6

The Full Cadence

THE ALGORITHM OF SYMMETRICAL EXCHANGE

Music, like the material world in which it finds its existence, is a most complex and beautiful thing. Over the centuries, humankind has unceasingly drawn inspiration from the beauty of the natural world, while in equal measure, seeking to understand the complex mechanisms and myriad interrelationships which lie behind its elegant facade. This quest is more than idle curiosity, for as well as the natural human 'delight in knowing' that so characterises mankind; the chances of survival for any individual creature or species are greatly enhanced by a capacity to model their environment and thereby anticipate or even change future events. All creatures must absorb energy from their environment to maintain and replenish their highly ordered living state. This they principally do by eating, drawing in low entropy in the form of food. To know where and when, and how, to harvest and even increase nature's bounty of free energy, has been the foundation stone upon which human success is built; and perhaps also our downfall, in not finding the wisdom to recognise nature's limits. Throughout this continuing quest for knowledge, guided by both instinct and experience, human beings in general, and philosophers and scientists in particular, have sought simple underlying principles and categories as a means to describe and understand the vast and varied wonders of the great material world.

In this chapter the same strategy is applied to those 'little worlds' of tonal relationships we call musical compositions. That is, one single principle and one single category are proposed and developed, as the means to understanding the underlying source of structure and coherence in traditional music. The single principle is the *algorithm of symmetrical exchange*, for which the musicians' term modulation has been appropriated: the *Modulation Algorithm*. And the single category is that of the *Harmonic Series*: the *natural modes of vibration* for an ideal elastic material body. By combining the two, the modulation algorithm and the harmonic series, a dynamical model of tonally organised music can be constructed: a model of a *Modulating Oscillatory System* – which being a bit of mouthful, is most often abbreviated to MOS. Now, to introduce the central MOS concept, or rather mechanism; to provide a real example, all that is required is to examine in some detail a full or perfect cadence – that most basic progression of tonal music, which closes almost every piece. Essentially in this chapter, the two chords of the full cadence are being treated as a tonal composition in their own right, an isolated (minimal) little world of just one relationship. Although pieces of music don't come much smaller than this, only two chords are needed to illustrate the mechanism by which change is achieved in these wholly relational modulating oscillatory systems: Just as a wheel would normally turn myriad times in the making of a journey, but only one revolution is necessary to illustrate the principle of the wheel itself.

The Full Cadence

Perhaps the most familiar and recognisable of chord progressions is that of a *full cadence* (V^7– I) where the dominant or dominant-seventh chord is succeeded by the tonic at the end of a phrase, section or piece. In the key of C major the chord progression consists of a G-major chord (with or without a minor-seventh) resolving to a C-major chord. Some other names are also applied to this cadential formula – prefect, authentic or final cadence. Almost every piece of tonal music, from the humblest nursery rhyme to the mightiest symphony, closes with some form of this progression: and also the majority of phrases within them as well. The chord progression also plays a crucial role in establishing a change of key, through the creation of a full cadence onto a new tonic chord. For example, in the changing of key center from C major to G major, the full cadence chord progression D-seventh to G-major establishes the new tonal center – but that is another story which we shall not pursue at present.

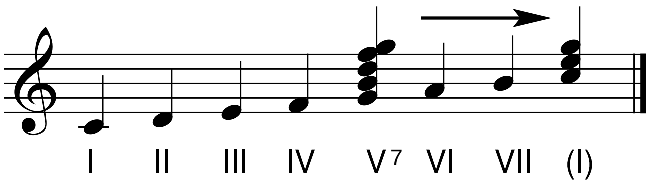

Figure 6.1 A dominant-seventh chord on the fifth degree of the C-major scale resolving to the common major chord on the first degree (tonic), a full or perfect cadence. Notice that in this configuration both chords share a common top note (G) which 'smoothes' the transition from one chord to the other. The scale degrees C, D, E, F, G, A, B, [C] are marked by Roman Numerals I through VII.

For a piece of music in the key of C major, this closing cadence would consist of two chords: a penultimate chord built on the 'root' note G – often with an added minor-seventh interval creating a dominant-seventh chord – followed by the tonic triad, the C-major chord. The perfect or full close cadence is illustrated in Figure 6.2.

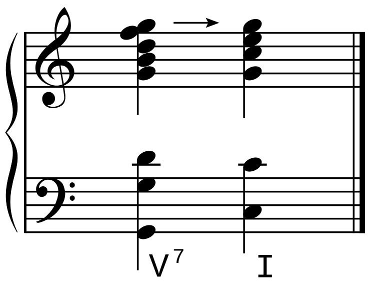

Figure 6.2 A full or perfect cadence. Notice that the lowest notes of the two chords, the root tones G and C, are separated by a perfect fourth. This interval makes the relationship between the note frequencies a ratio 3:4 (96Hz : 128Hz, at pitch level middle C = 256Hz).

Customarily the full cadence is illustrated and used with a configuration of a falling fifth between the bass root notes V and I. However, when viewed from the perspective of the modulating oscillatory systems model, as a structure in a pseudo-physical, this is not the cadence's essential form. The configuration shown in Figure 6.2 is the elemental form where entropy rises and complexity is lost from the system. The many other possible arrangements of notes within a full cadence are simply elaborations of greater or lesser extent of this core exchange, overseen by the Second Law of Thermodynamics.

Although the chords in Figure 6.2 form probably the most familiar of all chord progressions, it is worth playing them on a piano in the written configuration. The arrangement of notes has some important characteristics:

Firstly, the notes of both chords are in the configuration of harmonic series or overtones, that is, the bass or root note is in the relationship of fundamental tone to each chord's upper notes.

Secondly, both chords share the same top note – G – which connects or *conjoins* the chords, enabling a smooth transition to occur between the two chords.

Thirdly, the G-seventh chord in this progression contains eight notes while the C-major chord has only six notes, eight notes are exchanged for six – 8:6 notes, which reduces to a ratio of 4:3.

And fourthly, the frequency ratio between the two fundamental root tones (G to C, a perfect fourth) is 3:4, the exact reverse of the above ratio.

Thus, the perfect cadence chord progression illustrated in the particular arrangement given in Figure 6.2, represents a rather special transitional configuration between the two chords, in which they are joined by their common upper note and exchange a ratio of 4:3 tones of the harmonic series (eight for six notes) on fundamental roots which themselves have the reverse frequency relationship of 3:4. This is an example of the *algorithm of symmetrical exchange* at work – the *modulation* algorithm – a more complex rendering of the simple model of metrical change introduced in Chapter 4, pages 16–17. And, although this might appear rather a restrictive formula at first sight, it is capable of unlimited extension, in that *any whole numbers* could be substituted for the numbers three and four in the above example, e.g. 4:5, 8:9, 3:7, 13:23, etc.

A Wider Context

Returning to the arrangement of the notes within the two chords of the full cadence in Figure 6.2, as noted, these notes are not haphazardly placed; in fact, the notes in both chords describe harmonic series of their own: one series built on the note G-96Hz and one built on C-128Hz.

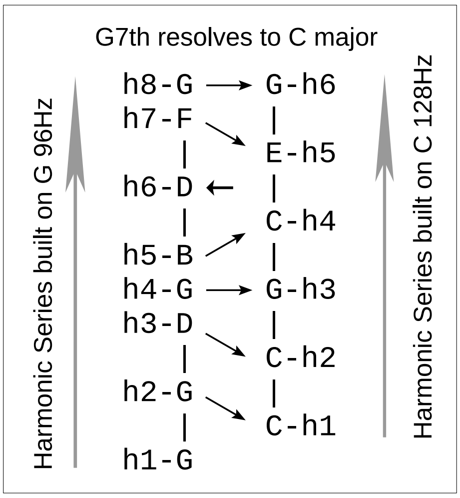

Figure 6.3 The two chords of the full cadence V^7– I (Figure 6.2) are in the configuration of harmonic series. Notice that the voice-leading in the upper section leaves no position in the C-major chord for D-h6 to fill; effectively D-h6 is ejected from the system (and as illustrated in Figure 6.5 G-h1 is left behind).

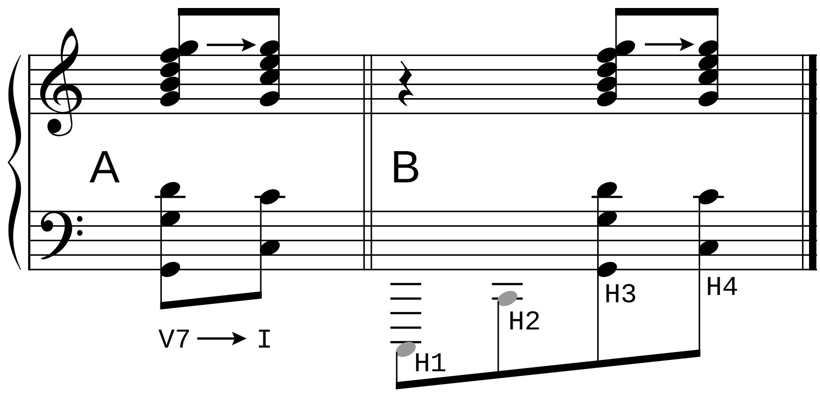

Figure 6.4 The wider context of nested harmonic series in the dominant-seventh to tonic chord progression, connected by the common ground of their upper notes, G.

Preceding the dominant and tonic chords in Figure 6.4B are two further notes labelled H1 and H2, and the notes that were labelled G-h1 and C-h1 in Figure 6.3 are now labelled H3 and H4! What has changed? The context has changed. In Figure 6.4B we are looking at the wider relationship between the two harmonic series taken together, which in Figures 6.3 and 6.4A are considered in isolation. These two series have a relationship by virtue of their common top notes (G-768Hz) and respective fundamental tones G-96Hz and C-128Hz which are connected by the proportion 3:4 – and this relationship has some interesting implications.

The notes labelled H1 and H2 in Figure 6.4B are implied by the full cadence, in that the root notes of the two chords, the bass notes G and C, have the frequency relationship of 3:4. Therefore, by actualising the step from G-96Hz to C-128Hz, the potential steps down to C-H2 and C-H1 are laid out as well. Through creating (or better computing) the relationship of 3:4 in the root tones of the full cadence chord progression, the value of the unit one is also being delineated. Thus the implications of the relationship of the objective dominant and tonic chords of a full cadence extend further than the notes themselves: describing other real or potential relationships, in particular the implied fundamental frequency C-H1 (32Hz) and more generally, all frequencies with whole number relationships to G-H3 and C-H4 – like for example G-H24, the top note in both chords. Indeed, what our full cadence has defined for us, is an *underlying harmonic series.* A harmonic series that must exist because of the common harmonic element(s) shared by both constituent series – the notes G-H12 (384Hz) and G-h24 (768Hz).

Now, viewed in this new light, the two isolated series of Figure 6.3 can be re-interpreted as harmonic series *nesting* within the ratios of a broader underlying series. What makes this re-interpretation possible is the relationship of 3:4. Any two isolated series with a whole number relationship between their fundamental tones, be it 4:5, 13:17 or whatever, will imply the existence of another underlying harmonic series which connects them together. In Figure 6.5 the isolated series formed of the chords of the full cadence are now illustrated dressed in the harmonic partials of their ‘parent’ underlying series. To distinguish between them, the underlying series is termed the *fundamental nesting series* – sometimes denoted with capital Hs as above – and the two formerly isolated ‘child’ series are termed *nested series* – marked as lower-case ‘hn’. There is, potentially, a great deal going on under the surface in tonally organised music, indeed the notes of the objective musical sound are just the tip of an iceberg of aural relationships.

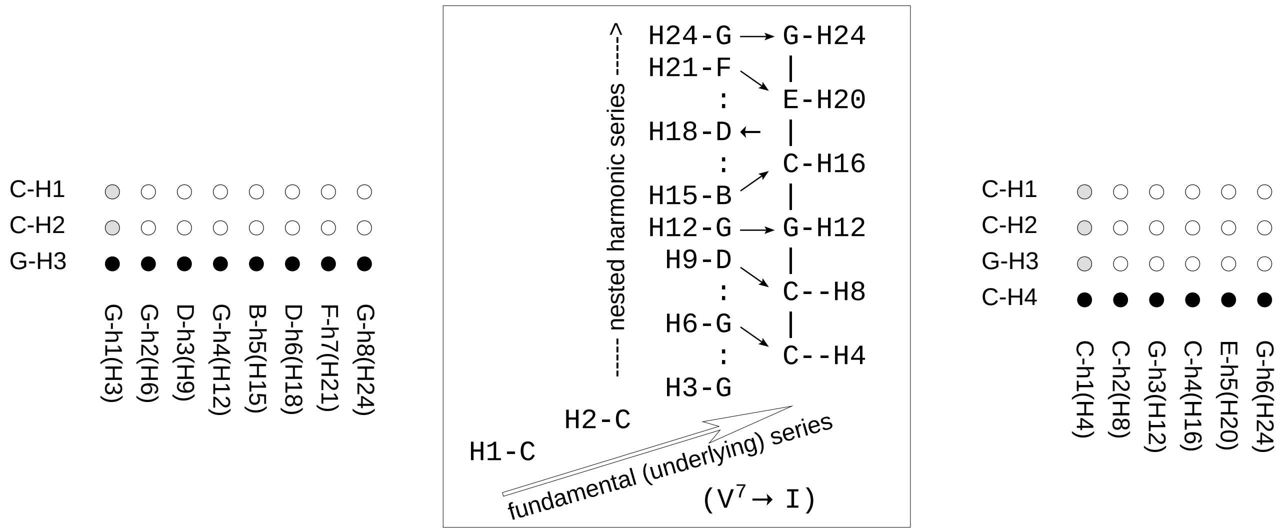

Figure 6.5 The wider context of Figure 6.3B, expressed as harmonic ratios of the fundamental series (using upper-case Hs) and note letters. Left and right are the two associated rectilinear number patterns of mutable twenty-four, the digit sequences MBN $8_3\,0_1$ and $6_4\,0_1$ respectively.

Nested Harmonic Series

The principle attribute of this network of underlying relationships is that one set of relationships nests within another, rather like Russian dolls. The complete harmonic series of eight and six notes/ratios derived from the objective chords of the full cadence, are in effect nested harmonic series set within the wider fundamental tones implied by their G-H3 to C-H4 step of a perfect fourth. That is to say no notes/ ratios will be found in the two chords (nested series) of eight and six notes respectively, that are not also present in the extrapolation of the fundamental nesting series built on C-H1. For example, the top note G is G-h8 of the ‘isolated’ dominant-seventh chord/series and G-h6 of the ‘isolated’ tonic chord/series, yet both of these are also G-H24 in the fundamental series.

Now with this model of the full cadence it becomes possible to understand and relate the chord progression – the succession of dominant and tonic – to two fixed points: the absolute fundamental C-H1 (32Hz) and the common ground of the conjunction between the two chords at G-H24 (768Hz). However, while the conjunction exists as an indisputable objective physical and aural fact, the absolute fundamental is implied by other structures and is of a rather more shadowy nature, perhaps even confined only to theory and the mathematics of the MOS model. Yet still, the ear and aural cognition by grasping the conjunction, effectively ‘know of’ the absolute fundamental implicitly, whether or not some mental process explicitly calculates its ‘value’. Ultimately, the conjunction alone is sufficient to construct the relationship between the two chords, to inform aural cognition and to articulate the MOS model.

What we have here is a way of viewing and describing the two most basic structural qualities of music’s harmonic core: identity and change – what notes are and how chords evolve. And this provides a mechanism for analysing the flow of tonally organised compositions, as the music steps from harmonic entity to entity. Indeed, with the MOS model the music is precisely numbers – mutable base digit sequences – forming the basis, in essence, for a new approach to harmonic theory. Of course this mechanism will not capture or describe all aspects of a composition, it has little to say of the emotional content for instance, but it does provide something solid and quantifiable to work with in terms of the basic parameters of frequency, wavelength, amplitude, energy and information. The name of this mechanism within the MOS model, the simple, yet subtle algorithm of symmetrical exchange between harmonic series, is *modulation* – used in its broader meaning of change or adjustment. It is a principle that may be extended to exchanges of discrete whole numbered units in any domain, and might perhaps yield a high-level approach to the understanding of periodic systems in general.

Algorithm of Symmetrical Exchange

To recap: a symmetrical relationship is evident in the chord progression of the full cadence illustrated in Figures 6.2 and 6.3. Eight notes of the dominant-seventh chord/series are exchanged for six notes of the common major tonic chord/series, a ratio of exchange of 4:3 (eight notes exchanged for six notes). This ratio of exchanged tones is mirrored, in reverse, by the frequency relationship of the fundamental root tones of the two chords/series (3:4) – an interval of a perfect fourth. Eight notes or ratios of the harmonic series built on the fundamental tone G-96Hz are exchanged for six notes or ratios built on the fundamental tone C-128Hz, with the frequency relationship between the fundamental tones G and C being that of 3:4. This is a most symmetrical chord progression – 4:3 (8:6) notes are exchanged on fundamental tones with a frequency relationship of 3:4!

Why this symmetry? The answer lies in the necessity of having common ground between the two chords/series for the exchange to take place at all. Without the shared top note G-H24 (768Hz), a *conjunction* in MOS terminology, the transition between the complex wave patterns generated by the two chords would be jerky, rough and disjointed (more on this aspect shortly under the metrical heading). In addition to which, the conjunction between the two chords of the full cadence, forces the exchange to consist of a relatively small number of notes/ratios, so as to produce fundamental tones in agreement with the degrees of the scale (and commensurable to the ear and aural cognition). Without the common ground of the top note G between the two chords/series such an exchange or chord progression would sound unnatural or forced.

For example, the chord progression F#-seventh to C-major has no common ground, no common note for a considerable distance above their respective fundamental tones. The first conjunction F#h16 and F#h11 respectively – here we are dealing with true harmonic ratios not tempered intervals – yields an awkward and complex exchange of ratio 16:11 – a diminished fifth. Though such an incommensurable exchange remains theoretically possible and no trouble for the algorithm of symmetrical exchange to encompass, a chord of fifteen unique intervals (true harmonic partials) is well beyond the ear's grasp. Indeed, the chords lying beyond eight 'note-partials' of the harmonic series (i.e. ninth and tenth chords) already prove difficult enough for the ear to interpret.

Also, to anticipate a question; in many natural-sounding chord progressions founded on simple ratios, the common ground may not actually be visible in the written notes of the score, as they are in the full cadence example. However, providing the conjunction is there in aural fact, care of the low order harmonics of timbre (or perhaps even via combination tones manufactured by the asymmetry of the ear) the transactions of the algorithm of symmetrical exchange are equally valid. Indeed over the past 300 years and more, instrument makers have been struggling to enhance and refine the tone quality of most instruments, intuitively shaping their harmonic spectra to conform to the whole number relationships of an ideal oscillatory body. Though our attention is naturally focused on the written notes of a composition, our ears are open to a far wider frequency range, which the processes of cognition work upon 'in the background', effortlessly yielding information about timbre, direction, reverberation, etc. The flow of tonally organised music, in a natural and convincing manner, depends upon the chords (extrapolated to nested harmonic series) providing the ear with suitable *conjunctions* between the harmonic steps in a tonal composition – *either in the written notes, but more often, amongst their harmonic partials*: The unwritten ‘notes’ of the music’s harmonic spectra to which the ear and aural cognition is so finely attuned. Thus the conjunction, G-H24, in the top notes in the two chords of the full cadence example we have examined, may be generalised over the range of chord progressions developed during the tonal era in western music.

Information and Energy

Viewed in terms of information content, the dominant chord in Figures 6.5 consists of eight *states* of the harmonic series built on the fundamental note G-H3, which is exchanged for the six *states* of the tonic built on C-H4 – by virtue of the underlying relationship of the fundamental series which links them together. It is the whole system from C-H1 to G-H24 which makes the exchange feasible; but what also makes it desirable, is that such an exchange will, in principle at least, release energy and complexity.

It is an unfailing attribute of the material world (encapsulated in the second law of thermodynamics) that all systems will release energy and order, where possible, so as to tend toward an equilibrium state. Manifestations of this all-pervading trend are as diverse as coffee growing cold over time and messages becoming scrambled in transmission: overall order within a system tends to decrease, to decay toward equilibrium with its wider environment. The dominant-seventh chord of eight notes possesses a given level of order and information; it is a more complex entity than the six notes of the common C-major chord. The all-pervading Second Law operates on the relatively complex arrangement of eight notes of the dominant-seventh chord/series seeking to reduce it to a simpler (and less energetic) state. The algorithm of symmetrical exchange – *modulation* – provides a mechanism by which this transformation can take place. Though, of course, this description is of the system ‘in principle’, music is not itself self-organizing: it is the composer and performer that ultimately choose at which point the stress of the dominant chord will relax to the tonic.

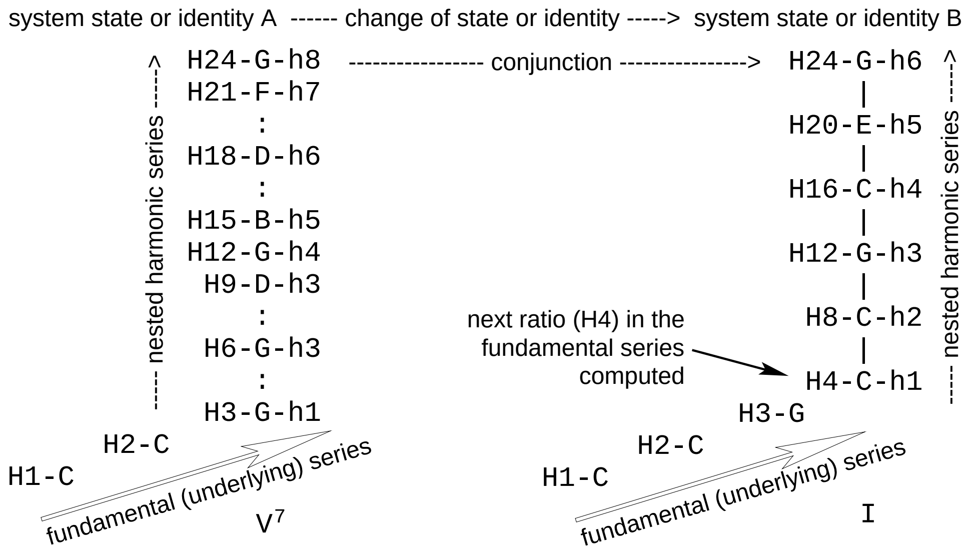

Figure 6.6 The full cadence chord progression can be viewed, in principle, as the computation of consecutive states in the structural evolution of an oscillatory system toward equilibrium.

In performance the dynamics of a composition are in constant flux: crescendo, diminuendo, sforzando, etc. Energy levels in ‘real life’ music making vary from moment to moment, both with changes in frequency (notes) and in amplitude (volume) of the oscillatory sources. However, while it would be possible (though rather complicated) to include this aspect of tonal music in the analysis presented in these documents, for the sake of clarity, energy levels are generally take to be uniform and ascending in an ordered way with increased frequency. Thus, if for argument's sake it is assumed that each note or oscillator within the system is of equal unitary amplitude (the energy of an oscillator is proportional to the square of its amplitude), it is clear from Figures 6.5/6 that, in principle, the energy embodied in the dominant chord/series is greater than that in the tonic. Indeed, as illustrated by the arrows and heard in the voice leading, the tritone B-H15 to F-H21 resolves to the major-third C-H16 to E-H20, squeezing out the note D-H18 in a symmetrical pincer movement which neither gains nor loses ratio-units in total (i.e. H15 + H21 = H16 + H20). However, the energy of oscillator D-H18 is lost, in principle, by the system. In contrast to this, in the lower section of the dominant chord/series (G-H6, D-H9 and G-H12) all notes have equal or lower energy slots to fill in the new tonic chord/series. The lower section reduces its energy by three ratio-units, while the upper section liberates eighteen ratio-units.

Overall, what the modulation mechanism of symmetrical exchange has achieved, in principle, through the resolution of dominant-seventh to tonic chord, is the handing-down of one note/ratio/ oscillator from the nested level to the fundamental level of the system, 'paying' for this work to be done by liberating the energy embodied in twenty-one (3+18) ratio-units to the wider environment. The process could be likened to a (physical) computation: the system has calculated/created, through the full cadence chord progression, the value of the next frequency in the fundamental nesting series: C-H4, Figures 6.5/6.

In playing and listening to music, perhaps the human mind, through the processes of aural cognition, is able to draw upon this low entropy, tonally organised, musical sound and garner a little of the relational dividend released through such exchanges. Indeed, as the typical musical phrase normally computes a passage from greater to lesser complexity, the entropy yield released by the operation of such a mechanism, provides nourishment for the mind – in an analogous way to the acquisition of energy and maintenance of low entropy that occurs through the human metabolism's absorption of food.

Changing Key (Tonal Center)

Now widening the focus of discussion from the minimal two-chord tonal 'composition' of dominant and tonic considered above, so as to take in some broader aspects of the algorithm of symmetrical exchange, shall examine changes of key and meter.

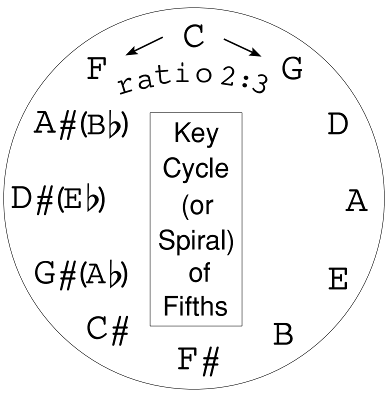

Figure 6.7 Key cycle – Twelve keys or tonal centers, each related to its clockwise neighbor by an upward interval of a fifth, the ratio 2:3, and to its anticlockwise neighbor by a downward interval of a fifth, the ratio 3:2.

In Chapter 2 a method of producing the notes of the western musical scale by squeezing twelve intervals of a fifth into seven octaves (with the 'fix' of tempered fifths of ratio 1:1.4983) was described. Nesting these twelve note scales within twelve key centers produces a closed cyclic system of tonal areas

– the cycle of fifths. (While using true fifths of ratio 1:1.5 results in a slight mismatch, the cycle doesn't quite close, producing an open spiral of key centers.) The mechanism for creating this structure is, essentially, the *modulation algorithm*. The 'measuring rod' of the twelve note scale, is being marked out using modulation 'dividers' of fifths and octaves. From a common starting frequency, seven dupla[1] modulations of 1:2 (octaves) are being drawn against twelve sesquialtera[1] modulations of 2:3 (fifths), so as to reach the point where the steps of the two dividers come back together again (almost). Once made, this measuring rod can be used to draw all sorts of extended musical shapes and patterns – melody, harmony, songs and sonatas; all manner of music. By moving through these structures sequentially, that is by playing and listening to music, we experience the process of modulation, viscerally, in its many layers... as exchanges of chords, meters and keys, and nowhere more clearly than where the reference frequency of key (tonal center) changes.

For the most part in *Journey to the Heart of Music* the term modulation is used 'generically', that is to denote the algorithm of symmetrical exchange applied at any and all frequency levels. However, in music, changing key or tonal center (changing to a scale built on a different note) is the normally accepted meaning of the term modulation. In this restricted usage, modulation most often involves moving to the key immediately above (dominant) or below (subdominant) the present key in the cycle of fifths (the tonic key). These three keys are closely related sharing many notes in common. By moving one step up or down the key cycle, the new key will have the frequency relationship of 2:3 (interval of a fifth, C to G) for upward movement or 3:2 (fifth, C to F) for a downward step. And here lies a puzzle, in that although the relationship between adjoining keys is 2:3, the actual chord progression generally used in music to step from one to the other is the sesquitertia[1] 3:4 relationship (dominant-seventh to tonic exchange), and not the obvious 2:3 move, the sesquialtera exchange. The explanation for this perhaps lies to some extent in the energy well underlying the eight ratios of a major-seventh chord (see Figure 1.9), which perhaps give the chord sufficient motive power to escape the tonal pull of the old key, as well as the reconfiguring of scale relationships inherent in the accidentals the chord introduces (i.e. Bflat or F# when the key of C changes to F or G, respectively). To step down the key cycle requires a seventh chord constructed on the previous tonic, I^7 in C, to become the dominant, V^7 of the new key of F: a straightforward release of stress – chords C-seventh to F-major. However, to move up the cycle from the home key of C is a more energetic task requiring a seventh chord constructed on the relationally more distant chord II (D, chord V in the dominant's key, G). Once the system has worked its way out to reach chord II^7, it can then very satisfyingly relax, via a sesquitertia 3:4 exchange, to chord V of the old key, which, from this vantage point has come to feel like chord I in the new key of G – which it now is.

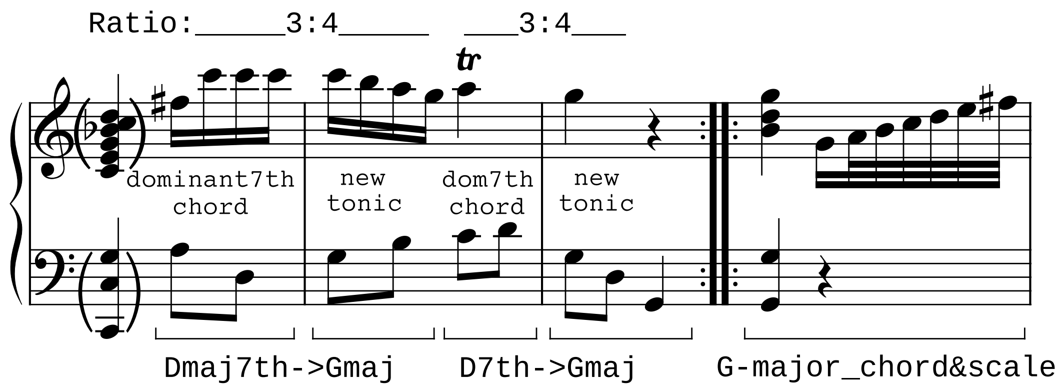

Figure 6.8 C-major to G-major modulation (2:3) – the 'escape' from C-major is achieved by two sesquitertia (3:4) exchanges, of D-seventh to G-major from within the key of Cmajor. (Haydn Piano Sonata No. 2 – Hob. XVI/7).

The movement illustrated in Figure 6.8 begins in the key of C-major, the home key, firmly established by the opening C-major G-major C-major chord sequence (see Figure 7.3). As the movement approaches the mid point, Haydn introduces the principal structural element, the relationship 2:3:2 between beginning, middle and end. However, the new tonal center of G-major, the arch suspended between two C-major pillars, is establish not by a straightforward 2:3 modulation but by twice moving out to the 'distant' (in terms of the key of C) chord of D-seventh (that is the dominant-seventh chord in terms of the key of G) and resolving this D-seventh chord onto a G-major chord. Thus the listener apprehends, probably unconsciously, the new tonal center or reference point from which to measure and relate the notes of the middle section, as well as placing them in relation to the overall structure of the movement. The effects of the modulation algorithm are generally more felt, like a 'force' arising from 'relational acceleration and inertia', rather than being something primarily apprehended by the intellect. Perhaps this feature partially accounts for tonal music's almost universal power and appeal?

By modulating to a second 'coordinate system' (the key of G) and then moving back to the home coordinates of C-major the system has created extra 'relational space' for itself. Haydn, through the process of modulation, has extended the range and variety of relationships that can exist within the 'little world' of this sonata movement. Interestingly, the mechanism by which the tonal area of the composition has been extended is that of a full cadence into the new key, and, the relationship between the new and the original tonal areas is that of the full cadence writ larger (i.e. middle section key of G, first and final sections key of C) – a nesting of the full cadence relationship within itself.

The Modulation Algorithm in Graph Format

It may also be helpful to visualise the process of *modulation* as two graphs (G-seventh above C-major) each consisting of the interference pattern of their respective whole systems – the complete set of ratios in state A and state B illustrated in Figure 6.6.

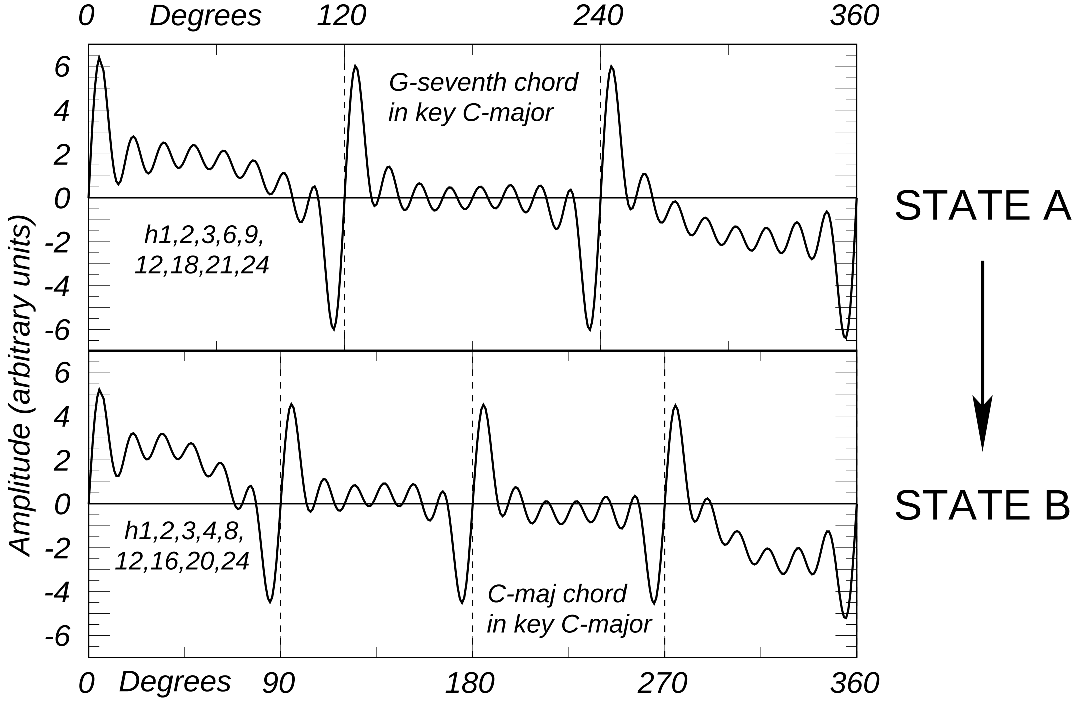

Figure 6.9 Graphs of the 'whole system' interference patterns of the dominant-seventh to tonic progression. Each note/oscillator has been given equal amplitude and uniform phase; all notes/oscillators of each respective system have been included.

Each complete system produces twenty-four fluctuations arranged as groups of eight and six small (nested) fluctuations, delineated by three and four larger (fundamental) fluctuations. The shared twenty-four fluctuations in total, being the common ground or conjunction of G-H24 (768Hz) and the overall period of the graphs, the common absolute fundamental frequency C-H1 (32Hz). In Figure 6.9, it can be seen that these two fixed features endure but what changes is the internal arrangement of strong and weak fluctuations – the internal metrical structure of the system is rearranged in the dominant-seventh to tonic full cadence.

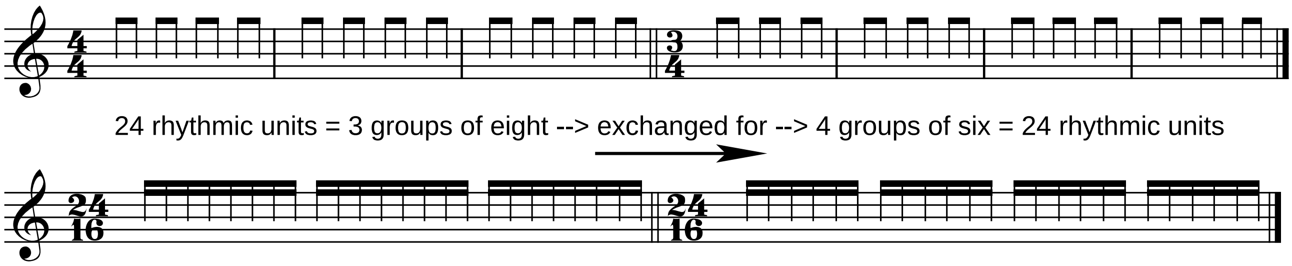

Figure 6.10 Two metrical reductions of the graphs in Figure 6.9 of the dominant-seventh to tonic chord progression.

These two graphs essentially provide a metrical description of the process of *modulation.* That is, the action of the algorithm of symmetrical exchange upon a physical system of whole number relationships, so as to compute one step of its structural evolution. Though perhaps at first unexpected, the aptness of a metrical interpretation of harmonic progressions only goes to highlight an underlying unity between the temporal and harmonic domains of tonal music. Whether the scale of measurement is metronome ticks per minute (MM) or cycles per second (Hz), the underlying principles which govern these two domains remain consistent.

Metrical Examples

J.S. Bach as usual provides a definitive example – in a courante from the English Suites discussed in the Introduction – where each bar mirrors the two metrical states expressed in the double graph of the full cadence in Figure 6.9 (i.e. 3×2^n for 3/2 time and $2^n \times 6$ for 6/4 time). In Figure 6.11 extra time signatures have been added in illustration. The dance maintains its metrical instability, the two voices oscillating between alternative energy levels (meters) until, approaching the cadences, they both relax to the lower state of 3/2.

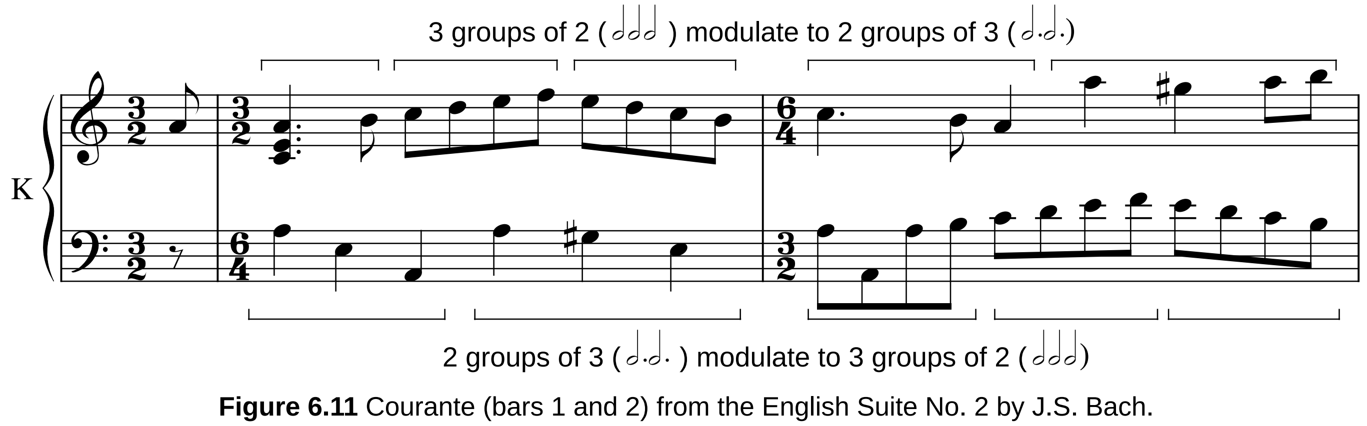

Figure 6.11 Courante (bars 1 and 2) from the English Suite No. 2 by J.S. Bach.

Where the sesquialtera 2:3 modulation exchange occurs in music, in metrical form, it is often given the name *hemiola*. Usually it's found approaching a cadence, where its function is analogous, in principle, to that of the reverse thrust used by jets after touchdown, that is, to give up energy to the surrounding environment so as to help bring the system to a halt. Figure 6.12 provides an example by Corelli.

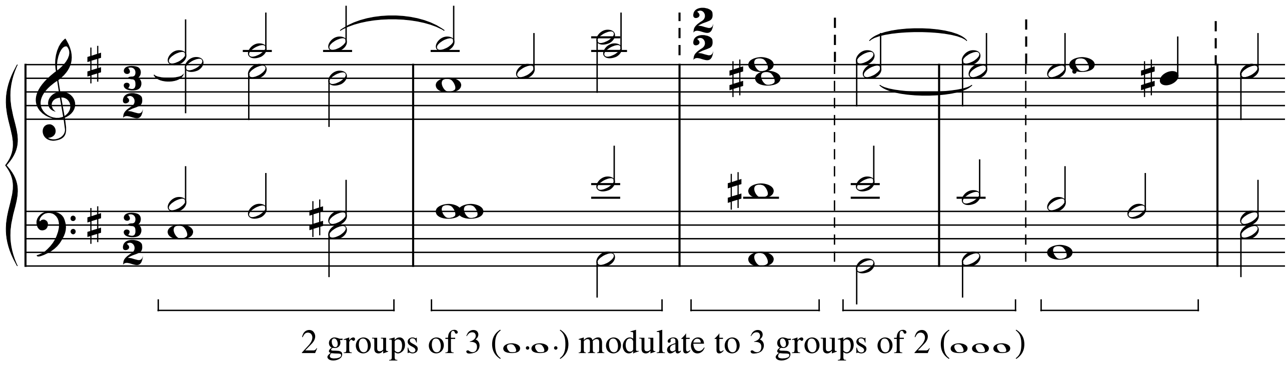

Figure 6.12 Hemiola at cadence (Adagio bars 29–33): Trio sonata Op. 3 No.7 by Arcangelo Corelli (1653–1713)

In the first two bars of Figure 6.12 the system could be described metrically in terms of the mutable number MBN $3_2 0_1$, that is, two measures each nesting three halfnote pulses. As the cadence approaches (the last chord in the example) the music breaks into three dotted-line measures of two halfnote pulses, that is 2/2 time – MBN $2_3 0_1$. Such ‘metrical modulations’ are most often not barred in the score, simply understood by musicians in practice. The process is familiar to drivers – approaching a turn, they change down to a gear with more cogs, to give the car’s motion more inertia to ‘push against’, similarly, the hemiola above is providing more metrical units (cogs) to resist the music’s forward motion and so better articulate a ritardando.

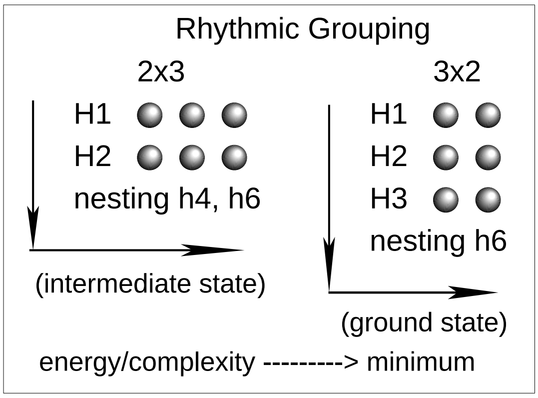

Figure 6.13 The changes of rhythmic and metrical grouping in Figures 6.11 and 6.12 can be expressed as number patterns as well as the mutable numbers: MBN $3_2\, 0_1$ <--> MBN $2_3\, 0_1$. (See also the simple model, Figure 4.14.)

BEYOND THE FULL CADENCE

To close this whistle-stop chapter introducing the modulation algorithm, in Figures 6.14 and 6.15, the exchange of dominant and tonic chords/series is extended, first to the ii–V^7–I cadential formula and then to a whole phrase, briefly illustrating the further extension of the principle of symmetrical exchange to multiple chord progressions. Interestingly the ii–V^7–I ‘dominant-of-the-dominant’ cadence shown in Figure 6.14 is constructed from the 3:4 modulation exchange nested within itself[2]. Also in Figure 6.15 notice that the phrase begins with the reversal of the dominant to tonic exchange (i.e. I–V). The algorithm of modulation works in both directions – forward, releasing energy and information from the system, and, in principle at least, in reverse absorbing energy and complexity from its surroundings in the process.

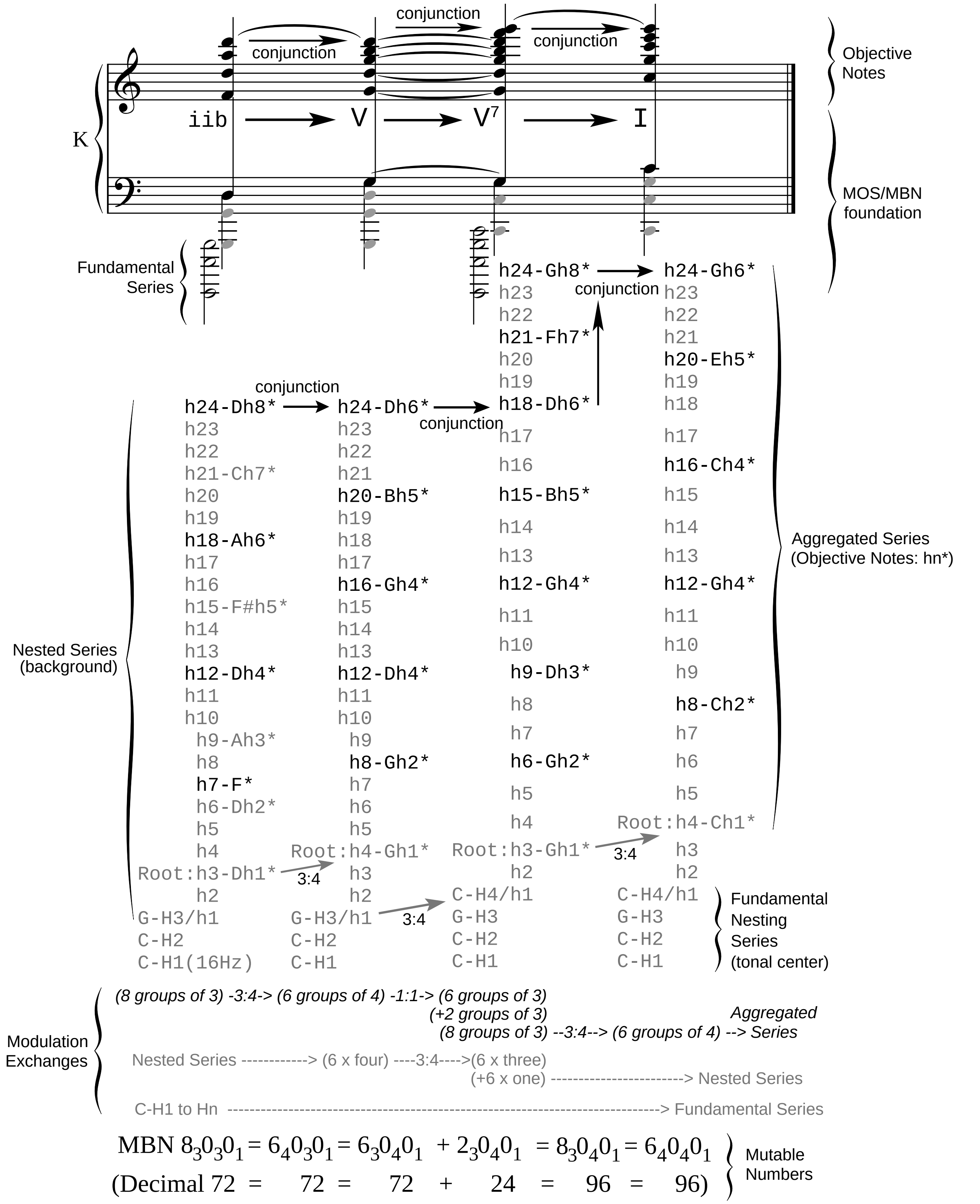

Figure 6.14 The familiar ii – V⁷ – I cadential formula in the key of C-major. The majority of the objective tones in the passage are held in the topmost aggregated series (black noteheads); however, one note, the low F of the first-inversion D-minor chord, falls outside this series and is scooped up in the net of the underlying nested series.

That a minority of notes cannot be accommodated within the surface layer of the aggregated[3] series, as is the case for F-h7 in Figure 6.14, should not be seen as a shortcoming but a positive necessity, encouraging the 'ear' to actualise the fainter background relationships. Though to what degree the processes of aural cognition penetrate beyond the surface level of the MOS model is a debatable question. However, when taken to the limit, where the ear encounters harmonies in which the objective notes fail to form any recognisable pattern or series in the upper aggregated layer – such as in some augmented and diminished chords – with these configurations the ear will often struggle to be able to divine a clear root and so all that is left to grasp at, is some vague sense of the underlying nested structure.

Conjunctions

```
:---------:---------:---------:---------:---------:---------:---------:---------:
:G768-h24 :G768-h32 :         :         :         :         :G780.2h32:G780.2h24:
:         :D576-h24 :D576-h16 :D576-h12 :         :F682.6h32:F682.6h28:         :
:---------:---------:         :         :C512-h16 :C512.0h24:---------:---------:
```

the Music (incomplete nested series)

```
 C512-h16 :         :---------:G384-h08 :G384-h12 :---------:         :C520.1h16:
:         :B480-h20 :         :---------:---------:         :B487.6h20:         :
:         :         :A432-h12 :         :         :A426.6h20:         :         :
:G384-h12 :G384-h16 :         :G384-h08 :G384-h12 :         :         :         :
:         :         :F#360h10 :         :         :         :         :         :
:E320-h10 :         :         :         :E320-h10 :         :         :E325.1h10:
:         :D288-h12 :         :D288-h06 :         :         :D292.6h12:         :
:         :         :C252-h07 :         :C256-h08 :C256.0h12:         :         :
:         :         :         :B240-h05 :         :         :         :         :
:         :         :         :         :         :A213.3h10:         :         :
:         :G192-h08 :         :         :         :         :         :G195.0h06:
:         :         :         :         :         :F170.6h08:F170.6h07:         :
:         :         :D144-h04 :         :         :         :D146.3h06:         :
:C128-h04 :         :         :         :C128-h04 :         :         :C130.0h04:
:         :         :         :G096-h02 :         :         :         :         :
```

Fundamental Tones

```
:---------:---------:---------:---------:---------:---------:---------:---------:
:         :         :         :G48Hzh1 3:2        :         :         :         :
:         :         :D36Hzh1 3:4        :         :         :         :         :
:C32Hzh1 4:3        :         :         :C32Hzh1 3:2        :         :C32.5Hzh1:
:         :G24Hzh1 2:3        :         :         :         :G24.4h1 3:4        :
:         :         :         :         :         :F21.3h1 7:8        :         :
:---------:---------:---------:---------:---------:---------:---------:---------:
```

Fundamental Series

```
:C16Hz H16:         :         :         :         :         :         :         :
:...
:C8Hz  H8 :         :         :         :         :         :         :         :
:...
:C4Hz  H4 :         :         :         :         :         :         :         :
:...
:C2Hz  H2 :         :         :         :         :         :         :         :
:---------:---------:---------:---------:---------:---------:---------:---------:
:C1Hz  H1 :C1.000Hz :C1.000Hz :C1.000Hz :C1.000Hz :C1.016Hz :C1.016Hz :C1.016Hz :
```

Figure 6.15 The opening phrase from a chorale set by J.S. Bach – *Ach Gott und Herr* – provides a simple illustration of a modulating oscillatory system, with 'the Music' expressed as parts of nested harmonic series modulating within the confines of 'the Key', a fundamental nesting series built on H1[1Hz]. Notes are shown with absolute frequencies and relative harmonic, e.g. the first melody note C512[Hz] is h16 of the nested fundamental tone C32Hz, which is itself H32 of the absolute fundamental tone C1Hz. In the last measure of the example the 'flexing' of relationships arising from the harmonic exchanges is accommodated by a slight change in the unit frequency. (A full MOS analysis of *Ach Gott und Herr* is given in Example T.)

Notes

1. It is useful to classify and refer to the proportions of modulation exchanges by their customary Latin names of sesquitertia 3:4 (i.e. three and one more), sesquialtera 2:3 and dupla 1:2 (see Figure 9.4 for other common ratios).

2. An alternative MOS reading of this cadence is given at the close of the Mozart example given in Chapter 12.

3. The term ‘aggregated series’ is introduced to clearly distinguish the upper layer of nesting in a system consisting of three levels of nested structure. The lowest layer in a system is termed the *fundamental series*, the middle layer the *nested series* and the top layer the *aggregated series*. These three levels of nested structure are illustrated and labelled in Figure 6.14.