundations. Continuing this upward extension finds unlimited numbers of conjunctions yielding ever more complex ‘tertiary’, ‘quaternary’, etc. exchanges in the THS. (Beyond a quaternary exchange the term *‘n’-fold* exchange is introduced, e.g. *five-fold* exchange.)

Computing Internal Arrangements

However, returning to the primary exchanges or modulations (following the dashed line in the THS, Figure 9.2) it is apparent that for as long as ratios are added to the top of the nested harmonic series, this sequence would compute the set of positive whole number ratios: h1C, h2C, h3G, h4C, h5E... hn – the fundamental series. Thus the upper nested harmonic series is extending the lower nesting harmonic series of the system by one ratio per modulation/transformation, with each step in the sequence achieved by an exchange of n –> (n – 1) ratios of the *nested* harmonic series based on the *nesting* fundamental frequencies h(n – 1) and hn. These primary exchanges could be viewed as the *primitive operations* of the model’s modus operandi. In Figure 9.3 the first eight primary modulations are illustrated in musical format (with each one adjusted so as to begin on a C-major chord).

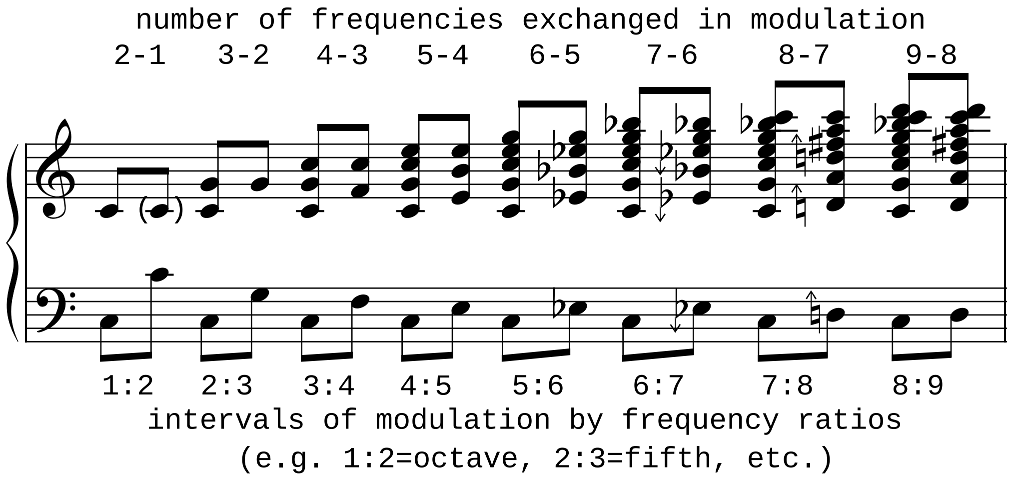

Figure 9.3 Primary modulations expressed in musical (harmonic) terms. Symmetrically, the interval of modulation (in the bass clef) is mirrored by the reverse number of notes exchanged, i.e. two notes exchanged for one produces an octave modulation, three notes for two a fifth, etc. The modulations 6:7 and 7:8 fall outside the fractions of the just intonation musical scale and the semitone modulation 15:16 is not reached in this illustration. While tempered scales have no place in a system based on 'natural' whole numbers, the modulation 17:18 gives a close approximation to the equal-tempered semitone i.e. 18/17 = 1.0588 against 1.0594 for the tempered interval (as noted by Vincenzo Galilei).

And below, in Figure 9.4, the modulation exchanges illustrated in Figures 9.3 are presented with their Latin names, proportions, harmonic ratios and constituent notes..

Name	Proportion	Ratios/Harmonics	Notes/Chords Exchanged
Dupla	1:2 exchange	h1, 2 --> h2.	2 notes C+C --> 1 note C
Sesquialtera	2:3 exchange	h2, 4, 6 --> h3, 6.	3 notes C+C+G --> 2 notes G+G
Sesquitertia	3:4 exchange	h3, 6, 9, 12 --> h4, 8, 12.	4 notes G+G+D+G --> 3 notes C+C+G
Sesquiquarta	4:5 exchange	h4, 8, 12, 16, 20 --> h5, 10, 15, 20.	5 notes C+C+G+C+E --> 4 notes E+E+B+E
Sesquiquinta	5:6 exchange	h5, 10, 15, 20, 25, 30 --> h6, 12, 18, 24, 30.	6 notes E+E+B+E+G#+B --> 5 notes G+G+D+G+B
Sesquisexta	6:7 exchange	h6, 12, 18, 24, 30, 36, 42 --> h7, 14, 21, 28, 35, 42.	7 notes G+G+D+G+B+D+F --> 6 notes A#+A#+F+A#+D+F
Sesquiseptima	7:8 exchange	h7, 14, 21, 28, 35, 42, 49, 56 --> h8, 16, 24, 32, 40, 48, 56.	8 notes A#+A#+F+A#+D+F+G#+A# --> 7 notes C+C+G+C+E+G+A#
Sesquioctava	8:9 exchange	h8, 16, 24, 32, 40, 48, 56, 64, 72 --> h9, 18, 27, 36, 45, 54, 63, 72.	9 notes C+C+G+C+E+G+A#+C+D --> 8 notes D+D+A+D+F#+A+C+D
Secondary Sesquitertia	3:4 exchange	h3, 6, 9, 12, 15, 18, 21, 24 --> h4, 8, 12, 16, 20, 24.	8 notes G+G+D+G+B+D+F+G --> 6 notes C+C+G+C+E+G

Figure 9.4 Table of the first eight primary modulation exchanges,
plus the secondary sesquitertia 3:4 (V^7- I) exchange.

In principle, such *modulation* exchanges might be interpreted as a process whereby a piece of music (or any oscillatory system) changes from one arrangement of internal relationships (constituent frequencies) to another. In tonal music, the rearrangements appear to operate at a number of levels: the levels of key/tonal center, chord/harmonic progressions, rhythm/meter, timbre/tone color. Most often the reconfiguration will be from a more complicated way of generating a given number of fluctuations per period to a less complicated internal arrangement which achieves the same goal with greater economy – as the Law of Increasing Entropy demands. A musical phrase typically launches from rest with a rapid increase in complexity, followed by a gentle release of tension and information as it drifts back down to a closing cadence. However, under particular conditions an oscillatory system might move in the opposite direction, absorbing energy from its environment and moving to a more complex internal configuration – reversing the small black arrows in the THS, Figure 9.2. Such reversal is not forbidden by the thermodynamic laws and through this feature the MOS model is able to encapsulate the ever changing eddies and flow of stress that underlies the topography of musical phrases. Modulation is a two-way thoroughfare. In general terms the modulation algorithm can be expressed in the form:

$$n_{H(\frac{m}{a})} \longleftrightarrow m_{H(\frac{n}{a})}$$

e.g. secondary sesquitertia modulation exchange
n = 8, m = 6, a = 2
dominant-seventh to tonic progression

for series of ‘n’ and ‘m’ harmonics, with integer values of ‘a’ representing primary, secondary, tertiary, etc. exchanges and ‘H()’ signifying the relative fundamental frequency of the respective series.

Historical Perspectives

Remarkably, medieval and renaissance scholars and composers from Franco of Cologne in the 1250s onward,[4] gradually worked out these modulation exchanges, in the form, principally, of rhythmic proportions in the *mensural system of notation*,[5] providing a thoroughly systematic scheme of classification. For example, proportio dupla 2/1, sesquialtera 3/2, sesquitertia 4/3, sesquiquarta 5/4, etc., using the Latin prefix to denote a numerator larger by one than its denominator – which is exactly what is needed to classify the conjunctions and exchanges in the THS. So, adopting their scheme, the primary exchange of four ratios based on h3 for three ratios based on h4 could be termed a *primary sesquitertia 3:4 modulation* and the fancy dominant-seventh exchange of eight ratios for six, also on h3 and h4, would be a *secondary sesquitertia 3:4 modulation* – the Figure 9.2 inset.

Like the first pedestrians to use the Millennium footbridge, once musicians strode forth in harmonic steps they became unknowing players in a metrical game, with trial and error, plus an evolutionary process, seeking out the optimal organizational principle for an oscillatory system – structure through nested modulation. The evolutionary path trod by western music was (is) I think, beyond the overall control of individuals and groups. The surface details no doubt were influenced by what might be termed ‘fashion drift’ and more directly by composers through innovative changes of style. However, the development of the underlying structure of the evolving system would essentially obey the laws of a musical form of natural selection controlled en masse by the musical ‘consumers’, the agents of selection, acting beyond prescriptive direction. The unseen hand of selection would, I believe, inevitably guide a system of music in harmonic parts (i.e. polyphony) toward the organizing principle of modulation – at every level. The point at which this system of harmonic modulation 'took off' is probably marked by the change from stepwise 'melodic' motion in the bass part to movement by interval leaps – as it started to track the whole number relationships of modulation from chord to chord (as well as key to key). This would put the dawn of the tonal era circa 1450 – 1500, halfway between the writing of the two examples in Figure 9.5.

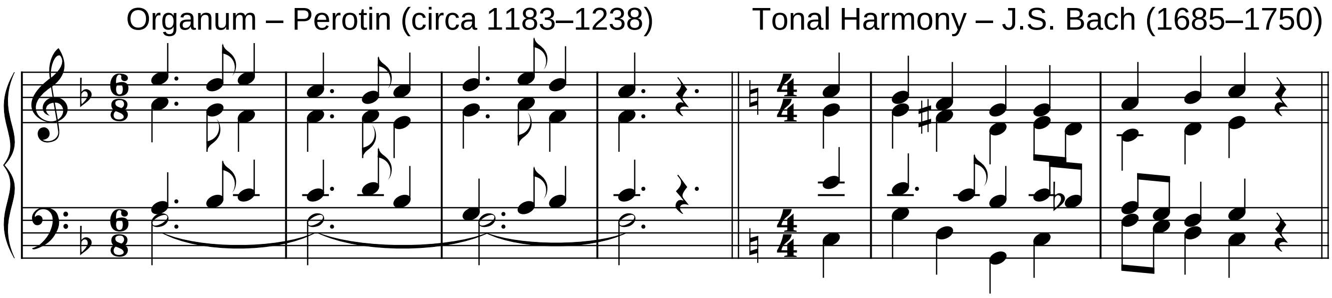

Figure 9.5 Whilst the ecclesiastical music of the middle ages evokes a vision of the unchanging certainties of heaven, the tonal music of J.S. Bach contains much of the character of a dynamic scheme of this world, a system of motion and computation.

The organum (an early form of music in parts) in Figure 9.5 is taken from *A History of Western Music* by Donald J. Grout and the accompanying text makes the point: "The sustained note sections, which sometimes involve hundreds of measures [of 6/8] over one unchanging bass note, form great blocks of fundamentally static harmony... [and] do not have the quality of harmonic movement to which we are accustomed in music of the eighteenth and nineteenth centuries, organized around clearly related tonal centers and working with dominant-tonic relationships; one cannot speak of chord progressions in Perotin, but only chord successions. The musical shape of a Perotin organum is defined by the design of the Gregorian Chant on which the piece is built..."

Gradually, over the past 500 years or more I suspect, musicians have been discovering and elaborating a system of music which contains its own internal logic – musical structure based on the whole number relationships of nested modulation. The need for 'external scaffolding' like cantus firmus chant melodies and the like, to hold the edifice up, fell away, as little by little, composers gained the confidence to build with the new materials of tonal relationships – which they found, miraculously, were self-supporting. It is, I think, the defining discovery of western music: *that tonal compositions – oscillatory little worlds – can stand on their own as self-coherent entities supported solely by their internal relationships*, given that the relationships are (broadly) those generated by the process of nested modulation: the algorithm of symmetrical exchange.

What also fell away over the years was the elaborate framework of medieval theory, much of it relevant, in terms of understanding ratio and proportion, but no longer explicitly required by composers for the new, viscerally intuitive, tonally organised music. The mathematics – i.e. the structural scaffolding – was still there holding up the building, but now internalised as chord progressions and key relationships, computed by the modulation algorithm, to produce self-sustaining musical structures – *little worlds*. However, a side effect of this shift from elaborate theorising about ratio and proportion to the actual *living out of the theory in sound*, in the form of tonal compositions, was that music became detached from its medieval sister disciplines of mathematics and astronomy, the quadrivium, drifting over time into the arms of 'the arts'. Just as musicians were discovering through practical usage, what I suspect may be an implementation of a more generally applicable scheme of nature – the generation of structure through the mechanism of nested modulation in oscillatory systems – the gap between the artists and the men of the new science proved too wide for this intuitive discovery to leap or leak across. The internalisation of medieval theory, in the form of the emerging language of tonal harmonic progression, perhaps cloaked a full understanding of it dynamical and numerical basis from view.

Of course the development was nowhere near clear cut. Music's little worlds of tonal relationships are by no means rigorous applications of the principle of modulation. Also medieval music contains a fair degree of tonal reference, and J.S. Bach, arguably the greatest tonalist of all, was extraordinarily fond of and adept at, adding extra (non-harmonic) contrapuntal elaboration. However, the fact that his music can be understood and enjoyed without the slightest awareness of these additional layers underlines the pre-eminence of tonal relationships as the primary source of form and structure.

The change in external style, which came after Bach, exemplified in the *idealised simplicity* of Haydn and Mozart, was to mark a classical phase of pure, internalised, tonal architecture. Musical structure and expression, form and function, as one:

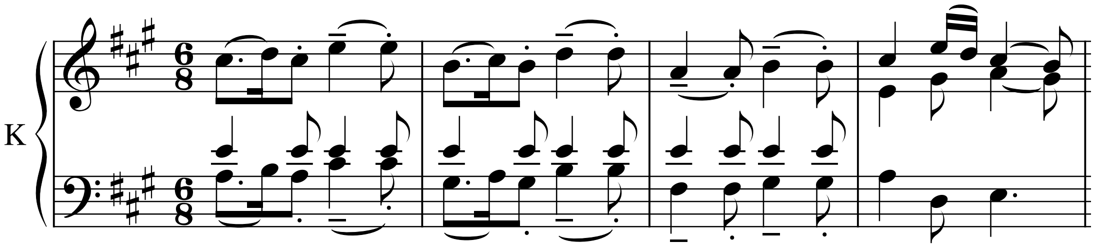

Figure 9.6 Theme from the Piano Sonata in A Major (K331/300) by W.A. Mozart.

Nesting and Nested Harmonic Series

Looking in more detail at nested structure: Illustrated below is a harmonic series, Figure 9.7, and to keep things simple only the first six harmonics of the series are illustrated. So in effect we are looking at columns 1, 2, 3 and rows 1:1 to 1:6 in the top left corner of the Table of Harmonic Series, Figure 9. (An expanded version of Figure 9.7 can be found in the CHPT19 folder, Quick Start Outline Figure E.5.)

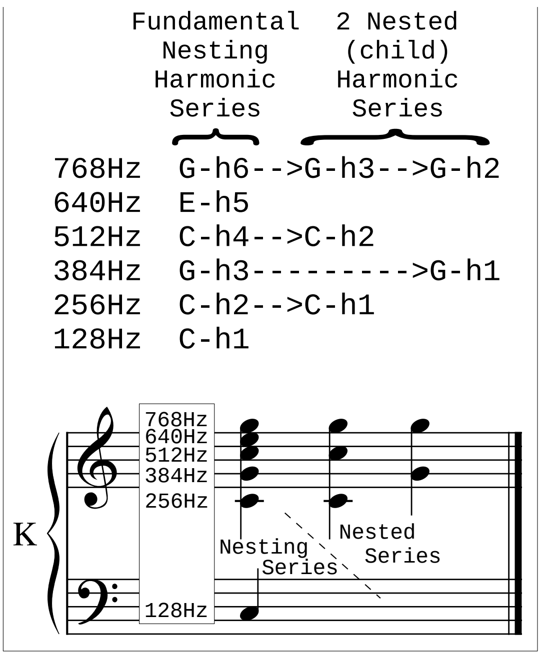

Figure 9.7 Extract from the opening of the Table of Harmonic Series expressed in ratios and chords.

Within a harmonic series there are potentially an unlimited number of nested harmonic series. On the left of Figure 9.7 is a fundamental nesting series consisting of the first six ratios of the harmonic series built on frequency C-128Hz. These six ratios form the common C-major chord expressed as a complete harmonic series, most often in music, chords appear in a less full and organised arrangement – though the overtones deriving from the lower notes in a chord will plug many of the gaps.[6]

To the right of the six ratios in Figure 9.7 are two *child* or nested series built on the frequencies C-256Hz and G-384Hz. The nested series are subsets of the nesting series, that is to say, there are no new ratios in the child series that are not also to be found in the *parent* series; and though we are considering here only six ratios, however far the parent and child series are extended, no ratios will be found in the child series which are not in the parent series. You can also see that C-256Hz, the h1 foundation of the first child series, is the second harmonic of the fundamental parent series and that G-384Hz, the h1 foundation of the second child series, is the third harmonic of the parent series. The THS above, Figure 9.2, illustrates the further expansion up to the start of the 36th child series. One might also draw a computational analogy between the parent/nesting series as hardware and the child/nested series as software.

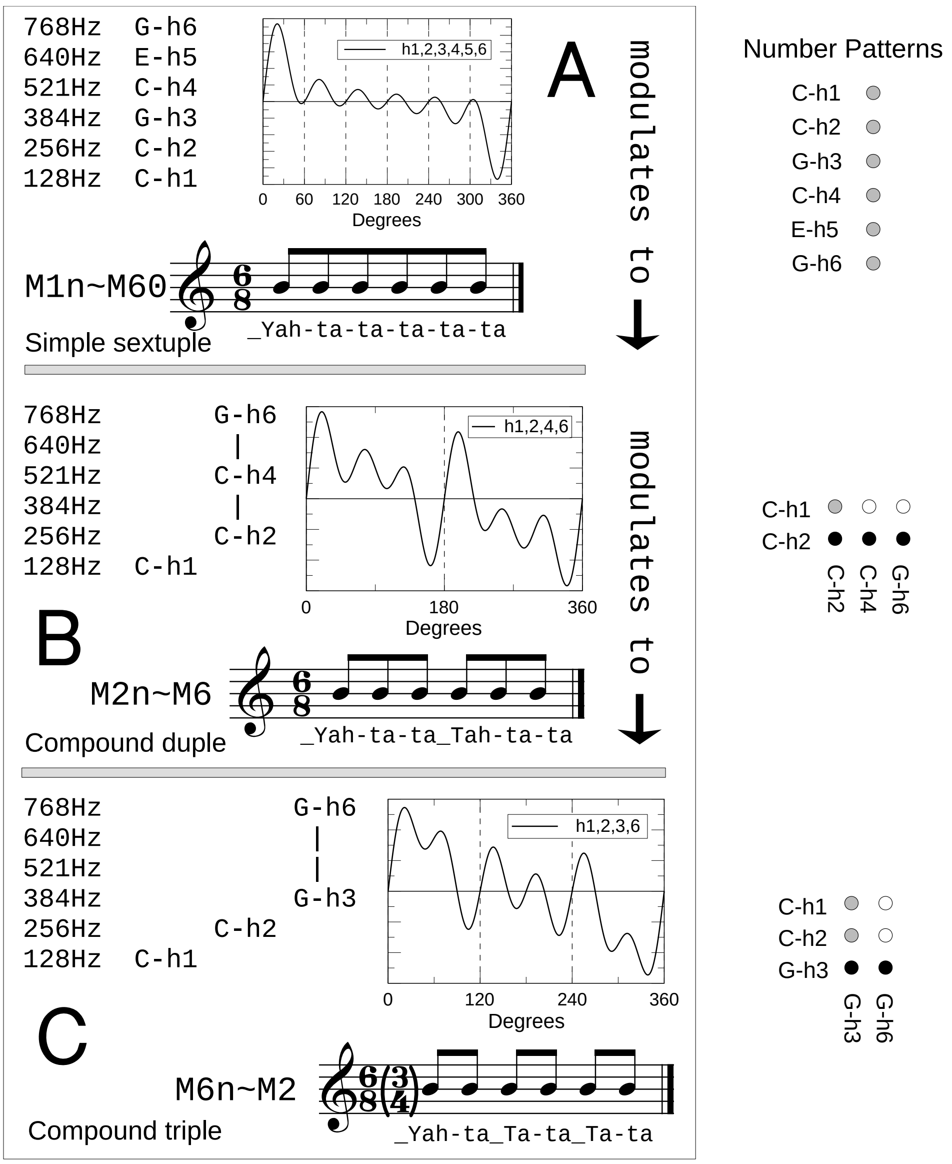

Figure 9.8 Four ways of describing internal arrangements: (top left) ratios in a table of harmonic series, (top right) as a graph, (bottom left) as nested Metrics and (bottom right) as time signatures and meters/rhythmic patterns.

As a vibrational system, the fundamental parent series is a rather labored and complex way of producing an interference pattern of period 128Hz containing six fluctuations – this is illustration A/A1 in Figures 9.8/9.9 (assuming roughly equal amplitudes and uniform phase). There are two less energetic alternatives. Illustrations B/B1 and C/C1 both produce a period of 128Hz and 6 fluctuations per period but in a more economical way. The A, B and C boxes in Figure 9.8 illustrate four different but equivalent ways of describing and thinking about the alternative arrangements of internal relationships, which a system can adopt to generate six fluctuations period.

In these four views of a system of nested harmonic relationships, the graphs of the interference patterns generated by the harmonics are aligned with the relevant time-signature/meter. Interpreting interference patterns, the *sum* of an oscillatory system's parts, in metrical terms, is helpful in understanding how a small and gradual change at the level of the whole system – a slight leaking of *metrical accent* – can be linked with a *step change* within the individual parts.

The existence of two (or more) entangled harmonic series is a crucial element in the process of modulation, as it allows a form of '*triangulation*' to occur. In the illustrations A1, B1 and C1, Figure 9.9, two vertices of a triangle of relationships, the period and the number of fluctuations per period, are held constant at h1 and h6 respectively in the interference pattern generated by the combined nesting and nested series. The third vertex is the point where the nested series/sub-system joins onto the fundamental nesting series. This joining point has some freedom to manoeuvre between 128Hz in illustration A1, 256Hz in B1 and 384Hz in C1 – which are the harmonics h1, h2 and h3 of the nesting series. No other frequencies (joins) are compatible with a period of 128Hz and six fluctuations per period (except the inverse of A – not illustrated).

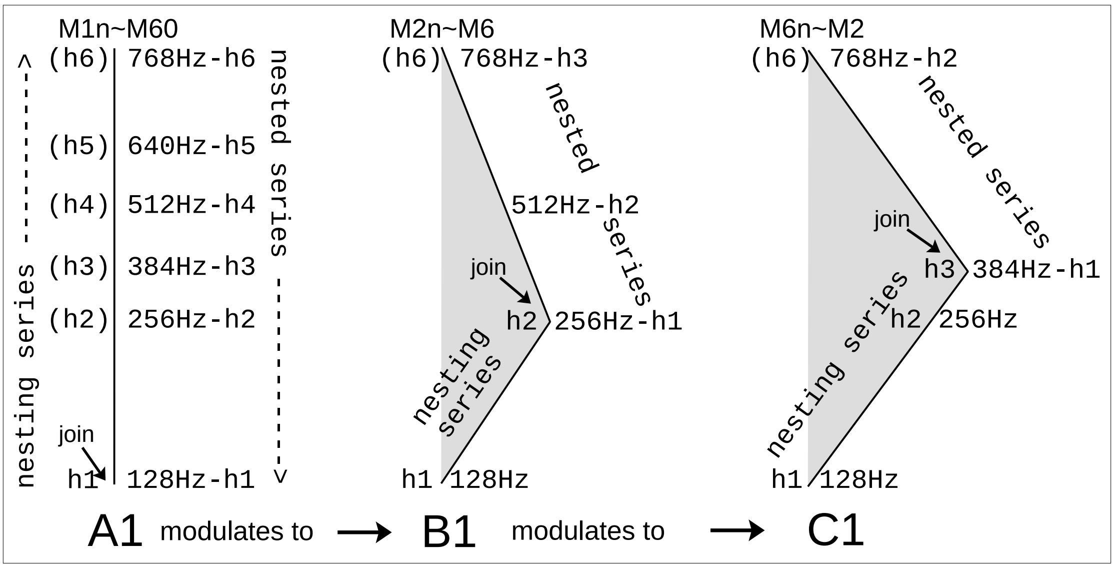

Figure 9.9 From left to right, the system relaxes, under the influence of the second law of thermodynamics, to the most economical internal arrangement, which it finds in the form: { { h1C + h2C + h3G } + h6G }.

After Modulation

Once the nested subsystem has exchanged three ratios based on 256Hz (illustrations B/B1) for two ratios based on 384Hz (illustrations C/C1), the upper vertex of the system is ready to absorb two more ratio/ oscillators, in the build-up to the next conjunction at row 1:12 in the Table of Harmonic Series. The sequence is illustrated in boxes D, E and F. This process is continuing the system's evolution as described by the dashed line in the THS, Figure 9.2, the process is moving the system forward through another energy cycle of gain, gain, modulation and loss.

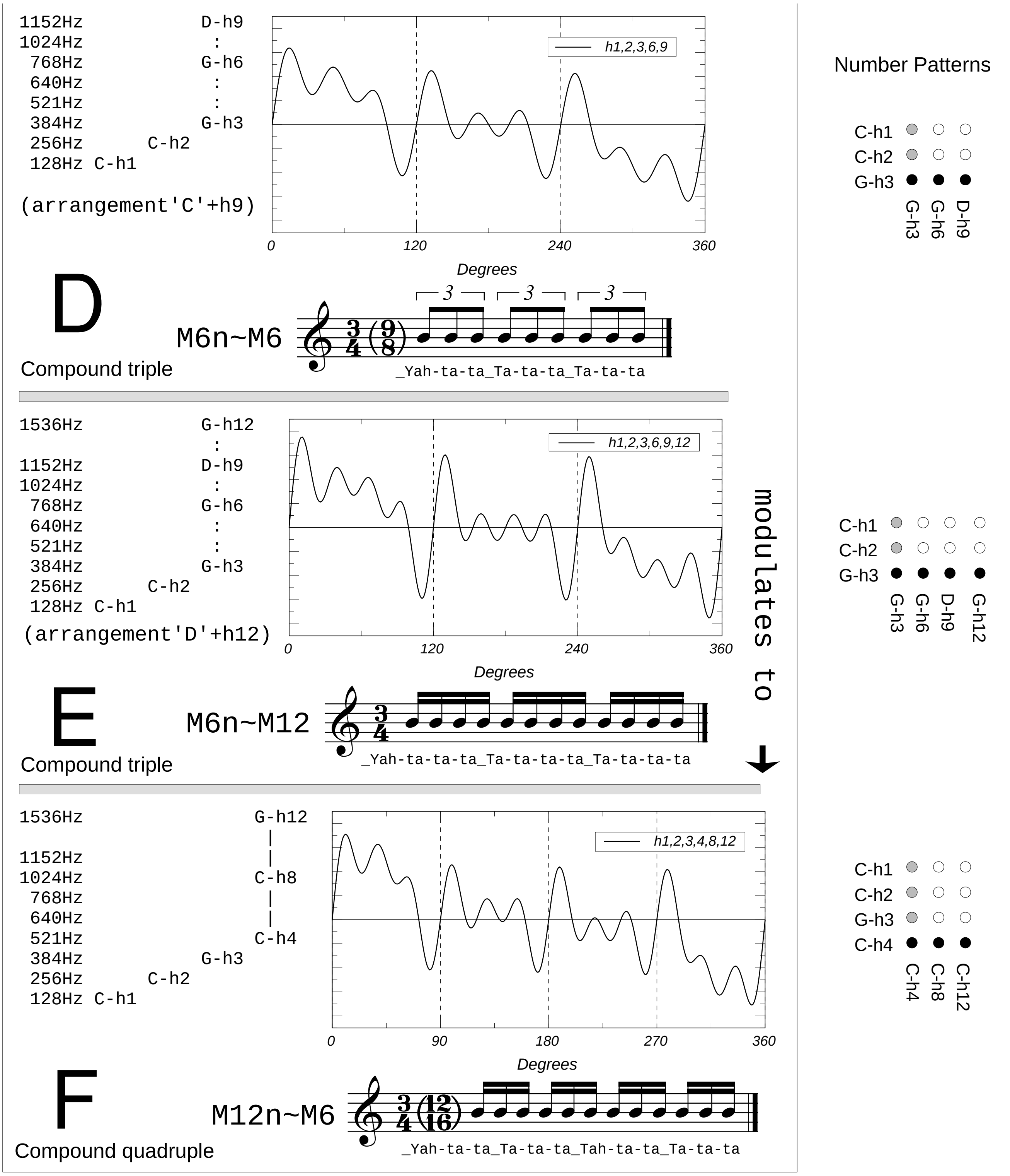

Figure 9.10 Four ways of describing internal arrangements, as in Figure 9.8. Illustrations D, E and F follow on from A, B and C, showing the further development of the system up to the next conjunction (h12) in the Table of Harmonic Series, where another modulation/exchange can occur. In both figures the relevant number pattern is appended on the right-hand side.

NUMBER PATTERNS IN THE MATERIAL WORLD

There is an interesting connection between such nested harmonic structures and probably the oldest form of mathematics. Counting using physical tokens to represent quantities and magnitudes, with pebbles, seeds, sticks, etc... among the most likely tokens beyond the ten finger-digits; and in such presumably purely additive number systems, the arrangements of tokens can fall into patterns. The ancient Greek mathematicians classified some of these arrangements as square and oblong, plus other shapes.

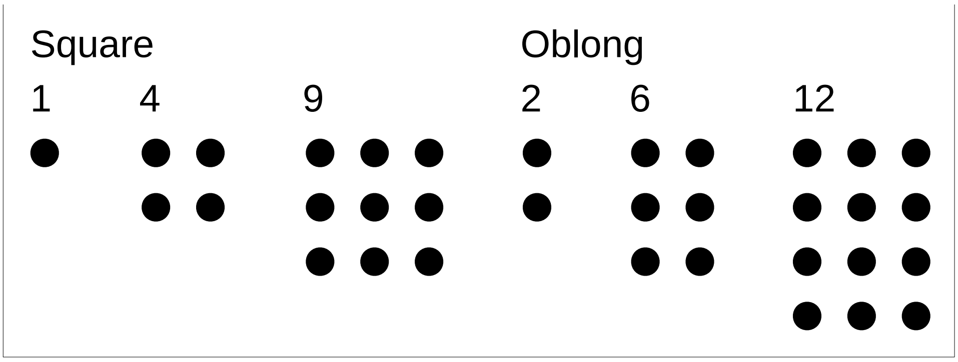

Figure 9.11 The first few square and oblong numbers.

The square and oblong numbers, viewed in the two coordinate dimensions of a vertical and horizontal array, have a particular connection with the modulation algorithm. Notably, the square and oblong numbers form an alternating sequence, which, beginning from unity, runs: 1, 2, 4, 6, 9, 12, 16, 20, 25, 30, ...; with each square number adding one column to form the next oblong number and each oblong number redefining its axes (effectively modulation), before adding another column to form the next square – illustrated in Figure 9.12. It is precisely this same sequence of numbers which is mapped out by the path of primary modulations in the Table of Nested Harmonic Series (THS Figure 9.2 dashed line).

Visually, the *rectilinear number patterns* mimic all the possible nested configurations of whole number oscillatory combinations and mutable numbers. In the discussion here, there are just two levels, the fundamental vertical dimension and a nested horizontal layer, and while up to three levels of nesting can be represented on the Cartesian ‘xyz’ axes graphically, beyond the cubic (Chapter 1, Figure 1.9), rectilinear number patterns lose their direct physical representation and so also their illustrative usefulness. Notwithstanding, fully filled rectilinear arrangements are conceivable in any number of (higher) dimensions and thereby they continue to match the unlimited possibilities of nesting harmonic series within themselves. But for now back to the two dimensions of squares and oblongs.

In Figure 9.12, the sequence of the first four primary modulations, beginning from unity (i.e. the first natural number), is charted in these visually illustrative rectilinear number patterns. Each step involves either addition or modulation. Addition occurs when the system attaches the next commensurable ratio/oscillator in the harmonic series to its ‘outer edge’ – its highest frequency. (Subtraction would involve the release of one or more ratios.) Addition continues until a rectilinear pattern is encountered, which has an equivalent but less energetic arrangement. At such points, a modulation or exchange may occur – a structural transformation mediated by the algorithm of symmetrical exchange, acting in accordance with the second law of thermodynamics. Basically, the system seeks out the configuration that yields its lowest viable level of energy and complexity – its ground state – that is, the most efficient internal arrangement capable of preserving the systems identity or integrity. This is a constantly recurring scenario as each oblong arrangement generated from the preceding square configuration will have a corresponding lower energy alternative.

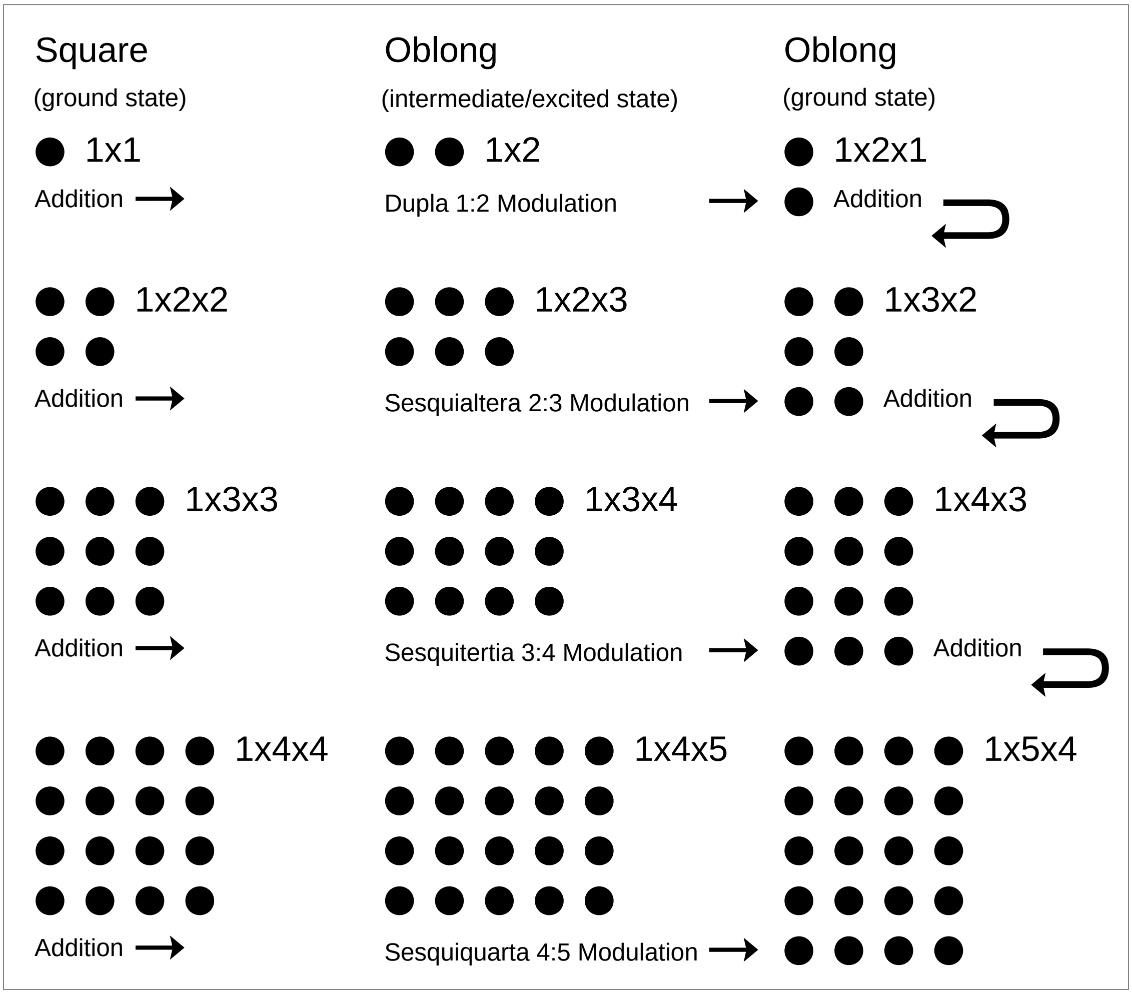

Figure 9.12 The first four primary modulations in the Table of Harmonic Series covering the numbers 1, 2, 4, 6, 9, 12, 16 and 20. The vertical axis represents the fundamental nesting series and the horizontal axis the nested series. Expressed as mutable numbers, the sequence of additions and modulations can be written:
MBN: 1_1, 2_1 --> $1_2\,0_1$, $2_2\,0_1$, $3_2\,0_1$ --> $2_3\,0_1$, $3_3\,0_1$, $4_3\,0_1$ --> $3_4\,0_1$, $4_4\,0_1$, $5_4\,0_1$ --> $4_5\,0_1$ ...

The first step is that of addition, H1 + H2, a single, one-dimensional harmonic series. Rather wonderfully, although this series defines an axis (which here is illustrated vertically) it doesn't preclude the existence of other axes, and with these other possibilities in play, the system has options. The question is what next to add, H3 or h4? If the system acquires H3 it continues its one-dimensional growth from MBN: 2_1 to form the prime state mutable number MBN: 3_1. Alternatively, the system could redefine itself (modulate) from MBN: 2_1 to MBN: $1_2\,0_1$, in other words become a two-dimensional entity, and then be open to the possibility of adding h4 rather than H3. The system doesn't have to decide which form to take, one or two dimensions, until the second step of addition looms, but, once the addition happens the die is cast, and the system follows one or the other route: single axis H1, 2, 3 or dual axes H1, 2 nesting h4. After this choice is made, the two-dimensional system's next step of addition would be to attach h6 which leads to another modulation exchange, from H1, 2 nesting h4, 6 to H1, 2, 3 nesting h6 (Figure 9.12, second row).

The alternative route of the one-dimensional harmonic series H1, 2, 3 next leads to the addition H4. However, the formation of the relatively complex prime state series, H1, 2, 3, 4 would provide another opportunity for the system to assume a two-dimensional form – this time via a secondary dupla 1:2 modulation from: H1, 2, 3, 4 to H1, 2 nesting h4. This exchange would involve the loss of h3, a large relaxation of energy and complexity and therefore highly attractive under the auspices of the Second Law. If the system again stubbornly resisted relaxation to a lower energy configuration, and continued its one-dimensional growth through H5, it would again find a route to relaxation proffered when it acquired the ratio/oscillator H6:

H1, 2, 3, 4, 5, 6 ------- tripla 1:3 modulation --------> H1, 2, 3 nesting h6 (MBN: 6_1 --> $2_3\,0_1$)
H1, 2, 3, 4, 5, 6 – tertiary dupla 1:2 modulation --> H1, 2 nesting h4, 6 (MBN: 6_1 --> $3_2\,0_1$)

The point illustrated here is that no matter how stubbornly a system resists a multi-dimensional structure, the modulation algorithm will incrementally offer an ever-increasing number of escape routes to a lower energy existence, while simultaneously, the Second Law will make the probability of the system taking one of these routes, increasingly large. In a low energy environment, only the prime number oscillatory patterns are safe from the siren call of modulation – as they already occupy their ground state configurations.

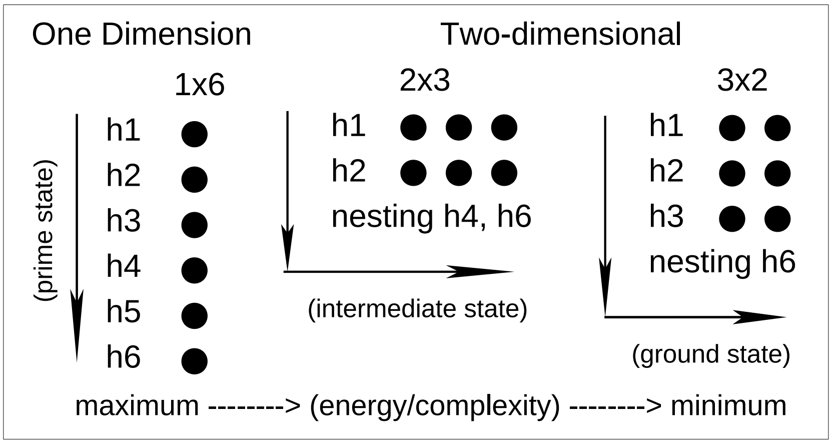

Figure 9.13 The number six has three possible rectilinear configurations: 1x6, 2x3 and 3x2, MBN: 6_1 , $3_2\,0_1$, $2_3\,0_1$.

All these scenarios lead to the addition of h6 to the system by one means or another. Six, as one of the set of fecund numbers. Formed of the factors two and three, six has two two-dimensional arrangements: 3×2 and 2×3. These two arrangements translate to the compound triple meter of time signature 3/4 and compound duple meter of 6/8 time – Figure 9.8B/C. Once a growing system has adopted a multi-dimensional structure, it will repeatably encounter points – mutable numbers like six – where it can release energy and complexity by rearranging its internal structure. In each of these modulatory steps between oblong number patterns (i.e. adjacent columns in the Table of Harmonic Series), the transformation is driven by the exchange of a higher energy oblong configuration for its corresponding lower energy equivalent, as illustrated in Figure 9.13 for six. The process fostered by the modulation algorithm (in principle, chord progression in tonal music) is powered by these transformations, as the Second Law relentlessly drives systems toward equilibrium.

The first step of modulation, the dupla exchange is perhaps special, somewhat subtle; in that the system is moving from a one-dimensional structure to an arrangement, which while remaining apparently one-dimensional, has become open to two dimensions. It has the potential to develop a nested structure by pairing ratios and growing by twos – e.g. H1, 2 nesting h4. Yet ultimately, the system still retains something of a one-dimensional existence, there are still four tokens in the square number pattern 2×2 and there are still four fluctuations in the interference pattern of the combined frequencies H1, 2 nesting h4 (assuming reasonably uniform amplitudes). The difference between H1, 2, 3, 4 and H1, 2 nesting h4 (number patterns 4 and 2×2) is one of energy and complexity. A multi-dimensional structure is more efficient. Figure 9.14 shows this simplification visually. By grouping the underlying ratios of the fundamental series into twos, and then threes, a less complex arrangement of the whole system is obtained, but one which still maintains its identity – the value of the mutable number. From the perspective of aural cognition, such a process of reduction or rationalisation by nesting would allow a relatively simple (and therefore more intelligible) nested harmonic series, to be extracted from a perhaps complex and extended underlying series implied by a particular objective chord or progression.

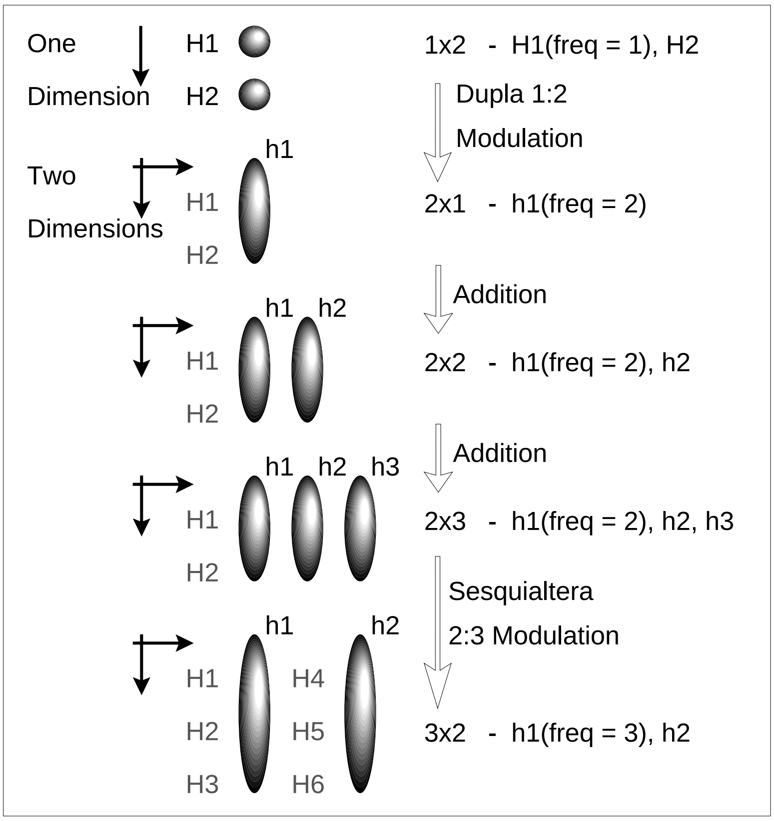

Figure 9.14 The simplified view of the complex fundamental series H1 through H6. (The color-scheme of gray and black for background and foreground ratios, is carried over into the Examples and Chapter 12.)

Computing Structure

Briefly moving now from the details of the modulation mechanism to the broader canvas again. In applying this oscillatory model to our appreciation of music, the experience of tonal music can be likened to a journey *through the structure* of the piece, with the passage from chord to chord and key to key understood (more or less rigorously) as a sequence of *computational steps*, taken by means of the *modulation algorithm*. In a modulating oscillatory system analysis, the chord progressions of the piece represent (parts of) the upper nested series in the model, and our internal sense of key/tonal center (generated from listening to the chordal exchanges) are represented by the nesting fundamental series. This then provides the backbone of tonal perception, all of which, combined with other sources of information (such as directional and visual inputs etc.) is processed to recover the full musical experience.

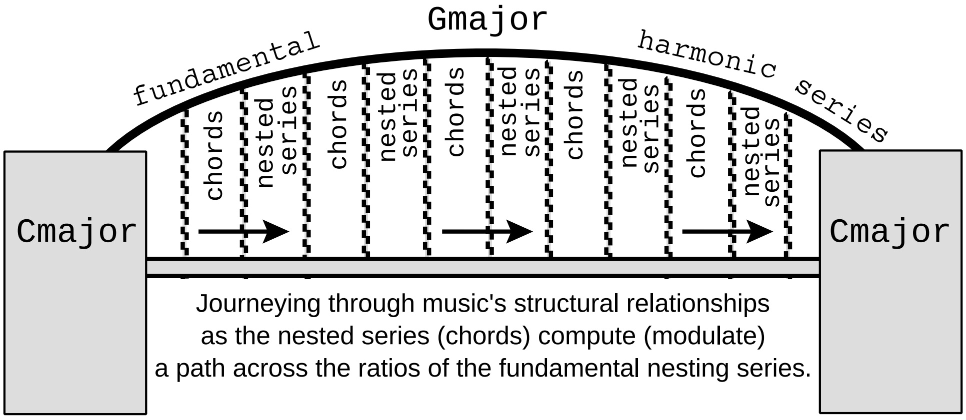

Figure 9.15 A musical performance is a journey through the structural relationships of a ‘little world’.

For musical compositions, we travel from one end of the structure to the other, experiencing the relationships sequentially, with each performance a rebuilding of the relationships anew. However, if the model were applied to suitable self-organising physical systems, the structure would be built once, with the modulation algorithm charting a course through a (perhaps complex) set of ‘cadences’, until eventually the system ran out of ratio/oscillators to compute. Thereafter, like the Sydney Harbour Bridge, such a physical system would maintain a permanent and stable existence. The underlying argument being pursued in regard to such hypothetical physical entities is simply that since systems constructed of waves may exhibit similar structural features, and, as tonal music is ultimately a system of (acoustic) waves, our music might share some structural features with other systems in the physical world that are also based on waves. So where might one look for stable systems built of waves and long lasting periodic behaviour?

In Chapter 15 a number of possible candidate structures are examined. For example, there is a striking parallel between the sequence of primary modulation exchanges (tabulated in Figures 9.4 and traced out by the dotted line in Figure 9.2) and the arrangement of a basic ‘shell’ structure observed in the atomic nucleus or, at the atomic level, the ‘cadence-like’ closure of electron ‘shells’ that produce the noble gases of the periodic table and stable molecular bonds. At the other extreme, in the macroscopic arena of celestial mechanics, one might cite the strikingly harmonic gravitational resonances of planetary periods in the solar system. However, this is to run ahead of the narrative, and for now, the discussion below is confined to how waves became part of science’s most fundamental theory of the physical world.

At the opening of the twentieth century the German scientist Max Planck introduced the concept of packets of radiation or ‘quanta’ to solve a long standing problem in classical thermodynamics concerning the emission of energy. Planck hypothesized that radiant emissions were made in discrete chunks of energy linked to the frequency of the emitter, and by this method he was able to limit the total amount of energy released to values that mirrored experimental measurements. (Previously theory predicted an infinite result!) Planck’s insight of 1900 marked the beginnings of quantum mechanics. Soon after, in 1905, Albert Einstein used Planck’s work to solve another nineteenth century problem in physics called the photo-electric effect: The observed relation between the frequency of light shone on photo-electric materials and the energy of electrons ejected by that beam of light. Essentially this was the absorption side of Planck's radiation theory, and it suggested that light waves had a particulate character.

Through the 1920's quantum mechanics became established as the best (though rather unsatisfying) explanation of the physical world at the smallest scale. In 1924 the physicist Louis de Broglie introduced his hypothesis of wave-particle duality. He proposed that particles such as the electron, and indeed subsequently all matter, possessed wave-like properties. That matter – solid materials – should possess wave characteristics was totally unexpected, because the wave nature of matter only becomes apparent at scales far to small for direct human experience. It was an inspired guess at the time it was made but soon after, in 1927, de Broglie's hypothesis was confirmed experimentally and in 1929 he received the Noble prize in physics for this work. However, even before de Broglie's hypothesis had been confirmed other scientists could see its potential. Careful measurement indicated that the motion and energy of electrons within the atom are limited or staged rather than freely variable. Matter waves or de Broglie waves as they were named, provided an explanation for this observed structure. If electrons behaved as matter waves bound by the positively charged nucleus, only integer multiples of a fundamental wave could exist within these confines. Just as only standing waves on a piano string can fit within the fixed vibrating length – all other non-integer wavelengths self-destruct. Louis de Broglie's discovery of the wave nature of matter opened the way ahead. The physicists Erwin Schrodinger and Max Born further developed de Broglie's idea to produce a dynamic, probability based understanding of the atomic scaled world. And although the resulting 'wave mechanics' sat uncomfortably with the older 'classical' instincts of many scientists, its predictions matched experimental measurements precisely. The wave-based quantization at the smallest scale of matter, introduced by Planck, and subsequently developed by Einstein, de Broglie, Schrodinger, Born, Bohr, Heisenberg and others would embed the *harmonic series* at the reductionist heart of physics.

Picture courtesy Wikipedia

Louis Victor Pierre Raymond de Broglie (1892–1987) was born into a noble family, the youngest son of the Duke de Broglie. The family lived in Dieppe on the Atlantic coast of France and had a long history of service to the French state. Louis had two sisters and two brothers, Maurice, and Phillippe who died in infancy. After finishing his secondary school education in Paris in 1909 Louis enrolled at the Sorbonne. Although at first attracted by the humanities, studying history, he later turned his attention to physics, perhaps influenced by his elder brother Maurice who was also a physicist. With the outbreak of war in 1914 Louis's scientific studies were interrupted by service in the French Army as a signals officer, which provided an enforced break from the 'tram-lines' of orthodox study and allowed a period of reflection upon recent scientific trends. Following the war Louis' development as a theoretical thinker in science was rapid, and by the summer of 1923 the insight into the wave nature of matter had formed in this mind. By 1924 in his PhD thesis, Louis de Broglie published his ground breaking hypothesis concerning matter: *any moving particle or object has an associated wave*. Louis work caught the eye of Albert Einstein who supported it enthusiastically and so it quickly entered the main stream of scientific thinking. After finishing this studies Louis took up a teaching post at the Sorbonne and in 1928 a professorship at the Institut Henri Poincare, and finally was appointed Professor of Theoretical Physics at the University of Paris in 1932. A position he held until retirements in 1962. Louis de Broglie was awarded the Nobel prize in physics in 1929 in recognition of his work on the wave nature of matter. Louis served on many committees and he published many books on waves mechanics over his long career. Like Einstein and many others, Louis de Broglie never fully accepted the statistical interpretation that came to dominate quantum mechanics. Louis de Broglie died in Paris on the 19th March, 1987.

MODULATION, METER AND TIME SIGNATURES

Concepts of meter, pulse and time signatures, can be usefully applied to oscillatory systems containing nested series because they follow the same logic of organising vibrational/rhythmic patterns into groups and hierarchies, that is to say, whole number patterns of strong and weak pulses. In particular, compound meters are able to encapsulate a separation of rhythmical levels which match the separation of nesting and nested harmonic series.

In illustration Figure 9.8A above – the 'labored' method of generating six fluctuations per period – the metrical expression is shown as a measure of 6/8, with the six eighth-notes all joined together by the 'beam'. This is indicating that the meter is simple sextuple – one strong beat followed by five weaker pulses. In performance this is very difficult to maintain. It's highly unstable. Almost any contour of melody or harmony would unbalance the music, tempting the players into adding secondary accents. (Like a pencil balanced upright on its end, any slight disturbance results in it falling to the more stable state of lying on its side.) The effect of adding one or more secondary accents, is to transform the 6/8 measure from simple sextuple into a compound form, with a secondary accent on the fourth eighth-note producing a 6/8 compound duple meter, or with accents on the third and fifth, a 3/4 compound triple meter – as shown in Figure 9.8B and 9.8C respectively. This is a practical example of the all-pervasive power of entropy increase, the second law of thermodynamics. Keeping to a simple sextuple meter demands of the player a constant effort, a high level of stress; relax for a moment and the simple meter is gone. More generally, in the durational domain this principle of relaxation – entropy increase – is felt in delightful hemiola effects, while in the domain of harmony it lurks behind the various cadences and many satisfying chord progressions – all of which are brokered by the modulation algorithm.

When an oscillatory system alters its internal arrangement, by the process of modulation, the external effect is no more than a subtle change of accent within the system's meter or interference pattern. For example, the slight relaxation from 6/8's _Yah-ta-ta_Tah-ta-ta to 3/4's _Yah-ta_Ta-ta_Ta-ta, in stepping from configurations B/B1 to C/C1 in Figures 9.8 and 9.9.

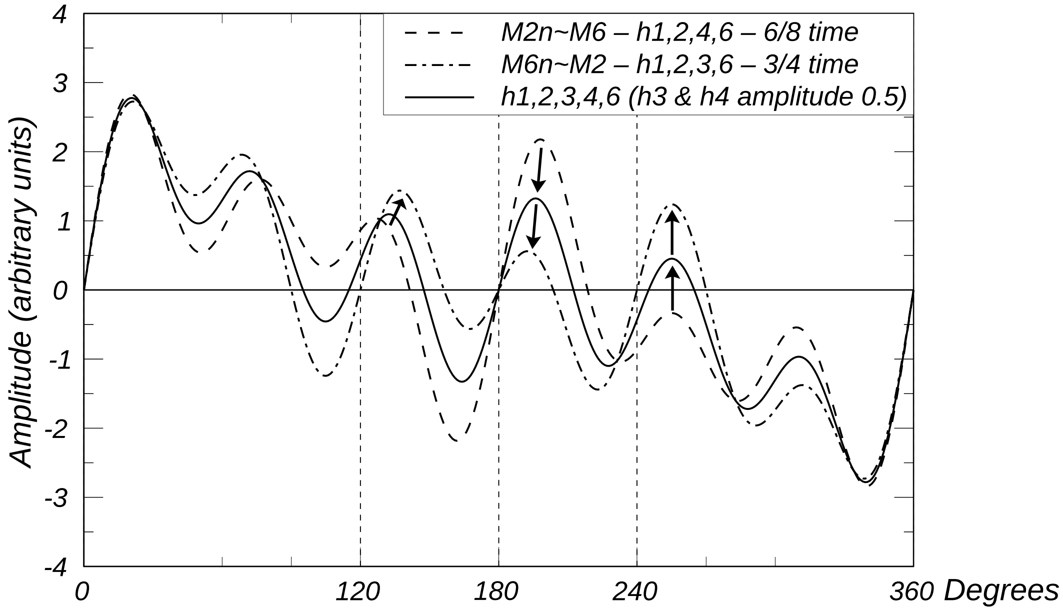

Figure 9.19 The full line marks the intermediate stage between a 6/8 and 3/4 meter interference pattern, with half of h4's amplitude/energy syphoned away into h3.

In a way a relational oscillatory system has two existences, one as a collection of separate parts and one as a whole unit. At the level of the whole system – the interference pattern – it only takes a small leakage of energy away from fourth fluctuation in the pattern of a 6/8 compound duple meter toward the third and (principally) fifth fluctuations, for the system to slip – that is modulate – into a 3/4 compound triple meter. However, this slight change of *accent* at the level of the whole system, a mere fluctuation or *degree of uncertainty* in wavelength and amplitude, results in a *step change* in the arrangement of the internal parts, as h4 'evaporates' from the system, to be replaced by h3.

Such a step change within a system, a re-calibration of the internal arrangements, might be viewed as a form of *oscillatory computation*: a mechanism whereby an oscillatory system could compute its internal relational evolution – its structural development. Naturally in tonal music this proposed computational development is not self-organising, compositions do not 'play themselves'. Yet still, in the mind of the composer or improviser something akin to 'tonal computation' is taking place, unconsciously, in that they are choosing a sequence of chords and meters based upon the common logic of tonality – the modulation algorithm.

Euler's Metric

Metric with a capital 'M', first introduced in Chapter 4, is essentially a concept borrowed from Leonhard Euler's theory of consonance and dissonance, and can be defined as: *the lowest common multiple of the constituent frequencies in a system, expressed in simple whole numbers*. Euler's Metric is written with a capital 'M' when not preceded by the great mathematician's name so as to distinguish it from the more general meaning of metric; and, the term is most often abbreviated to a simple capital 'M' plus the LCM (lowest common multiple) of the frequencies in the system – for example:

M6 (h1+h2+h3), M2[f=3] (h3+h6), M12 (h1+h2+h3+h4) or M2520 (h1+h2+h3+h4+h5+h6+h7+h8+h9).

M	Metric followed by the lowest common multple (LCM) of the constituent frequencies of the system or sub-system, expressed in relative terms. e.g. M12 (h1,2,3,4 - LCM=12)
n~	abbreviation for 'nesting' e.g. M6n~M6 (9/8 time)
->	arrow abreviation for 'modulating to' e.g. M2n~M6->M6n~M2 (6/8 -> 3/4 time)
f'n'	frequency of h1 of Metric e.g. M2[f1.5]n~M6 thus:h1[freq=1.5],h2[f=3]n~h1[f=3],h2[f=6],h3[f=9]
( )	indicate the division and/or grouping of nesting and nested portions of a system e.g. M6n~(M2n~M2)
[ ]	enclose any other information relating to that system or sub-system.

Figure 9.20 A table of symbols and examples of 'M-number' usage in expressions of Euler's metric.

Originally these 'M numbers' were developed as a useful and compact way of handling multiple levels of nesting in oscillatory systems, and of expressing changes in, and exchanges between, nested systems. Later came the realisation that this metrical description was a step upon the way to a variable base number system – mutable numbers – and, that Euler's metric and conjunction value are expressions of the same basic idea. Notwithstanding this development in the MOS model, when thinking in terms of physical or pseudo-physical systems, as we are in this chapter, this ‘metrical algebra’ remains an appropriate and useful method. So, for example, 4/4 time could be simple quadruple or compound duple meter, one cannot tell; but the former would have a Metric 12 (h1, 2, 3, 4) ascribed to it, and the latter Metric 2 nesting Metric 2 (h1, 2, 4) – M12 and M2n~M2 for short. Below is a chart of ‘M syntax’.

The expressions are read in order from left to right, proceeding from the most fundamental level of nesting outward, through each succeeding layer of nesting, in turn; and are in all essentials mutable numbers in factor format. There is a notional, usually unwritten, M1 at the beginning of each expression. Although it might look like algebra at first sight, it is just a straightforward shorthand for nested patterns – thus the expression: M2n~M6 –> M6n~M2 translates to Figure 9.21

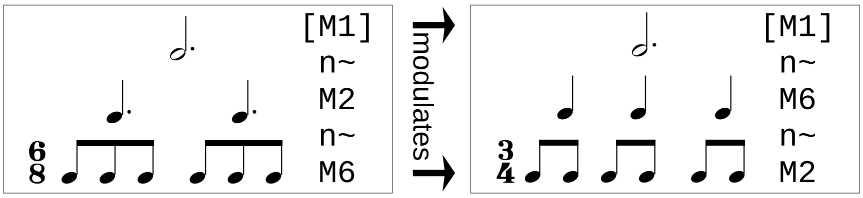

Figure 9.21 A diagrammatic/rhythmic representation of the transformation of nested patterns in the primary sesquialtera 2:3 modulation – M2n~M6 –> M6n~M2.

Overall the metrical patterns formed by modulating oscillatory systems, when acting strictly in accordance with the second law of thermodynamics, appear to produce a factorisation of the total number of fluctuations per period, arranged in order, so that the rhythmic grouping of largest magnitude occupy the (first) lowest frequency and so forth in ascending order, thus minimising the energy embodied in the system.

Nested Metrics of Maximum Economy

Fluctuations_per_Metrical_Period	Nested Metrics	Harmonics.........	Full Metrics..	Factors
1	M1............	h1................	M1............	1......
2	M2	h1,2	M2	2
3	M6	h1,2,3	M6	3
4	M2n~M2	h1,2,4	M12	2×2
5	M60	h1,2,3,4,5	M60	5
6	M6n~M2	h1,2,3,6	M60(6)	3×2
7	M420	h1,2,3,4,5,6,7	M420	7
8	M2n~M2n~M2	h1,2,4,8	M840	2×2×2
9	M6n~M6	h1,2,3,6,9	M2520	3×3
10	M60n~M2	h1,2,3,4,5,10	M2520(10)	5×2
11	M27720	h1-11	M27720	11
12	M6n~M2n~M2	h1,2,3,6,12	M27720(12)	3×2×2
13	M360360	h1-13	M360360	13
14	M420n~M2	h1,2,3,4,5,6,7,14	M360360(14)	7×2
15	M60n~M6	h1,2,3,4,5,10,15	M360360(15)	5×3
16	M2n~M2n~M2n~M2	h1,2,4,8,16	M720720	2×2×2×2
17	M12252240	h1-17	M12252240	17
18	M6n~M6n~M2	h1,2,3,6,9,18	M12252240(18)	3×3×2
19	M232792560	h1-19	M232792560	19
20	M60n~M2n~M2	h1,2,3,4,5,10,20	M232792560(20)	5×2×2

Figure 9.22 Nested Metrics in their most economical arrangement (left-hand column) with their constituent harmonics. The ‘full’ Metrics echo the situation of box A in Figure 9.8 with all harmonics present, e.g. M12 (h1, 2, 3, 4) compared to M2n~M2 (h1, 2, 4).

Finally, for the last conjunction on the dashed line, at column h9 of the THS (Figure 9.2), the metrical expression for the whole system of harmonics, the D^7-major chord in the key of C major, could be written – M2520[f=1] n~ M840 – which equates to the harmonics/ratios:

{ { h1C, h2C, h3G, h4C, h5E, h6G, h7A#, h8C, h9D }, h18D, h27A, h36D, h45F#, h54A, h63C, h72D }

SPIRAL OF FIFTHS

In Figure 9.23, building upon the above D^7-major configuration, the 'start' position represents the last conjunction shown on the dashed line in the THS (Figure 9.2) at h72, and at this point we depart from the path of *primary* modulation exchanges to pursue a less energetic recursive pattern of exchanges. The system, nominally in the key of C major, has reached this point through a sequence of eight primary exchanges (involving sixteen ratios), which has taken the upper tonal structure (nested series) from a solitary h2C to the eight ratios of the D^7 chord, the dominant-seventh of the key of G major:

h9D + h18D + h27A + h36D + h45F# + h54A + h63C + h72D – Metric 840[f=9].

This D^7-major chord marks the point in harmonic terms, where the system might step across a tonal boundary to a new set of coordinates based on the tonal center of G. By taking such a step, an oscillatory system is beginning to etch out the familiar structure of key relationships that we call the cycle or spiral of fifths. Familiar, except that here in Figure 9.23 exclusively whole numbers are being employed, which produces in strict terms, a spiral of twelfths.

In its most elementary form, the change of key from tonic to dominant, from C major to G major, is signalled by one or more conspicuous perfect cadences (D^7-major –> G-major) onto the new tonic chord. Through this progression, music forces a *jump or boundary* upon the model's formerly smooth evolution. By jumping from an upper nested series of eight ratios built on D-h9 to six ratios built on G-h12, (a secondary sesquitertia 3:4 exchange) the chord progression provokes the 'collapse' or rationalisation of the fundamental nesting series into two levels of nesting. From the relatively complex and unstable fundamental Metric 2520[f=1] (h1, 2, 3, 4, 5, 6, 7, 8, 9 + h12) the system moves seamlessly through M6n~M6[f=3] (H1, 2, 3 nesting h6, 9 + h12) before adding h12 and coming to rest in the configuration of M6n~M12 (H1, 2, 3 nesting h6, 9, 12). This recalibration of base units into rhythmic groups of three, under the ever-watchful eye of the 2nd law of increasing entropy, is another manifestation of the principle first seen in the primary exchanges, but with a multiplier of 3 (i.e. 9 ratios exchanged for 3 ratios between fundamentals h1 and h3). A tripla 1:3 modulation exchange.

Continuing the addition of harmonics/ratios to the top-most of the nested series reproduces the key cycle jump for every eight ratios, as energy and complexity are periodically relinquished by the system. In this way, eventually, the whole key cycle/spiral is generated, Figure 9.23 illustrates three cycles of the process. From this 'computation' of the raw twelve-note chromatic scale, by the application of the modulation algorithm of symmetrical exchange, it is clear that the harmonic or overtone series is the fundamental entity of tonal music, and that the scales of twelve, seven and five notes are secondary derivatives of this elementary object.

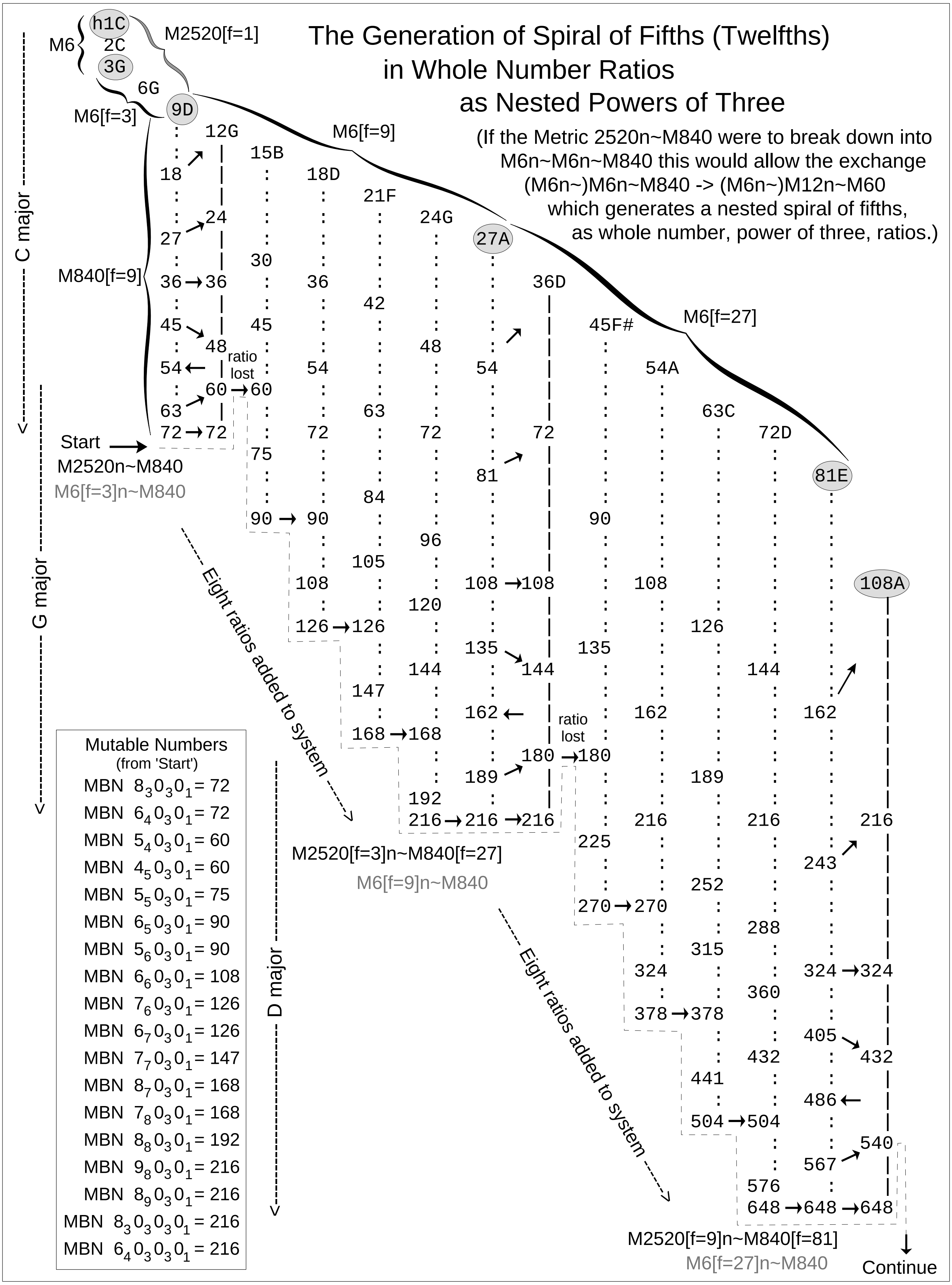

Figure 9.23 The breakdown into nested groups of three, forming key 'areas' and tonal boundaries. Inset box: the equivalent mutable base numbers for the G major tonal center, from 'start' h72 to h216.

Introducing the Aggregated Series

The aggregation of ratios into sub-groupings, as seen above in the generation of the key cycle, is an important feature of the MOS model, and could be viewed in terms of an additional layer of nesting. (Aggregated series are crucial for encapsulating the minor chord and mode – discussed in Chapter 11.)

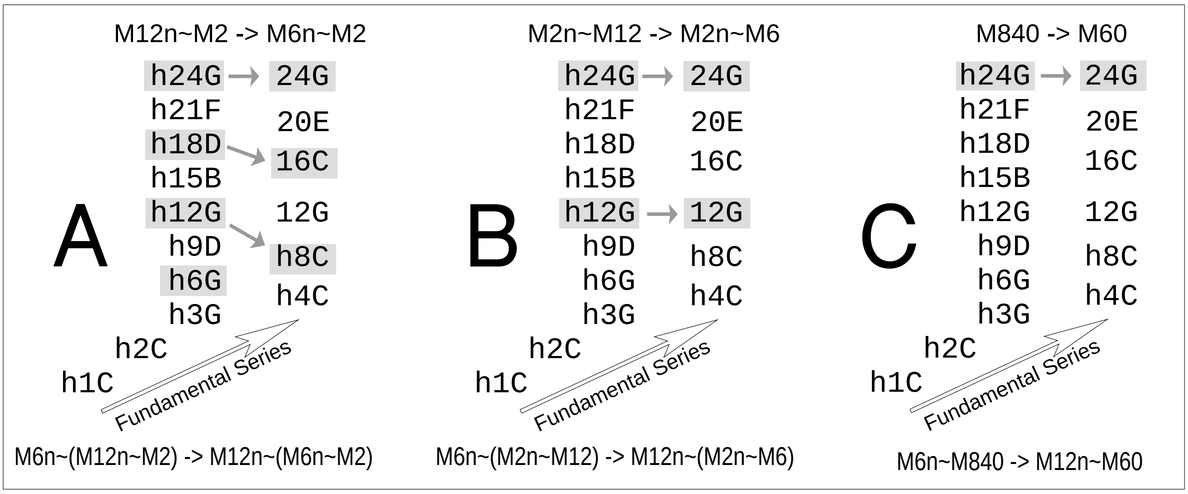

Figure 9.24 Three alternative groupings in the secondary sesquitertia 3:4 exchange's computation of the dominant-seventh to tonic chord progression. A: Four groups of two for three groups of two (4x2 –> 3x2), B: Two groups of four for two groups of three (2x4 –> 2x3) and C: One group of eight for one group of six (1x8 –> 1x6).

Already in Figures 9.23 and 9.24B, the secondary sequitertia modulation gives hints of aggregation within the upper level nested series, into sub-groups of two (2×4 –> 2×3). However, this is a rather special case, as the first chord contains, potentially, an additional layer (2×2×2), providing an extra flexibility; that is, the possibility of changing the number of aggregations from four to three (4×2 –> 3×2), or the actual units of aggregation from four to three (2×4 –> 2×3), in addition to the *full* exchange at the level of eight for six (1×8 –> 1×6). Here, it can be helpful to think in terms of the various different ways in which a Metric could be *parsed*.

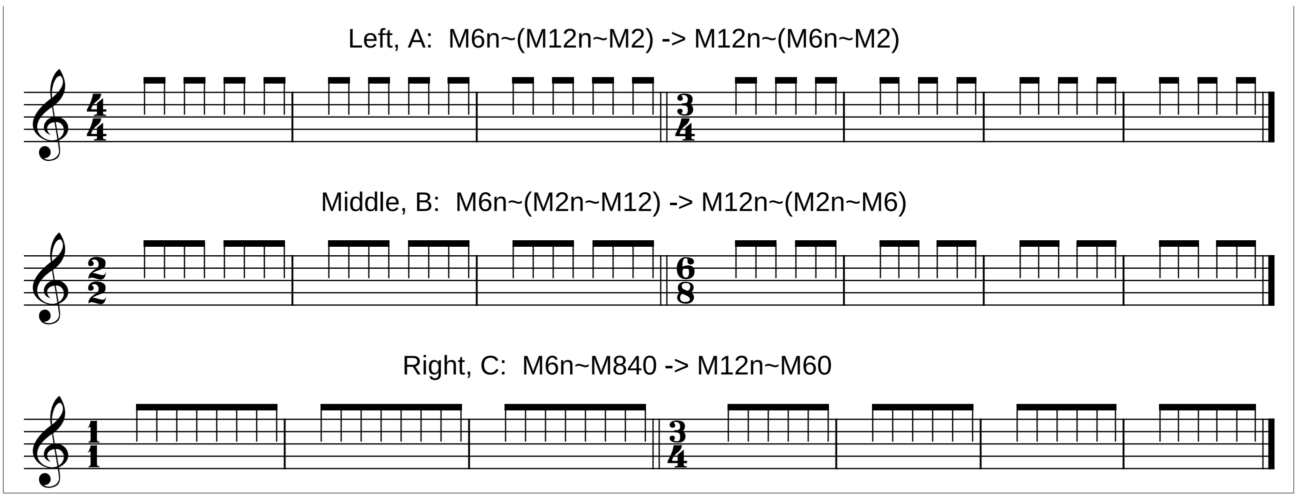

Figure 9.25 The modulation exchanges shown in Figure 9.24A/B/C, expressed rhythmically (Metrics written above).

The second law of thermodynamics implies that more complex Metrics should break down into nested groups where possible. However, in Figure 9.24C, the full Metric 840 version of the exchange is prevented from breaking down into the aggregated forms of Figure 9.24A and 9.21B (M2n~M12 or M12n~M2) by the presence of the major-third B-h15, and particularly the seventh F-h21. The addition of the seventh to a common major chord forces the ear to encompass the relatively complex (and unstable) full Metric 840, which relaxes by modulation to a Metric 60 common major chord. (Though it would be possible for one or other of the configurations Figures 9.24A/B to be 'contained' within the more energetic right-hand arrangement, but not both simultaneously.)

One particular advantage of using the 'Metric' notation rather than mutable numbers is that it can express complex agglomerations of oscillators rather more clearly as illustrated in Figure 9.26.

```
              [ M12n~M2
[M1]n~M6      [ M840
              [ M6n~M6  [ M2n~M2
                        [ M2n~M6

              [ h6,9,12},h24}
{{{{{H1,2,3}, [ h6,9,12,15,18,21,24}
              [ h6,9},h18,27}, [ h54},h108}
                               [ h54},h108,h216}
```

Figure 9.26 A complex system of oscillatory relationships expressed in Metric notation (top) and as an equivalent agglomeration of oscillators (bottom).

END NOTES

Though focused on traditional tonally organised music, the model of modulating oscillatory systems might have wider connections to other oscillatory/periodic system – as hinted at by the shell structure of the atomic nucleus. Also, via *the unifying perspective of information and computation* it is possible to contemplate stepping back from the details of individual systems, to higher levels of abstraction, facilitating a wider inclusive view of music, as one member of a perhaps broad set of oscillatory/periodic phenomena found in the material world.

The development of this essentially computational approach to tonal music has been influenced by the thoughts and insights of many scholars, from a broad range disciplines, and is most certainly a work in progress, rather than a polished finished product. Therefore no doubt at this early stage in the development of the model there will be many omissions, weaknesses and flaws, which, in the light of further thought and the most welcome counsel and advice from colleagues and friends, will require amendment, together with considerable elaboration and augmentation. It is a model that can only ever loosely describe pieces of music in their entirety (though hopefully capturing their structural essence) as there are so many arbitrary and disparate elements in such complex, externally driven, little worlds. However, when the principles of the model are applied rigorously, as if in a truly self-organising system, the outcome appears to take on the character of *digit sequences* in a *position value counting structure* – that is, mutable base numbers. Likewise, while not a rigorous example, tonal music should perhaps in principle, at its most fundamental level, be viewed as a form of *positional number system* generated and governed by the *algorithm of modulation*: though ultimately of course, we relish these mutable base computations more for their intrinsic sound qualities, than the arithmetic results themselves.

The MOS model takes within its purview all units of frequency, from a perhaps notional absolute fundamental (H1) to the highest objective partials arising from musical performance. The model seeks out conjunctions between such complex extended harmonic series and by so doing links the succession of musical sounds into a sequence of logically explicable transitions. To begin with, my own use and understanding of the model was predominantly from the 'bottom up', and something of this early approach remains in the text of this and some other chapters. Indeed, the model was originally developed with regard to 'hard' physical systems containing the full range of frequencies, as much as it was with the more elusive material of tonal music. Thus the 'metrical algebra' presented above suggests an equality of reality for both the absolute fundamental tone and the conjunction frequencies, when in regard to musical sound, only the latter is an objective observable fact. In the early days, when trying to apply the MOS model to tonal music, I gave much attention to low frequency combination tones and considered how subjective aural perceptions might play a role. However as time went on, I began to see that for musical sound the difficulties of working at such a low frequency range (i.e. the fundamental nesting series) was not such a stumbling block as it first appeared: In that the existence and perception of the conjunction frequency, bundled up within the audible sound mass, implied the 'deep tail' of structure down to an absolute fundamental frequency, without the necessity of it actually being objectively present in the sound, or, arising in some process of aural cognition. Essentially, the objective musical sound (which includes the conjunction frequency) is sufficient for aural cognition to make explicable the transitions of melody and harmony in tonally organised music. The deep mathematical structure added by the MOS model, underpins and amplifies this visceral surface. Thus my understanding and use of the model, in regard to the analysis of tonal music, has turned around to a 'top down' approach, now viewing the objectively present conjunction frequency and its accompanying sound mass as the firm basis from which a notional structure may be projected downward, to find an absolutely fundamental frequency level that helps elucidate both the musical structure and the listeners appreciation of it.

Notes

1. Hall, A.R., *From Galileo to Newton,* (Wm. Collins, 1963; Fontana, London, 1970) page 33.

2. Wolfram, S., *A New Kind of Science*, (Wolfram Media Inc., Champaign, IL, USA, 2002) pages 54–6, 909.

3. Viewed from the perspective of mathematical group theory the nesting of one harmonic series within another is an automorphism of the fundamental series and the set of all possible nestings of the harmonic series within itself might be construed as an automorphism group.

4. Grout, D.J., *A History of Western Music*, (J.M. Dent, London, 1960) page 101.

5. Morris R.O., *Contrapuntal Technique*, (OUP, 1922) page 27; Apel W, *Harvard Dictionary of Music*, (Harvard University Press, Cambridge, Massachusetts, 1966) page 439.

6. Sethares, W.A., *Tuning, Timbre, Spectrum, Scale*, (Springer, London, 2005) Chapter 2 – What Is a Spectrum, Fig. 2.5.