

Journey to the Heart of Music

Philip Perry

Copyright © P.J.Perry 2003, 2006, 2009. All rights reserved.

The right of Philip J. Perry to be identified as the author of this work has been asserted by him in accordance with the UK Copyright, Designs and Patents Act, 1988.

Chapter 6 – The Full Cadence

THE ALGORITHM OF SYMMETRICAL EXCHANGE

Music, like the material world in which it finds its existence, is a most complex and beautiful thing. Over the centuries, humankind has unceasingly drawn inspiration from the beauty of the natural world; while in equal measure, seeking to understand the complex mechanisms and myriad interrelationships which lie behind its elegant facade. This quest is more than idle curiosity, for as well as the natural human ‘delight in knowing’ that so characterises mankind; the chances of survival for any individual creature or species are greatly enhanced by a capacity to model their environment and thereby anticipate or even change future events. All creatures must absorb energy from their environment to maintain and replenish their highly ordered living state. This they principally do by eating, drawing in low entropy in the form of food. To know where and when, and how, to harvest and even increase nature’s bounty of free energy, has been the foundation stone upon which human success is built, and perhaps also our downfall, in not finding the wisdom to recognise nature’s limits. Throughout this continuing quest for knowledge, guided by both instinct and experience, human beings in general, and philosophers and scientists in particular, have sought simple underlying principles and categories as a means to describe and understand the vast and varied wonders of the great material world.

In this chapter the same strategy is applied to those ‘little worlds’ of tonal relationships we call musical compositions. That is, one single principle and one single category are proposed and developed, as the means to understanding the underlying source of structure and coherence in traditional music. The single principle is the *algorithm of symmetrical exchange*, for which the musicians’ term modulation has been appropriated: the *Modulation Algorithm*. And the single category is that of the *Harmonic Series*: the *natural modes of vibration* for an ideal material body. By combining the two, the modulation algorithm and the harmonic series, a dynamical model of tonally organised music can be constructed: a model of a *Modulating Oscillatory System* – which being a bit of mouthful, is most often abbreviated to MOS. Now, to introduce the central MOS concept, or rather mechanism; to provide a real example, all that is required is to examine in some detail a full or perfect cadence: that most basic progression of tonal music, which closes almost every

piece. Essentially in this chapter, the two chords of the full cadence are being treated as a tonal composition in their own right, an isolated (minimal) little world of just one relationship. Although pieces of music don't come much smaller than this, only two chords are needed to illustrate the mechanism by which change is achieved in these wholly relational modulating oscillatory systems (MOS). Just as a wheel would normally turn myriad times in the making of a journey, but only one revolution is necessary to illustrate the principle of the wheel itself.

The Full Cadence

Perhaps the most familiar and recognisable of chord progressions is that of a *full cadence* (V^7-I) where the dominant or dominant-seventh chord is succeeded by the tonic at the end of a phrase, section or piece. In the key of C major the chord progression consists of a G-major chord (with or without a minor-seventh) resolving to a C-major chord. Some other names are also applied to this cadential formula – perfect, authentic or final cadence. Almost every piece of tonal music, from the humblest nursery rhyme to the mightiest symphony, closes with some form of this progression: and also the majority of phrases within them as well. The chord progression also plays a crucial role in establishing a change of key, through the creation a full cadence onto a new tonic chord –i.e. the chord progression D-seventh to G-major in the change of key (tonal center) from C major to G major – but that is another story which we shall not pursue at present.

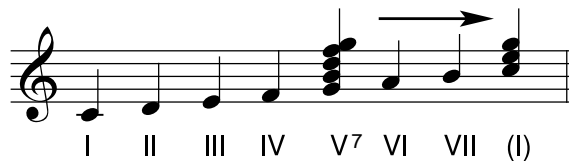


Figure 6.1 A dominant-seventh chord on the fifth degree of the C-major scale resolving to the common major chord on the first degree (tonic), a full or perfect cadence. Notice that in this configuration both chords share a common top note (G) which 'smoothes' the transition from one chord to the other. The scale notes C, D, E, F, G, A, B, [C] are marked by Roman Numerals I through VII.

For a piece of music in the key of C major, this closing cadence would consist of two chords: a penultimate chord built on the 'root' note G – often with an added minor-seventh interval creating a dominant-seventh chord – followed by the tonic triad, the C-major chord. The perfect or full close cadence is illustrated in Figure 6.2.



Figure 6.2 A full or perfect cadence. Notice that the lowest notes of the two chords, the root tones G and C, are separated by a perfect fourth. This interval makes the relationship between the note frequencies a ratio 3:4 (96Hz : 128Hz, at pitch level middle C = 256Hz).

6.3 - THE ALGORITHM OF SYMMETRICAL EXCHANGE

Although the chords in Figure 6.2 form probably the most familiar of all chord progressions, it is worth playing them on a piano in the written configuration. The arrangement of notes has some important characteristics:

Firstly, the notes of both chords are in the configuration of harmonic series or overtones, that is, the bass or root note is in the relationship of fundamental tone to each chord's upper notes.

Secondly, both chords share the same top note – G – which connects or *conjoins* the chords, enabling a smooth transition to occur between the two chords.

Thirdly, the G-seventh chord in this progression contains eight notes while the C-major chord has only six notes, eight notes are exchanged for six – 8:6 notes, which reduces to a ratio of 4:3.

And fourthly, the frequency ratio between the two fundamental root tones (G to C, a perfect fourth) is 3:4, the exact reverse of the above ratio.

Thus, the perfect cadence chord progression illustrated in the particular arrangement given in Figure 6.2, represents a rather special transitional configuration between the two chords, in which they are joined by their common upper note and exchange a ratio of 4:3 tones of the harmonic series (eight for six notes) on fundamental roots which themselves have the reverse frequency relationship of 3:4. This is an example of the *algorithm of symmetrical exchange* at work – the *modulation* algorithm – a more complex rendering of the simple model of metrical change introduced in Chapter 4, pages 16–18. And, although this might appear rather a restrictive formula at first sight, it is capable of unlimited extension, in that *any whole numbers* could be substituted for the numbers three and four in the above example, e.g. 4:5, 8:9, 3:7, 13:23, etc.

A Wider Context

Returning to the arrangement of the notes within the two chords of the full cadence in Figure 6.2, as noted, these notes are not haphazardly placed; in fact, the notes in both chords describe harmonic series of their own: one series built on the note G-96Hz and one built on C-128Hz.

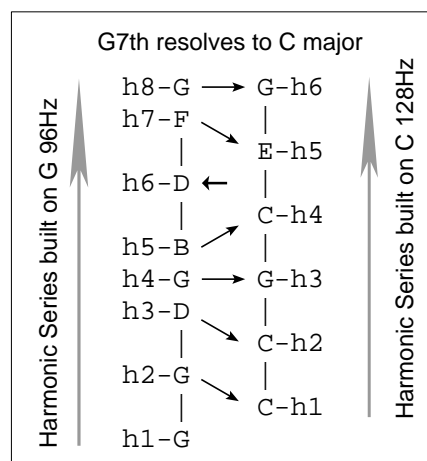


Figure 6.3 The two chords of the full cadence V7– I (Figure 6.2) are in the configuration of harmonic series. Notice that the voice-leading in the upper section leaves no position in the C-major chord for D-h6 to fill; effectively D-h6 is ejected from the system.

6.5 - THE ALGORITHM OF SYMMETRICAL EXCHANGE

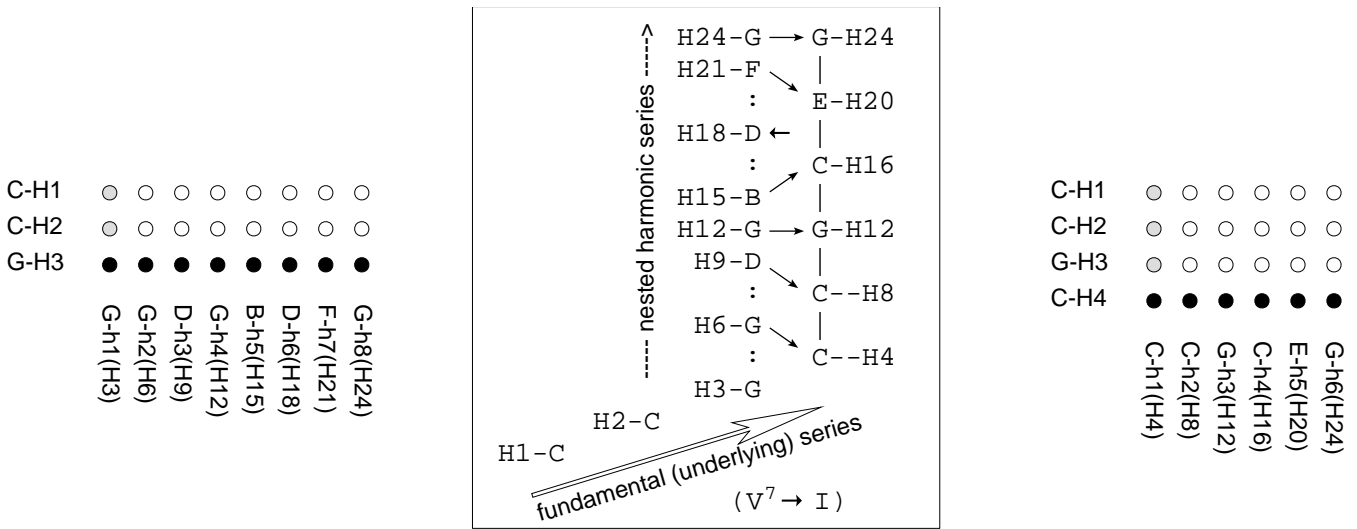


Figure 6.5 The wider context of Figure 6.3, expressed as harmonic ratios of the fundamental series (using upper-case Hs) and note letters. Left and right are the two associated rectilinear number patterns of mutable twenty-four, the digit sequences $MBN_3 0_1$ and $6_4 0_1$ respectively.

Nested Harmonic Series

The principle attribute of this network of underlying relationships is that one set of relationships nests within another, rather like Russian dolls. The complete harmonic series of eight and six notes/ratios derived from the objective chords of the full cadence, are in effect nested harmonic series set within the wider fundamental tones implied by their G-H3 to C-H4 step of a perfect fourth. That is to say no notes/ratios will be found in the two chords (nested series) of eight and six notes respectively, that are not also present in the extrapolation of the fundamental nesting series built on C-H1. For example, the top note G is G-h8 of the ‘isolated’ dominant-seventh chord/series and G-h6 of the ‘isolated’ tonic chord/series, yet both of these are also G-H24 in the fundamental series.

Now with this model of the full cadence it becomes possible to understand and relate the chord progression, the succession of dominant and tonic, to two fixed points: the absolute fundamental C-H1 (32Hz) and the common ground of the conjunction between the two chords at G-H24 (768Hz). What we have is a way of viewing and describing the two most basic structural qualities of music’s harmonic core (and perhaps other physical oscillatory systems as well): identity and change – what notes are and how notes evolve, a mechanism for analysing the flow of tonally organised compositions in precise numerical terms, as the music steps from harmonic entity to entity. Indeed, with the MOS model the music is precisely numbers – mutable base digit sequences – in essence a new harmonic theory for music. Of course this mechanism will not capture or describe all aspects of a composition – it has little to say of the emotional content for instance – but it does provide something solid and quantifiable to work with, in terms of the basic parameters of frequency, wavelength, amplitude, energy and information. The name of this mechanism, the simple, yet subtle algorithm, of symmetrical exchange between harmonic series, is *modulation* – used in its broader meaning of change or adjustment. The principle may be extended to exchanges of discrete whole numbered units, in all frequency domains, and might perhaps yield a general, high-level approach, to the understanding of periodic systems.

Algorithm of Symmetrical Exchange

To recap: a symmetrical relationship is evident in the chord progression of the full cadence illustrated in Figures 6.2 and 6.3. Eight notes of the dominant-seventh chord/series are exchanged for six notes of the common major tonic chord/series, a ratio of exchange of 4:3 (eight notes exchanged for six notes). This ratio of exchanged tones is mirrored, in reverse, by the frequency relationship of the fundamental root tones of the two chords/series (3:4) – an interval of a perfect fourth. Eight notes or ratios of the harmonic series built on fundamental tone G-96Hz are exchanged for six notes or ratios on fundamental tone C-128Hz, with the frequency relationship between the fundamental tones G and C being that of 3:4. This is a most symmetrical chord progression – 4:3 (8:6) notes are exchanged on fundamental tones with a frequency relationship of 3:4!

Why this symmetry? The answer lies in the necessity of having common ground between the two chords/series for the exchange to take place at all. Without the shared top note G-H24 (768Hz), a *conjunction* in MOS terminology, the transition between the complex wave patterns generated by the two chords would be jerky, rough and disjointed (more on this aspect shortly under the metrical heading). In addition to which, the conjunction between the two chords of the full cadence, forces the exchange to consist of a relatively small number of notes/ratios, so as to produce fundamental tones in agreement with the degrees of the scale (and commensurable to the ear and aural cognition). Without the common ground of the top note G between the two chords/series such an exchange or chord progression would sound unnatural or forced.

For example, the chord progression F#-seventh to C-major has no common ground, no common note, for a considerable distance above their respective fundamental tones (the first conjunction F#h16 and F#h11 respectively – here we are dealing with true harmonic ratios not tempered intervals) yielding an awkward and complex exchange ratio of 16:11 – a diminished fifth. Though such an incommensurable exchange remains theoretically possible and no trouble for the algorithm of symmetrical exchange to encompass, a chord of fifteen unique intervals (true harmonic partials) is probably well beyond the ear's grasp. Indeed, the chords lying beyond eight 'note-partial' of the harmonic series (i.e. ninth and tenth chords) are already difficult for the ear to interpret.

Also, to anticipate an question, in many natural-sounding chord progressions founded on simple ratios, the common ground may not actually be visible in the written notes, as they are in the full cadence example. However, providing the conjunction is 'there in spirit', care of the harmonics of timbre and combination tones manufactured by the asymmetry of the ear, the transactions are still valid. Indeed over the past 300 years and more, instrument makers have been struggling to enhance and refine the tone quality of most instruments, intuitively shaping their harmonic spectra to conform to the whole number relationships of an ideal oscillatory body. Though our attention is naturally focused on the written notes of a composition, our ears are open to a far wider frequency range, which the processes of cognition work upon 'in the background', effortlessly yielding information about timbre, direction, reverberation, etc. Thus more broadly, for tonally organised music to flow in a natural and convincing manner, the chords (extrapolated to nested harmonic series) must for the most part, find this common ground of *conjunctions* between the harmonic steps in a tonal composition – either in the written notes, or more often, amongst their harmonic partials. And so the necessary conjunction of the top notes of the two chords G-seventh and C-major, which forces the frequency relationship of the fundamentals to be 3:4, may be generalised over the range of chord progressions developed during the tonal era in western music.

Information and Energy

Viewed in terms of information content, the dominant chord in Figures 6.5 consists of eight *states* of the harmonic series built on the fundamental note G-H3, which is exchanged for the six *states* of the tonic built on C-H4 – by virtue of the underlying relationship of the fundamental series which links them together. It is the whole system from C-H1 to G-H24 which makes the exchange feasible; but what also makes it desirable, is that such an exchange will, in principle at least, release energy.

It is an unailing attribute of the material world (encapsulated in the second law of thermodynamics) that all systems will release energy and order, where possible, so as to tend toward an equilibrium state. Manifestations of this all-pervading trend are as diverse as coffee growing cold over time and messages becoming scrambled in transmission: overall order within a system tends to decrease, to decay toward equilibrium with its wider environment. The dominant-seventh chord of eight notes possesses a given level of order and information; it is a more complex entity than the six notes of the common C-major chord. The all-pervading Second Law operates on the relatively complex arrangement of eight notes of the dominant-seventh chord/series seeking to reduce it to a simpler (and less energetic) state. The algorithm of symmetrical exchange – *modulation* – provides a mechanism by which this transformation can take place. Though, of course, this description is of the system ‘in principle’, music is not itself self-organizing: it is the composer and performer that ultimately choose at which point the stress of the dominant chord will relax to the tonic.

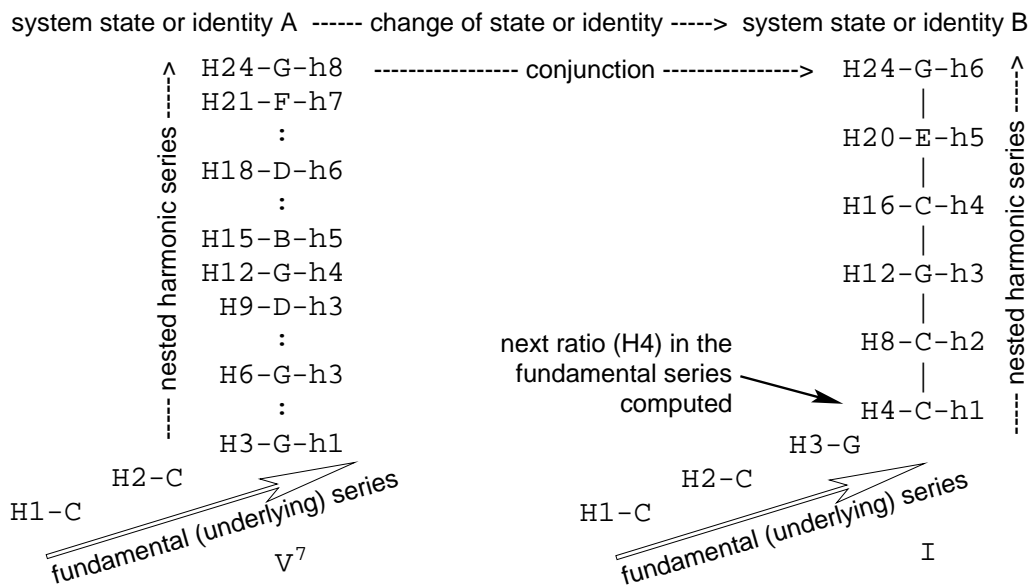


Figure 6.6 The full cadence chord progression can be viewed, in principle, as the computation of consecutive states in the structural evolution of an oscillatory system.

If for argument’s sake it is assumed that each note or oscillator within the system is of equal unitary amplitude (the energy of an oscillator is proportional to the square of its amplitude) it is clear from Figure 6.5 that, in principle, the energy embodied in the dominant chord/series is greater than that in the tonic. Indeed, illustrated by the arrows and heard in the voice leading, as the tritone B-H15 to F-H21 resolves to the major-third C-H16 to E-H20, it squeezes out the note D-H18 in a symmetrical pincer movement which neither gains nor loses ratio-units in total (i.e. 15+21 = 16 + 20). However, the energy of oscillator D-H18 is

lost, in principle, by the system. In contrast to this, in the lower section of the dominant chord/series (G-H6, D-H9 and G-H12) all notes have equal or lower energy slots to fill in the new tonic chord/series. The lower section reduces its energy by three ratio-units, while the upper section liberates eighteen ratio-units.

Overall, what the modulation mechanism of symmetrical exchange has achieved, in principle, through the resolution of dominant-seventh to tonic chord, is the handing-down of one note/ratio/oscillator from the nested level to the fundamental level of the system, 'paying' for this work to be done by liberating the energy embodied in twenty-one ratio-units to the wider environment. The process could be likened to a (physical) computation: the system has calculated, through the full cadence chord progression, the value of the next frequency in the fundamental nesting series: C-H4.

In playing and listening to music, perhaps the human mind, through the processes of aural cognition, is able to draw in this low entropy, tonally organised, musical sound and garner a little of the relational dividend released through modulation exchanges. Perhaps, as the typical musical phrase normally computes a passage from greater to lesser complexity, the entropy yield released by the operation of such a mechanism, provides nourishment for the mind – in a way similar to the absorption of energy and low entropy that occurs in the body, through eating.

The Algorithm in Graph Format

It may also be helpful to visualise the process of *modulation* as two graphs (G-seventh above C-major) each consisting of the interference pattern of their respective whole systems – the complete set of ratios in state A and state B of Figure 6.6.

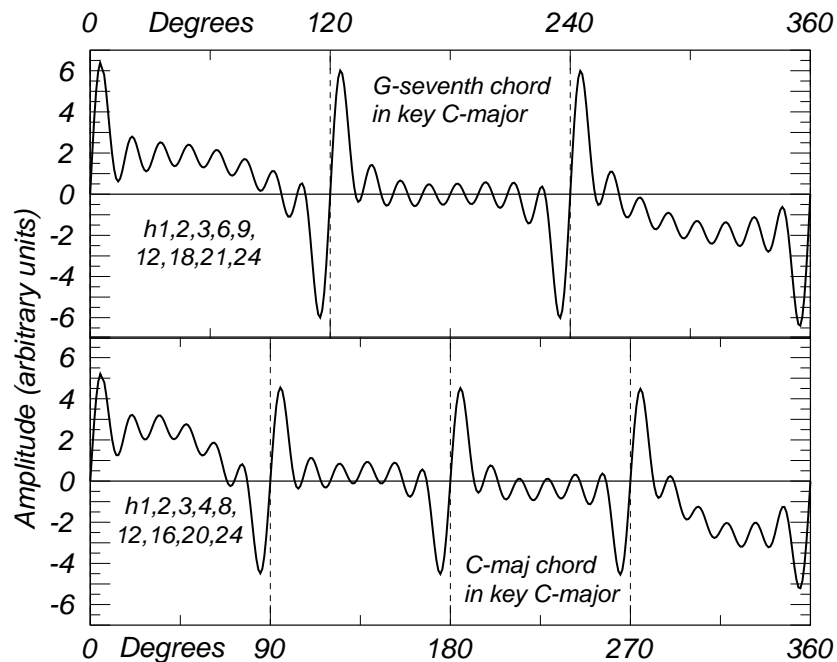


Figure 6.7 Graphs of the 'whole system' interference patterns of the dominant-seventh to tonic progression. Each note/oscillator has been given equal amplitude and unitary phase; all notes/oscillators of each respective system have been included.

Each complete system produces twenty-four fluctuations arranged as groups of small (nested) fluctuations, delineated by larger (fundamental) fluctuations. The shared twenty-four fluctuations in total, being the

6.9 - THE ALGORITHM OF SYMMETRICAL EXCHANGE

common ground or conjunction of G-H24 (768Hz) and the overall period of the graphs, the common absolute fundamental frequency C-H1 (32Hz). As can be seen in Figure 6.7, these two fixed features endure but what changes is the internal arrangement of strong and weak fluctuations – the internal metrical structure of the system is rearranged in the dominant to tonic full cadence.

These two graphs essentially provide a metrical description of the process of *modulation* – the action of the algorithm of symmetrical exchange upon a physical system of whole number relationships, so as to compute one step of structural evolution. Though perhaps at first unexpected, the aptness of a metrical interpretation of harmonic progressions only goes to highlight an underlying unity between the temporal and harmonic domains of tonal music. Whether the scale of measurement is metronome ticks per minute (MM) or cycles per second (Hz), the underlying principles which govern these two domains remain consistent.

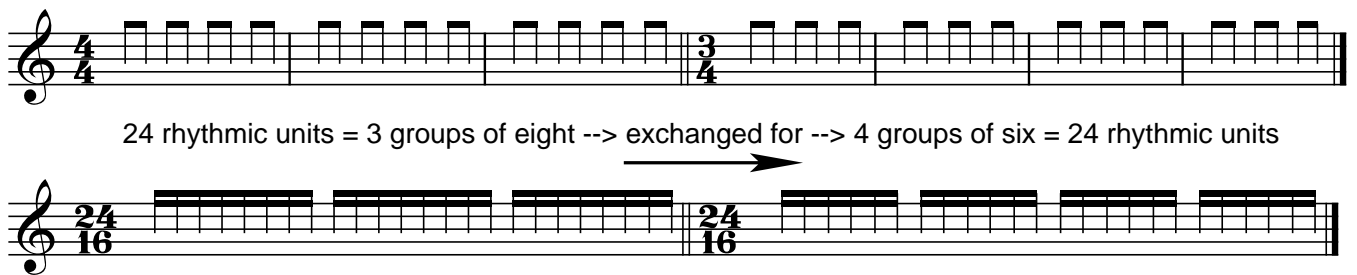


Figure 6.8 Two metrical reductions of the graphs in Figure 6.7 of the dominant-seventh to tonic chord progression.

Changing Key (Tonal Center)

Now widening the focus from the minimal two-chord tonal composition of dominant and tonic considered above, so as to take in some broader aspects of the algorithm of symmetrical exchange, that is, changes of key and meter. In Chapter 2 a method of producing the notes of the western musical scale by squeezing twelve intervals of a fifth into seven octaves (with the ‘fix’ of tempered fifths of ratio 1:1.4983) was described. Nesting these twelve note scales within twelve key centers produces a closed cyclic system of tonal areas – the cycle of fifths. (While using true fifths of ratio 1:1.5 results in a slight mismatch, the cycle doesn’t quite close, producing an open spiral of key centers.)

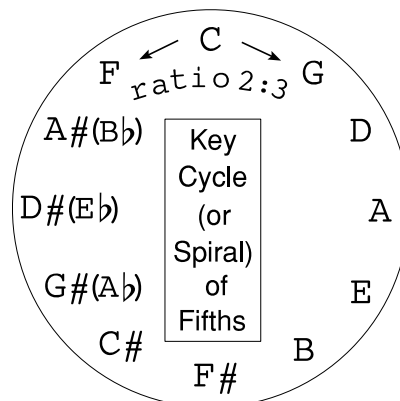


Figure 6.9 Key cycle – Twelve keys or tonal centers, each related to its clockwise neighbor by an upward interval of a fifth, the ratio 2:3, and to its anticlockwise neighbor by a downward interval of a fifth, the ratio 3:2.

6.10 - THE ALGORITHM OF SYMMETRICAL EXCHANGE

The mechanism for creating this structure is, essentially, the *modulation algorithm*. The ‘measuring rod’ of the twelve note scale, is being marked out using modulation ‘dividers’ of fifths and octaves. From a common starting frequency, seven dupla¹ modulations of 1:2 (octaves) are being drawn against twelve sesquialtera¹ modulations of 2:3 (fifths) – which marks the point where the steps of the two dividers come back together again (almost). Once made, the measuring rod can be used to draw all sorts of extended musical shapes and patterns – melody, harmony, songs and sonatas; all manner of music. By moving through these structures sequentially, that is by playing and listening to music, we experience the process of modulation, viscerally, in its many layers... as exchanges of chords, meters and keys, and nowhere more clearly than where the reference frequency of key (tonal center) changes.

For the most part in *Journey to the Heart of Music* the term modulation is used ‘generically’, that is to denote the algorithm of symmetrical exchange applied at any and all frequency levels. However, in music, changing key or tonal center (changing to a scale built on a different note) is the normally accepted meaning of the term modulation. In this restricted usage, modulation most often involves moving to the key immediately above (dominant) or below (subdominant) the present key (the tonic key) in the cycle of fifths. These three keys are closely related sharing many notes in common. By moving one step up or down the key cycle, the new key will have the frequency relationship of 2:3 (interval of a fifth, C to G) for upward movement or 3:2 (fifth, C to F) for a downward step. And here lies a puzzle, in that although the relationship between adjoining keys is 2:3, the actual chord progression generally used in music to step from one to the other is the sesquitertia¹ 3:4 relationship (dominant-seventh to tonic exchange), and not the obvious 2:3 move, the sesquialtera exchange. The explanation for this probably lies to some extent in the energy well underlying the eight ratios of a major-seventh chord (see Figure 1.9), which perhaps give the chord sufficient motive power to escape the tonal pull of the old key. Also, the reconfiguring of scale relationships inherent in the accidentals this chord introduces (i.e. Bflat or F# when the key of C changes to F or G, respectively). To step down the key cycle requires a seventh chord constructed on the previous tonic, I⁷ in C, to become the new V⁷ of the key of F: a straightforward release of stress – chords C-seventh -> F-major. However, to move up the cycle from the home key of C is a more energetic task requiring a seventh chord constructed on the relationally more distant chord II (D, chord V in the dominant’s key, G). Once the system has worked its way out to reach chord II⁷, it can then very satisfyingly relax, via a sesquitertia 3:4 exchange, to chord V of the old key, which, from this vantage point has come to feel like chord I in the new key of G – which it now is.

Ratio: _____ 3:4 _____ _____ 3:4 _____

dominant7th chord new tonic dom7th chord new tonic
 Dmaj7th->Gmaj D7th->Gmaj G-major_chord&scale
 KEY:C-major-----RatioChanges->2:3---G-major----->

Figure 6.10 C-major to G-major modulation (2:3) – the ‘escape’ from C-major is achieved by two sesquitertia (3:4) exchanges, of D-seventh to G-major from within the key of Cmajor. (Haydn Piano Sonata No. 2 – Hob. XVI/7).

The movement illustrated in Figure 6.10 begins in the key of C-major, the home key, firmly established by the opening C-major G-major C-major chord sequence (see Figure 7.3). As the movement approaches the mid point, Haydn introduces the principal structural element, the relationship 2:3:2 between beginning, middle and end. However, the new tonal center of G-major, the arch suspended between two C-major pillars, is established not by a straightforward 2:3 modulation but by twice moving out to the ‘distant’ (in terms of the key of C) chord of D-seventh (that is the dominant-seventh chord in terms of the key of G) and resolving this D-seventh chord onto a G-major chord. Thus the listener apprehends, probably unconsciously, the new tonal center or reference point from which to measure and relate the notes of the middle section, as well as placing them in relation to the overall structure of the movement. By modulating to a second ‘coordinate system’ and then moving back to the home coordinates of C-major the system has created extra ‘relational space’ for itself. Haydn, through the process of modulation, has extended the range and variety of relationships that can exist within the ‘little world’ of the sonata movement.

The effects of the modulation algorithm are more felt, like a ‘force’ arising from ‘relational acceleration and inertia’, rather than being something primarily apprehended by the intellect: Perhaps this feature partially accounts for tonal music’s almost universal power and appeal?

Metrical Examples

J.S. Bach as usual provides the definitive example – in a courante from the English Suites already alluded to, where each bar mirrors the two metrical states (6/4 & 3/2) of the double graph Figure 6.7. In Figure 6.11 extra time signatures have been added in illustration. The dance maintains its metrical instability, the two voices oscillating between alternative energy levels (meters) until, approaching the cadences, they both relax to the lower state of 3/2.

The musical score for the Courante (bars 1 and 2) from the English Suite No. 2 by J.S. Bach is shown. The score consists of two staves, treble and bass clef, with a grand staff bracket labeled 'K'. The time signatures are 3/2, 6/4, and 3/2. Annotations above and below the staves describe metrical groupings: '3 groups of 2 (d d d) modulate to 2 groups of 3 (d d d)' and '2 groups of 3 (d d d) modulate to 3 groups of 2 (d d d)'. The notes are: Treble: G4, A4, B4, C5, B4, A4, G4; Bass: G2, F2, E2, D2, C2, B1, A1.

Figure 6.11 Courante (bars 1 and 2) from the English Suite No. 2 by J.S. Bach.

Where the sesquialtera 2:3 modulation exchange occurs in music, in metrical form, it is often given the name *hemiola*. Usually it’s found approaching a cadence, where its function in principle is like that of the reverse thrust used by jets after touchdown, that is, to give up energy to the surrounding environment so as to help bring the system to a halt. Figure 6.12 is an example by Corelli.

6.12 - THE ALGORITHM OF SYMMETRICAL EXCHANGE

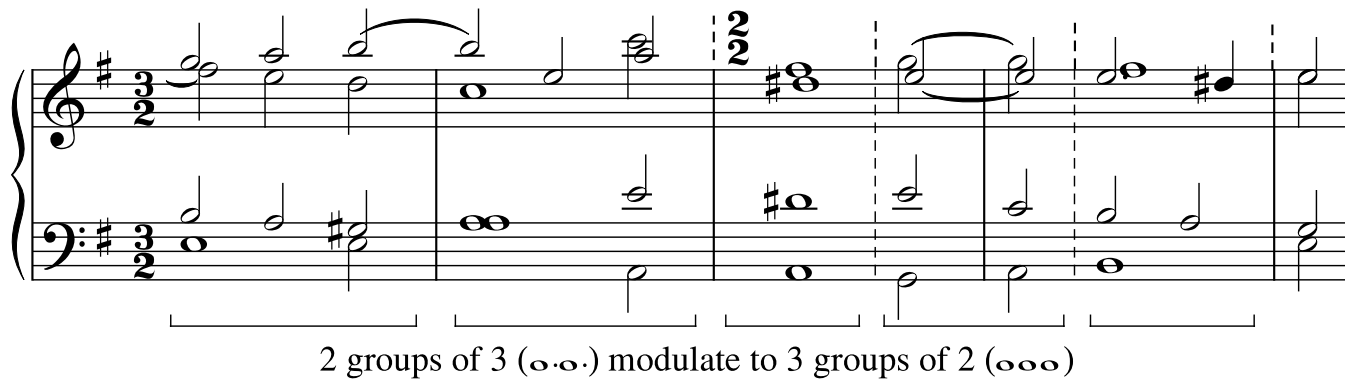


Figure 6.12 Hemiola at cadence (Adagio bars 29-33) – Trio sonata Op. 3 No.7 by Arcangelo Corelli (1653-1713)

In the first two bars of Figure 6.12 the system could be described metrically in terms of the mutable number MBN $3_2 0_1$, that is, two measures each nesting three halfnote beats. As the cadence approaches (the last chord in the example) the music breaks into three dotted-line measures of two halfnote beats, that is $2/2$ time – MBN $2_3 0_1$. Such ‘metrical modulations’ are most often not barred in the score, simply understood by musicians in practice. The process is familiar to drivers – approaching a turn, they change down to a gear with more cogs, to give the car’s motion more inertia to ‘push against’, similarly, the hemiola above is providing more metrical units (cogs) to resist the music’s forward motion and so better articulate a ritardando.

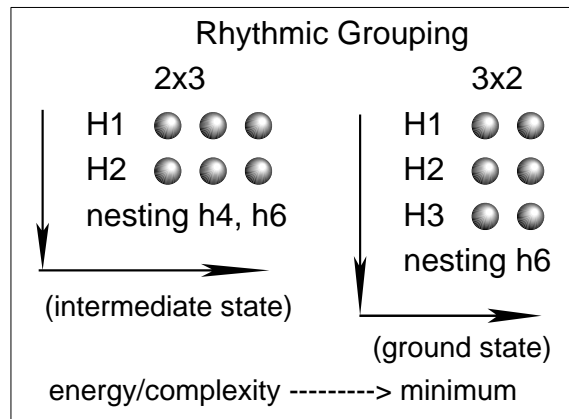


Figure 6.13 The changes of rhythmic and metrical grouping in Figures 6.11 and 6.12 can be expressed as number patterns as well as the mutable numbers: MBN $3_2 0_1$ <-> MBN $2_3 0_1$. (See also the simple model, Figure 4.14.)

To close this whistle-stop chapter introducing the modulation algorithm, below in Figures 6.14 and 6.15, the exchange of dominant and tonic chords/series is extended, first to the $ii \rightarrow V^7 \rightarrow I$ cadential formula and then to a whole phrase, briefly illustrating the further elaboration of the principle of symmetrical exchange to other chord progressions. Notice also in Figure 6.15 that the phrase begins with the reversal of the dominant to tonic exchange of the full cadence (i.e. $I \rightarrow V$): the algorithm of modulation works in both directions – forward, releasing energy and information from the system, and in reverse, in principle at least, absorbing energy and complexity from its surroundings in the process.

6.13 - THE ALGORITHM OF SYMMETRICAL EXCHANGE

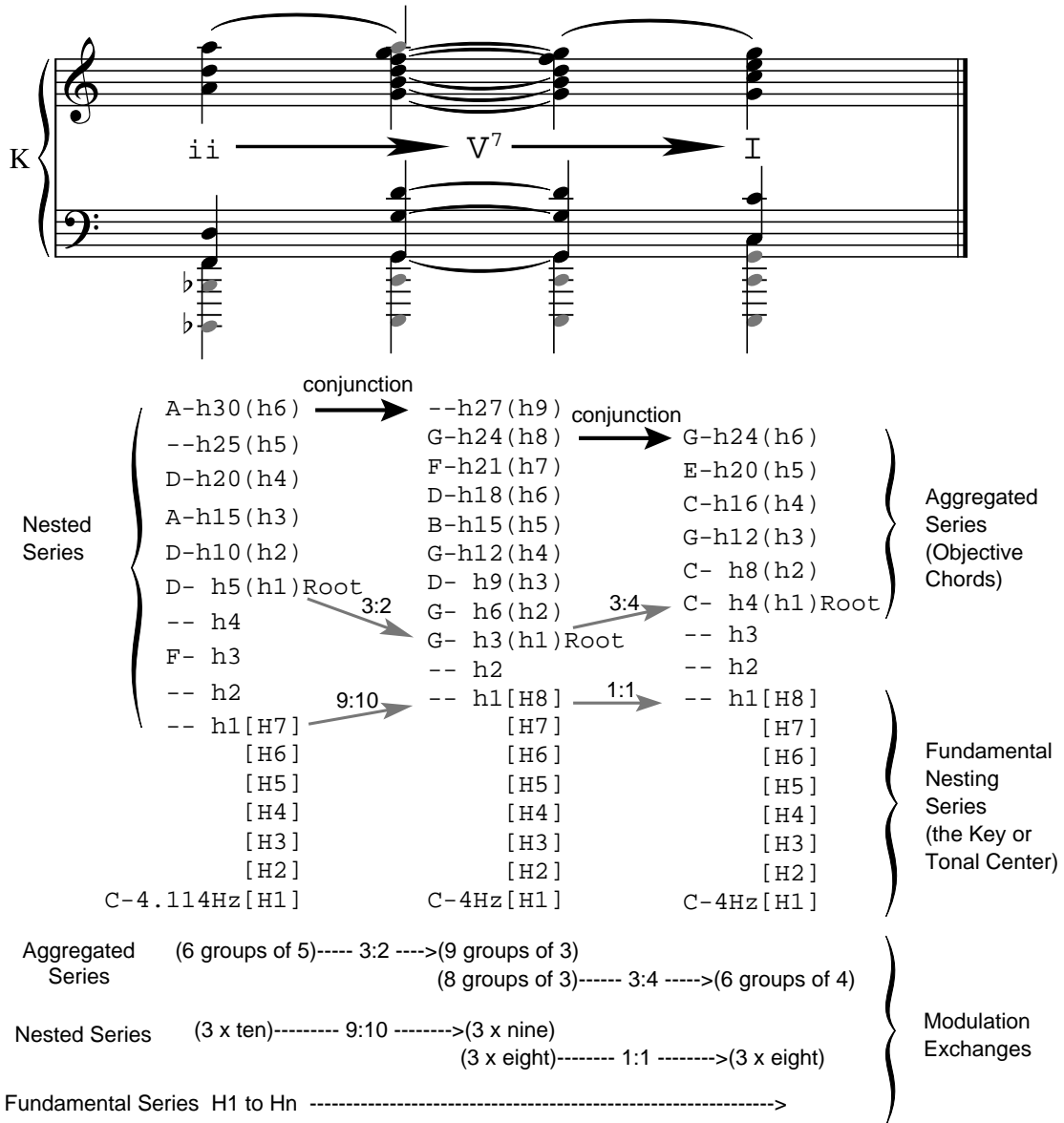


Figure 6.14 The familiar ii – V⁷ – I cadential formula in the key of C-major. The majority of the objective tones of the passage are held in the topmost aggregated series (black noteheads); however, one note, the low F of the first-inversion D-minor chord, falls outside this series and is scooped up in the net of the underlying nested series (beginning in gray notes). The fundamental nesting series is not shown on the staff.

That a minority of notes cannot be accommodated within the surface layer of the aggregated series, as in Figure 6.14, should not be seen as a shortcoming but a positive necessity, encouraging the ‘ear’ to actualise the fainter background nested relationships. Indeed, occasionally, the ear will encounter harmonies where the majority of objective notes fall outside the upper aggregated layer, such as in some augmented and diminished chords. With these configurations the ear will often fail to be able to distinguish a clear root. To what degree the processes of aural cognition penetrate beyond the nested level down to that of the fundamental/tonal center/key series probably depends in some degree on individual acuity, experience and training.

6.14 - THE ALGORITHM OF SYMMETRICAL EXCHANGE

Conjunctions	:-----: :-----: :-----: :-----: :-----: :-----: :-----: :-----:
	:G768-h24 → G768-h32 : : : : :G768-h32 → G768-h24 :
	: :D576-h24 :D576-h16 → D576-h12 :F672-h21 → F672-h32 :F672-h28 : :
	:-----: :-----: :-----: :-----: :-----: :-----: :-----: :-----:
	:C512-h16 : : : : : : :C512-h16 :
	: :B480-h20 : : : : :B480-h20 : :
	: : : :A432-h12 : : : :A416-h20 : : :
	:G384-h12 :G384-h16 : :G384-h08 :G384-h12 : : : :
	: : : :F#360h10 : : : : : :
	:E320-h10 : : : :E320-h10 : : : :E320-h10 :
	: : : :D288-h12 : : : :D288-h06 : : : :D288-h12 : :
	: : : :C252-h07 : : : :C256-h08 :C256-h12 : : :
: : : : : :B240-h05 : : : : : :	
: : : : : : : :A208-h10 : : : :	
: : : :G192-h08 : : : : : : :G192-h06 :	
: : : : : : : : : :F168-h08 :F168-h07 : :	
: : : :D144-h04 : : : : : : :D144-h06 : :	
:C128-h04 : : : :C128-h04 : : : :C128-h04 :	
: : : : : :G096-h02 : : : : :	


Fundamental Tones	: : : :G48Hzh1 3:2 : : : :
	: : : :D36Hzh1 3:4 : : : :
	:C32Hzh1 4:3 : : : :C32Hzh1 3:2 : : : :C32Hzh1 :
	: : : :G24Hzh1 2:3 : : : : : :G24Hzh1 3:4 :
	: : : : : : : :F21Hzh1 7:8 : : :
	:-----: :-----: :-----: :-----: :-----: :-----: :-----: :-----:
the Music (incomplete nested series)	
K	
Fundamental Series	:C16Hz H16
	:... :
	:C8Hz H8
	:... :
	:C4Hz H4
	:... :
	:C2Hz H2
:... :	
:C1Hz H1	

Figure 6.15 The opening phrase from a chorale set by J.S. Bach – *Ach Gott und Herr* – provides a simple illustration of a modulating oscillatory system, with ‘the Music’ expressed as parts of nested harmonic series modulating within the confines of ‘the Key’, a fundamental nesting series built on H1[1Hz]. Notes are shown with absolute frequencies and relative harmonic, e.g. the first melody note C512[Hz] is h16 of the nested fundamental tone C32Hz, which is itself H32 of the absolute fundamental tone C1Hz.

[8/08/09]

Note

1. It is useful to classify and refer to the proportions of modulation exchanges by their customary Latin names of sesquitertia 3:4 (i.e. three and one more), sesquialtera 2:3 and dupla 1:2 (see Figure 9.4 for other common ratios).