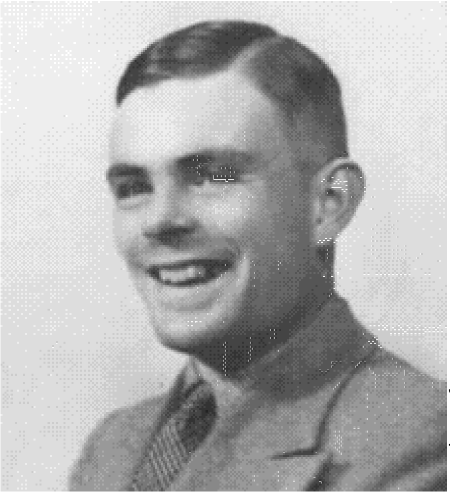
3

Numbers in the Material World

COMPUTATION, INFORMATION AND ENTROPY

The opening two chapters of our journey have concerned themselves with aspects of mathematics and music viewed from the perspective of oscillation in the material world. We now move on to investigate the related topics of information and computation, which at first sight might appear somewhat remote and tangential to the themes of earlier chapters; however, once through the initial detail, it hopefully will emerge that there is a great deal of common ground to be found between the practice of music and the world of computers and information. Indeed, though music might appear the most ethereal of the arts and information seem rather intangible in character, it is a solid material existence that binds them together.

The operation of a computer, when stripped of all its finery, reduces down to a simple basic process: *input* data (information) is converted to *output* data by the application of an *algorithm* – a program or set of rules. This general procedure was described by the distinguished mathematician and computer pioneer Alan Turing for the forerunner of modern computers: the Turing machine. The process has often been likened to cooking, where the basic ingredients are transformed into a finished dish, by following a set of instructions, the recipe – an algorithm.



Picture courtesy Wikipedia

Alan Mathison Turing (1912–1954) was born in London, England, the son of an official in the Indian civil service, Julius Mathison Turing, and his wife Sara, daughter of an engineer also working in India. During his childhood his parents were for the most part away in India, while Turing and his elder brother remained in England with friends, relatives and at boarding school. This practice, not uncommon at that time, protected childrens' health and education but sometimes left them devoid of close family affection. At school, Turing's precocious mathematical talent was evident to his teachers, some of whom failed to encourage it fearing, it might 'unbalance' the rounded public school virtues they were attempting to instil. From Sherbourne public school Turing went on to Cambridge University to study mathematics at King's College obtaining his degree in 1934. At that time David Hilbert, a mathematician of immense talent and standing, and heir of the great University of Gottingen tradition had posed twenty-three problems in mathematics. Turing was fascinated by one item on Hilbert's list of unsolved problems, given in his lecture to the International Congress of Mathematicians in 1900 – the 'decision problem'. Was there some finite mechanistic process which could be applied to any mathematical statement to decide its truth? Alan Turing's answer, echoing similar work by Kurt Godel and Alonso Church, and given in terms of conceptual 'machines', *Turing machines* as they are now called, was no. Turing also realised that each Turing procedure, devised to operate on one particular mathematical statement, could in fact be mimicked by a universal machine, which first loaded an appropriate procedure and then applied it to the problem in hand. Effectively, this described the stored program computer of the present day. During the second world war Alan Turing worked at Bletchley Park developing electronic machines for code breaking and after the war moved to Manchester University's computer laboratory. Alan Turing was a homosexual, never marrying for cover and in a period when such practices were illegal, this aspect of his life led to many difficulties, and most probably, his early death from cyanide poisoning.

INFORMATION AND STRUCTURE

But what are these ingredients and products, what exactly *is* data: what is information made of, and how is it made? To investigate this question, it might be helpful to examine a little data – some text for instance. Clearly, information can be found in a book, that is, in the *matter* composing a book – the physical object. Ultimately, for the purposes of this discussion, a book is a collection of atoms and molecules; it consists of specific arrangements of atoms, the components of ink on paper, which en masse amount to letters, words, sentences: a material encoding of information – perhaps a shopping list or maybe a sonnet. It is the *structure*, the relative positions of the components, which holds the informational content. Run the book through a shredder, or chop it up with an axe and the very same components will be given a very different arrangement relative to each other. The message held within the former structure, the neatly printed letters, words, sentences, etc., can no longer be deciphered. The original structure has been transformed and so too the information it contained. The new arrangement of (shredded) components still contains stacks of information, that is lots of structural relationships of ink and paper in a tangled mass; however, in their new configuration the components no longer convey the original message. The information – that is the structure – has simply been reorganized, but the meaningful message contained within the book has effectively been obliterated. (Though at a price, but only with the expenditure of a great deal of effort, the original message could be recovered, by carefully piecing all the threads of paper back together!)

When you think about it, all meaningful information, even the most ephemeral thought, however abstract, eventually reduces to a *material* something – neurons, synapses, molecules, electrons, waves, etc., arrangements of matter/energy – physical structures. Indeed, fundamentally it could be argued, information is simply *arrangements of physical objects* – arrangements of electrons in the random access memory of a computer, arrangements of magnetized molecules on a disk surface, arrangements of ink on paper, arrangements of tiny electrical impulses in the brain.

So what we are ultimately doing with our computers by converting input data into output data, is *rearranging physical objects* in accordance with a given set of rules – a program. We take a physical structure as input (i.e. data) and transform it into a different physical structure – the output. And following from this, conversely, if meaningful information is no more than an appropriate arrangement of physical objects, then any arrangement of physical objects, *any physical structure*, from a tiny atom to enormous galaxies, will also embody some *raw* information about themselves, in that their physical form is specifically one thing, particular accumulations of matter/energy arranged in a specific way. (This is analogous to the state of our shredded book, with loads of 'jumbled' information – the product of its 'history' in the shredder.) However, attempting to read the raw information in a particular structure would be somewhat boring and bizarre in most circumstances: rather like trying to read a shredded book, where there is still structure to be found in the specks of black ink on the strands of white paper but meaningful letters can no longer be distinguished.

From this viewpoint, information could be understood as an aspect or attribute of the physical world, not something, abstract, separate or divorced from hard reality; and if information is inextricably linked to the material world, then without persistent physical structure to express it, information cannot exist and persist. Indeed, if information cannot exist without physical forms to articulate it, and all physical structure embodies at least some raw information, it would appear that information and physical structure are, perhaps ultimately, one. So returning to the original question: What is information? Ultimately, it would appear to be *relationships*, expressed in the form of connections between physical entities – that is, material structure.

The raw information an object embodies about itself, its physical form, sometimes characterised as *syntactic* information, is somewhat akin to a system of numbers or an alphabet – a fundamental or lowest level of information content: the default level of meaning where the (raw) information simply describes itself. This is rather like each letter of an alphabet embodying a fixed quality. An Xness, Yness, Zness, etc., or the basic character of oneness, twoness, threeness, of digits in a number system and thereby each letter also possesses a settled relationship to all the other letters in its particular alphabet. For example, ‘X’ is itself and not ‘A’, or ‘B’, or ‘C’.

Meaningful Information

A scheme of fixed structures such as letters, numbers or indeed the chemical units (bases) in DNA, can express or *host* further layers of structure – which is often characterised as *semantic* information. You are reading some *sematic* information now! Semantic layers of structure/information derive meaning by reference to a particular context, such as an English speaker, a living cell or a computer chip. And these semantic layers of information might be likened to *software* running on the *hardware* of the syntactic structure which hosts their very existence. One might draw an analogy between the hardware level of letters of the alphabet and the atoms of the material world, and software layers of words (i.e. sentences, paragraphs and chapters, etc.) and molecules (i.e. alloys, solutions, complex materials). DNA, the recipe for making and maintaining living cells, has specific arrangements of its own four-letter alphabet ‘A, G, C, T’, the nucleotide bases: adenine, guanine, cytosine and thyamine, which host the ‘instructions of life’ written across two long molecular ‘pages’.

The ‘hard-wired’ syntactic information represented by atoms usefully defines a base level of structure upon which many further layers of ‘soft-wired’ structure are be built. Soft-wired or semantic information though is fragile, almost magical stuff; devoid of its context, it’s not there – with the system of semantic meaning falling back to a default syntactic state. Four pebbles lying in the sand, are no more than four chunks of rock exhibiting a particular spacial arrangement, until a percipient eye discerns in them a square number. Break into a living cell, extract the contents and you find ordinary molecules, not special ‘living’ molecules that ‘know’ how to synthesise the building blocks of life. However, what does distinguish the molecules when actually in the cell, is their particular arrangement, *the structure*. This structure does not come cheaply: it is the product of a vastly long chain of events – the evolutionary history of the organism, probably greater than three and a half thousand million years of sustained effort, in total. And what a price has been paid, over aeons of relentless struggle, for the complexity which allows humankind to ponder!

Tonal music is a particularly interesting form of information in that it is written in the universal, and perhaps eternal, alphabet of numbers – mutable base numbers. Tonally organised music possesses characteristics both syntactic and semantic. Though Shakespeare words will one day be as remote as those of Homer, the music of Bach will never become similarly inaccessible to the ears of humanity for as long as evolution maintains the present form of the mammalian hearing mechanism. Information about some part of J.S.Bach's mind and living essence has passed into the computations recorded in his scores and subsequent generations have valued this information so highly that millions of copies of his music have now been reproduced. Bach has successfully found another channel, beyond biology (where he was also conspicuously successful) by which to perpetuate information concerning his own being. And even more remarkable, if the semantic context provided by humankind were in time to disappear from the material world, who is to say that some other intelligence might not find interest in Bach's music – because being written in the universal code of numbers, it would also be intelligible, on some level, to them.

Elaborating and accumulating semantic information appears to be very hard work in comparison with the production of syntactic forms, though fundamentally a common thread runs through both types: information processing. The processing of information would appear to enter the material world at its lowest level, like a motif, and, building on itself through countless layers, emerges to become a commanding theme. There is something verging on the miraculous in the apparent 'complexification' achieved in the material world over time – perhaps a miracle of software?

Ultimately, it is difficult to make meaningful distinctions between information and material structure or information processing and change in physical systems. Indeed, one could speculate, should the whole story of consciousness ever come to be told, that even this precious attribute might turn out to be no more than a byproduct of information processing, an *emergent quality* capable of achieving self-knowing when a sufficient amount of the right type of processing, in a sufficiently integrated form, is carried out. (Though equally, there might well turn out to be much more to the story.) This emergent characteristic would be somewhat analogous to the role of heat in the macroscopic world of everyday existence, where it appears a real and vital element; however, on closer examination, at the scale of atoms and molecules, heat is found to be no more than the product of the motion (energy) of these tiny particles.

From the beginning of the scientific era, the road of discovery has appeared to steadily lead away from a human-centric view of nature, which placed mankind and his world at the focus. Perhaps, the growing informational methodologies in science might some day reverse this trend, with today's human *observers* of nature re-entering the limelight, in a more holistic approach based on an informational continuity stretching from basic processes out to the most complex of structures? But enough, to close this section, something a little more authoritative – a quote from a leading scientist in the field of thermodynamics, J.D. Bekenstein, writing in the Scientific American:

> "... a century of developments in physics has taught us that information is a crucial player in physical systems and processes. Indeed, a current trend, initiated by John A. Wheeler of Princeton University, is to regard the physical world as made of information, with energy and matter as incidentals."

and he concludes the article with words by a distinguished colleague, Lee Smolin:

> "... a final theory must be concerned not with fields, not even with spacetime, but rather with information exchange among physical processes. If so, the vision of information as the stuff the world is made of will have found a worthy embodiment."

Information and Numbers

Our decimal system of numbers, so familiar that we seldom think about how it works, is one ubiquitous example of semantic information. We write the digits down on paper, sometimes handle them in our heads and often in today's world, encode all manner of information as numbers in electronic form within computers. However the numbers are processed, the procedures always rely on some material underpinning; ink and paper, interconnected neurons, differing electrical potentials in circuits or whatever other *physical* digits come to hand.

Almost all number systems today use a particular spacial arrangement of physical symbols, termed *position or place-value notation*, that is the familiar columns of digits: units, tens, hundreds, thousands, etc. noted in Chapter 1. Position notation attributes significance to the order (i.e. positional relationship) of symbols, in addition to their unique meaning. Here already the structure – the position in space (and time) of one *physical* digit relative to the other tokens in a number – can be seen to be crucial. Indeed, perhaps it might be possible to take this connection between numbers and material structure further, and view the invention of positional notation as the discovery of a most profound analog or model of the material world at large. A duality, which the Pythagoreans so long ago grasp at, and one which lies at the heart of the MOS model's interpretation of tonal music: where musical sound in the form of dynamic physical structures in performance reveal themselves to be, also, formal mutable base numbers.

Energy also, which is indissolubly linked with matter by a very famous equation, is an important ingredient in the nexus between information processing and the evolution of physical structures.

Information, Energy and Entropy

During the 19th century, the age of the steam power, the study of the flow of energy through these 'heat engines' led to the foundation of a branch of science quite logically called *thermodynamics*, the principles of which are encapsulated in four fundamental laws concerning the nature of energy. The application of these thermodynamic principles has enabled many apparently disparate areas of science to be drawn together. The second law of thermodynamics confirms the everyday experience that a cup of coffee will get cold if left for too long before drinking (that energy flows from a hot to cold) and that without constant housework our homes become untidy (disorder tends to increase). Underlying these everyday experiences is a most powerful and wide reaching concept; it can be expressed in many different ways, but is essentially encapsulated in the idea that, overall, disorder or randomness tends to increase in a 'system' over time.

The great nineteenth century Austrian physicist Ludwig Boltzmann developed these ideas into a statistical method of understanding processes which involve uncountable numbers of individual units, such as atoms: introducing the concept of entropy – the probability of any given state (i.e. arrangement) of a system. Overall, entropy is the measure of a system's level of disorder or randomness. A system with a *high level of entropy*, close to its equilibrium state, contains little structure and differentiation, and tends toward a uniform, random and disordered character, whereas *low entropy* denotes an interesting, ordered and complex structure with significant information content.

Rather confusingly some writers use the terms 'high' and 'low' in regard to entropy with the opposite interpretation and sometimes the term negative entropy is used to describe systems far from equilibrium. Whichever nomenclature is used the essential concept to hold on to is that *entropy is a* *measure of the number of distinguishable structural arrangements a system can occupy.* That is, given that all distinguishable arrangements are equally likely to occur, the probability of the system being found in one of these arrangements is proportional to the total number. If a system has many different and distinguishable ‘states’ or structural arrangements open to it, then it can potentially express through this variety of complex structural states a quantity of information. However, if a system is close to its equilibrium state or restricted with only one or a few arrangements that it can possibly adopt, the quantity of information it can hold or express is severely limited. A flask of homogenous arrayed gas molecules would possess millions upon millions of separate arrangements of the individual (though identical) molecules; however, none of these configurations would be particularly distinguishable. The information content would be small. Under normal circumstances, such systems tend to lose orderliness to their environment, until they reach equality with their surroundings, or until they reach an equilibrium state of maximum internal disorder and randomness. Entropy is equivalent to a measure of the amount of information in a system.

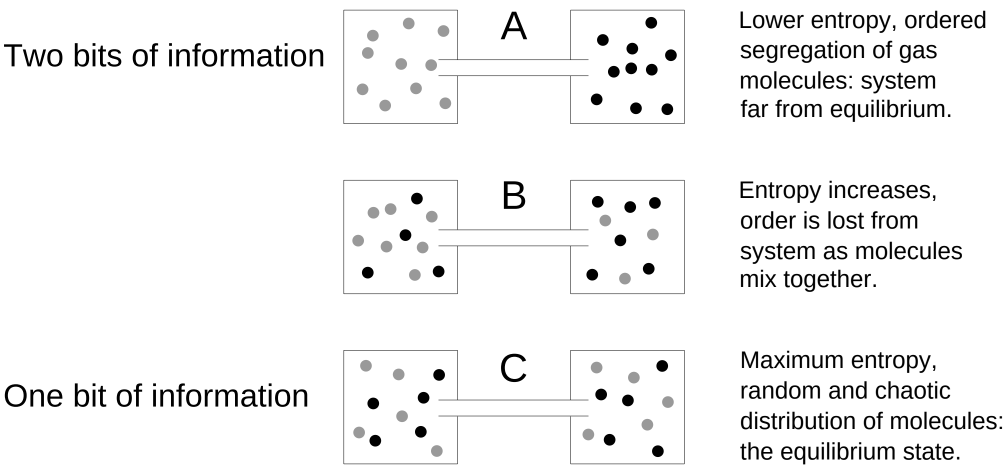


Figure 3.1 The transition from lower to higher entropy. A) Ordered segregated arrangement: there are two clearly distinguishable arrangements, ten blacks to the right, ten grays to the left (or the reverse arrangement) – two bits of information; B) Orderliness leaks away as the particles begin to mix; and C) the system reaches maximum disorder when it can be described, overall, as ‘any random distribution’ of ten black and ten gray particles which in reverse looks exactly the same and so constitutes only one distinguishable state or piece of information.

To bring one facet of entropy into focus, let’s consider the state of my desk, which is as usual, a mess! I see books, manuals, CDs, floppy disks, music, pens, pegs, photos, a fine weave of cables, etc. scattered haphazardly with little system or order. Looked at broadly, there would be an innumerable number of individual arrangements of this archipelago of chaos, which are roughly equivalent. Swap a pen here for a CD there, a writing pad for a book and overall, on average, nothing has changed. The arrangement of my desk displays a relatively high level of entropy, order is lacking in the random scattering of objects.

It is a sad truth that there are many more states of disorder, ways of being disorganised... and far fewer states of order. Just imagine what bliss housework would be if all things tended to fall naturally into ordered states, rumpled beds became smooth linen, shirts fell off the washing line into a neatly folded pile, etc. Because disordered or random states are far more numerous and so more likely on average to occur, it therefore follows that the probability of an ordered state existing is rather lower than the probability of a less ordered state. It would be statistically very unlikely (but not impossible) that one day an earth-tremor or similar event might shake my desk into a neat and tidy state... but probably not any time soon.

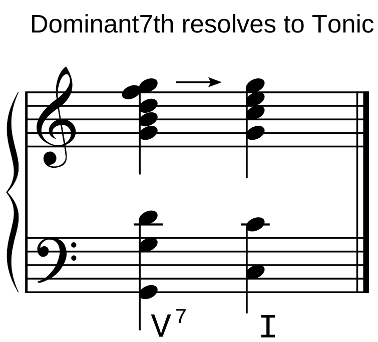
Now, I could tidy up my desk myself (I might need to hire a forklift), sort all the books alphabetically, stack up the CDs neatly in cases, put the disks in a container and so on. This requires work to be done, lots of work and much heat flows (in accordance with the second law of thermodynamics) from my sweating brow into the surrounding environment. Order reigns over the desk, but at a price. That charge or cost of the clear-up is the additional random motion of molecules, kinetic energy, imparted to the environment (mainly) by heating through my strenuous efforts to impose order. The entropy of my desk has gone down – it is in a more ordered state – but at the expense of increasing the entropy (disorder) of the wider environment in the form of a very slight increase in random molecular motion – heat. And because I'm less than 100% efficient at turning work energy into order, the total entropy of the world – desk plus environment – is now slightly higher. Thus, order doesn't always have to degrade, it can increase in some part of a system – my desk for instance – but only at the expense of greater disorder elsewhere. However, when the whole system is taken into account, the second law of thermodynamics decrees, the entropy of any whole (closed or isolated) system cannot go down and will tend to increase. So, ultimately, the second law is asserting that the entropy of the universe, the ultimate whole system, will tend to go up (or stay the same) but cannot go down.[1]

Picture courtesy Wikipedia

Ludwig Eduard Boltzmann (1844–1906) was born in Vienna, the son of a minor government official with family roots in Germany; his mother was a country girl from Salzburg. After high school at Linz near Salzburg, Boltzmann studied science in Vienna, completing his PhD in 1866. During the late 1860's and early 1870's he furthered his studies and held a number of posts, as postgraduate assistant, lecturer and from 1869 a professorship at the university in Graz. In 1871 he was working with Helmholtz in Berlin and from 1873 to 1876 Boltzmann was professor of mathematics at the University of Vienna. Returning to Graz as professor of experimental physics, he married and settled down. With his wife Henriette, a teacher, he had five children, two boys and three girls. These were the best years, seeing him make great advances in thermodynamics by finding ways to apply the methods of mathematical probability to describe changes in physical systems composed of many parts by introducing the concept of entropy. Boltzmann, the grandson of a clockmaker, even saw that ultimately his approach to the evolution of suitably complex systems could perhaps be linked with the mysterious uni-directionality of time. Boltzmann was working at a time when the very existence of atoms was controversial and since the whole foundation of his statistical approach to the physics of systems composed of uncountably numerous particles rested on this assumption, his work was often ignored or rejected. Such criticism was hard to bear for a man subject to periodic fits of depression, as Boltzmann was, and in the end one of these attacks of depression led to his taking his own life in 1906.

The connection between heat and work was first noted during the eighteenth century in the manufacture of guns and cannons, where the considerable effort of boring out gun barrels made the drills very hot. Work produces heat and conversely heat flowing through an engine can do work. This is the practical observation that the study of thermodynamics began with; however, gradually over time the implications of this insight spread out to encompass a host of different fields. From chemistry to cosmology and communications to computers, the concept of entropy proved a fruitful means of explaining how change occurs in the material world – for understanding what it is that drives the systems of the great world.

Equally, thermodynamic decay and the concept of entropy-increase – the tendency for disorder to increase as systems change by becoming simpler and more homogeneous – may be applied to the little world of music: both in terms what drives harmonic progression and what underlies changes in meter. Although the topic is only lightly touched on here (reappearing in later chapters), the tendency for entropy in a system to increase can be employed to explain the underlying ‘force’ that animates tonally organised music. For example, in the dominant-seventh to tonic (V^7–I) full cadence, taken to be ultimately the exchange between two harmonic series of eight and six elements, the relatively more complex dominant chord loses two oscillators (D-h6 and G-h1) as the system moves to the simpler arrangement of a tonic chord consisting of only six elements. In accord with the second law of thermodynamics, the aurally agreeable and satisfying chord succession of the full cadence represents a natural evolution in the system (a tonal composition) to a higher level of entropy – illustrated in Figure 3.2. Similarly a change in meter from 6/8 to 3/4 time would involve a relaxation in the subdivision of the pulse from groups of three to groups of two. This metrical aspect is investigated further below and in Chapter 4 – see Figures 3.7–10 and 4.14.

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h8-G ⟶ G-h6
h7-F ↘  |
   |    E-h5
h6-D ←  |
   |    C-h4
h5-B ↗  |
h4-G ⟶ G-h3
h3-D ↘  |
   |    C-h2
h2-G ↘  |
   |    C-h1
h1-G
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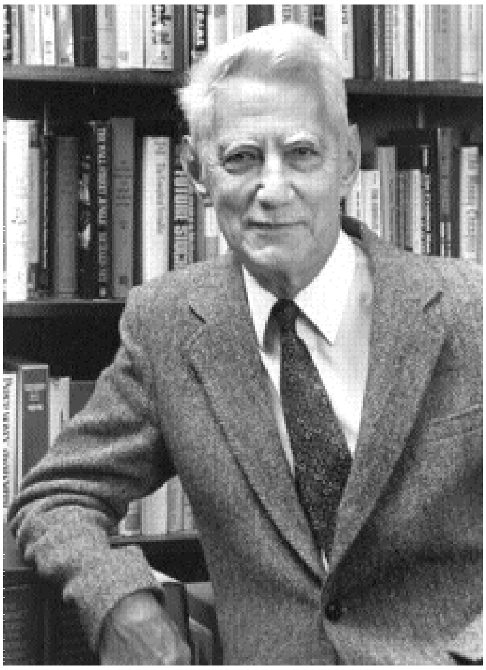
Figure 3.2 Entropy increases: the dominant-seventh to tonic chord progression viewed as the evolution of a system of harmonic oscillators from a more complex arrangement of eight to the less complex arrangement of six.

Different chord progressions present varying degrees of restructuring in their internal arrangements. The dominant-tonic exchange involves a considerable loss of order and thus produces a strong sense of directionality. For many chord progressions the amount of change is less marked and the resulting forward impetus less powerful (e.g. I–IIIflat, six to five oscillators); while for other progressions (e.g. IV–I^7, six to eight oscillators), the process is reversed with energy nominally flowing into the system as its entropy decreases and the chord’s internal arrangements becomes more complex. This produces the aural impression of pushing against the natural flow, of marching uphill or, at least, standing still; and in principle, it is the same process as tidying up my desk described above. In tonal music any extended sequence of chords will represent an ebb and flow of energy and entropy, as characterised in Figure 1.20.

Information Theory

Claude Shannon, a mathematician and founding father of the field now known as *information theory*, postulated that the degradation of information (in a signal) – the receiver's uncertainty as to which of all the possible arrangements of symbols had actually been sent – was a process involving the evolution of a system toward higher levels of entropy. Shannon was employed by the US military during the 1940s to investigate the nature of communication shortcomings which became evident over the vast distances of the Pacific theatre of war. This work, then dubbed communications theory, led him to the startling and original realisation that the information content of a message appears to behave in accordance with the second law of thermodynamics, that, all things being equal, disorder will tend to increase, and useful, meaningful information will be lost from the signal[2]. This discovery is a prime example of the way the second law has spread into disparate fields over the years.



Picture courtesy Wikipedia

Claude Elwood Shannon (1916–2001) was born and bred in up-state Michigan, USA, growing up in Gaylord, near the Canadian border, where he attended the local high school and worked part time for the Western Union telegraph company. From 1932 to 1936 Shannon studied electrical engineering and mathematics at the University of Michigan, afterwards going on to the Massachusetts Institute of Technology to work with early forms of computers. In his Masters' degree thesis at MIT, Shannon brought together a practical knowledge of electrical switching with a deep mathematical insight based on what he had learnt at Michigan of the nineteenth century mathematician and logician George Boole's symbolic logic. Essentially, Claude Shannon demonstrated that electrical switching could be used as a physical analog of (Boolean) logic – based on a system of binary numbers. During the second world war Shannon worked at AT&T Bell Labs, where he met his wife to be, Betty. After the war, in 1948, his seminal article, *A Mathematical Theory of Communication,* was published in a Bell Labs' journal (later appearing in book form, co-authored by W. Weaver), effectively establishing the scientific discipline of information theory. In his article, Shannon introduced the concept of entropy into the field of information processing and transmission, measuring the amount of disorder in a signal or message. For example, a perfectly reproduced signal or message represents a highly ordered, low entropy state and is therefore rather unlikely, given the 'random accidents' that might befall the information in transmission. In contrast the 'white noise' of a completely obscured message could be seen as analogous to thermodynamic equilibrium, a totally random and meaningless array of informational units. Claude Shannon had a most fertile mind, working in other related fields and pursuing many hobbies and amateur inventions – one of which was a flame-throwing trumpet! Returning to MIT in 1956, Shannon remained a faculty member until retirement in 1978. Claude Shannon died in 2001.

Semantic information represents a special and improbably ordered scheme which is always tending (statistically) to degrade toward random white noise, its equilibrium state. Any informational system, with a given level of entropy (order/disorder) will tend on average to increase its level of entropy or stay the same but not to spontaneously reduce it – looked at over a statistically sound sample. Just as there are a vast number of equivalent states of disorder which my desk might display, so equally, a signal, a message or this paragraph, has innumerable garbled states, but only one perfectly correct state. The often hilarious gibberish produced in a game of chinese whispers encapsulates the immutable trend toward higher entropy.

A message – an ordered scheme of relationships – reaches its maximum information density when the sequence of syntactic units or structures hosting the message is random but still meaningful in the given context. Any repetition of sequences or parallel patterns in the representation of information implies some level of redundancy, a degree of inefficiency. For example, the long but predictable sequence ‘1234123412341234...’ could be shortened to ‘write 1234 n times’ where n could be any number. Equally, in contrast to an information rich random sequence, the lattice-like structure of crystals, repeating a simple structural pattern over and over again exhibits a high level of informational inefficiency: in that nothing new can be discovered by examining more of the pattern. Their form, like ‘1234’ above, can be encapsulated in a short description or formula. Looking ahead somewhat, similarly, the output of simple computer programs – ‘cellular automata’ – discussed in Chapter 5 (see Figure 5.1), are also relatively information poor by this definition, in that it only takes thirty-two pieces of information (32 bits or binary digits) to specify a set of relationships, plus a short general-purpose set of rules (the program), to produce unlimited amounts of output. Meanwhile, in music, which generally contains many nested patterns and deep repetitions, though of a rather fluid and unpredictable nature, stands in the mid ground of informational density or efficiency. A piece of music must give us enough repetition to allow us to keep our bearings, while also feeding our appetite for novelty – a balancing act. Indeed, there appears to be an optimal *range* of informational density and flow in music, which is illustrated in the comparison of, for example, a Bach solo (unaccompanied) sonata with a piece of music in many parts, the rate of musical *events* over similar spans of measures are broadly the similar. As a general rule, as the number of parts increases in a composition, so the level of activity in each part tends to decline, illustrated in Figure 3.3.

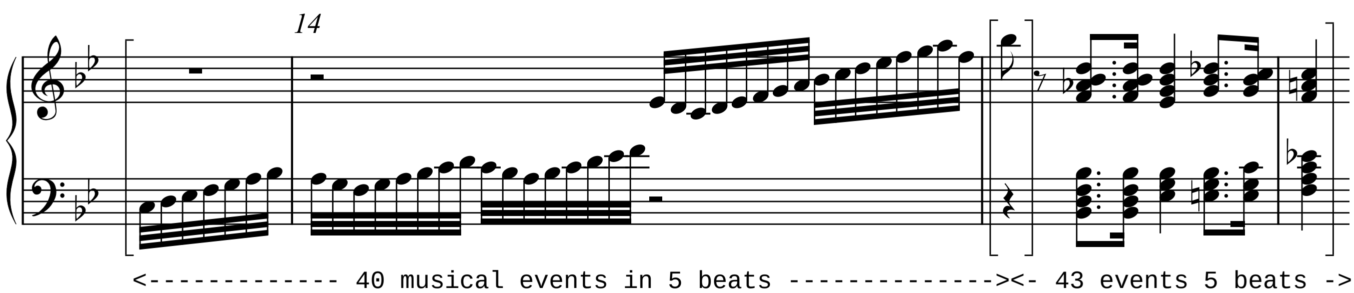


Figure 3.3 Two consecutive five beat periods of melodic and chordal texture from the Well-tempered Clavier by J.S.Bach reveal a roughly even flow of events – each note being considered as a separate musical event. (Prelude XXI, Book I, Bars 13–15.)

Recent work in the fields of thermodynamics and information theory has revealed a most surprising and unexpected relationship: that there is an upper limit to the amount of information which can exist within a given volume of space, and most surprising of all, the relationship is proportional to the surface area of the region, *not its volume*. That is to say, the density of information contained within a three-dimensional volume is limited by the volume's two-dimensional surface area. This amazing discovery is sometimes termed the holographic principle, the ramifications of which are as yet not fully understood. As with many other discoveries associated with the laws of thermodynamics, it has the essence of a basic physical law about it. Also it perhaps points towards a discrete, informational or information processing layer lying beyond or below the familiar world of everyday experience. Similarly in other areas of theoretical physics, avenues of exploration such as string theory, twistor theory and quantum gravity, look to processes working at levels far removed from the smooth continuous variability of the 'classical' world of human experience and intuition. There, in regions murkily distant, other unfamiliar principles hold sway and simple whole numbers appear to govern many relationships and activities.

The discovery of an *entropy bound* to the spatial distribution of information stems in part from the study of the thermodynamics of black holes, in cosmology, where the surface area of the event horizon shrouding a black hole has been found to be proportional to the quantity of information/structure that has fallen into it – expressed in fundamental units of area, the Planck length squared. Black holes are objects which have maximal entropy for their given volume – a volume delineated by the surface of the event horizon. There are no rearrangements of a black hole's constituents which are observable from outside and if extra material (information) is added to the object, the event horizon must expand by more fundamental units of area to accommodate the increase. In essence, all the information (structure) which goes to make up a black hole leaves a trace, a minimal trace, on the outer boundary of the object.

Black hole cosmology lies far distant from music theory, granted, but there are similarities and points of connection in that a prime state mutable number – a complete, single, harmonic series – like MBN 6_1 or MBN 7_1 (h1, 2, 3, 4, 5, 6, 7) share some similar properties. For one, rearranging the internal constituents of a prime state mutable number produces no observable change, mutable numbers in their prime state have maximal entropy. And if one unit of information is added to a prime state mutable number, its interference pattern grows by one oscillation (all other factors being equal). In addition, the face that a prime state mutable number presents to the outside world (i.e. its sum, the interference pattern), like an event horizon, is that of an even distribution of information over its external 'surface'. (This last feature assumes equal unitary amplitude for each constituent wave.) In Figure 3.4 two versions of the number six are drawn; firstly prime state MBN 6_1 in the continuous line of six even 60 degree oscillations and secondly ground state MBN 2_3 0_1 which also describes six (uneven) oscillation per period, but subdivided into three 120 degree groups of two. While the even, simple sextuple meter of MBN 6_1 betrays a uniform internal structure (one group of six), the compound triple meter (three groups of two) presented by MBN 2_3 0_1 tells of greater internal complexity, and possible variety in the structural arrangements of its constituent parts.

Finally in this section, a question to ponder: Where and how does order arise, and by what mechanism? The ultimate *source* of order in the macroscopic material world, as distinct from the mechanism by which it is made manifest, is believed to stem from gravity[3] acting upon the 'concealed' entropy implicitly contained within a region of uniformly distributed matter. Gravity squeezes the order out of a uniform mass, a body of gas for example, depositing the order in the form of explicit physical structure – the stars and associated planetary objects and debris. Inherent within the context created by the force gravity, information in the form of physical structure is processed, transformed. Looking out on the great, wide, world we find Nature adorned in amazing, intricately crafted works – objects of information – but how do such marvels come to be, when the second law of thermodynamics decrees that, overall, all things should fall into disorder? Perhaps one day they shall, with information processing dying away in an exhausted featureless universe, devoid of energy and difference. However, in the meantime, and disregarding such gloomy prognostication, what algorithm, one might ask, is keeping the cosmos in tune?

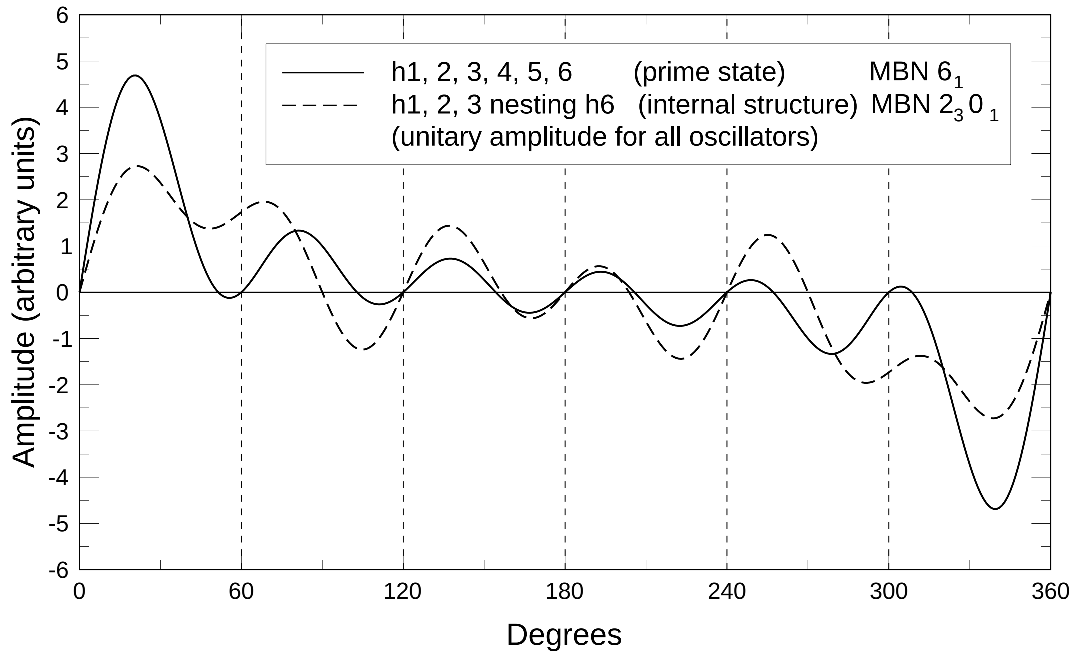


Figure 3.4 Mutable base number six in prime and ground states: MBN 6_1 and MBN $2_3\,0_1$.

ALGORITHMS AND COMPUTATION

Having spent some time in this chapter delving into the foundations of information and computation, the time has come to try a little light practical work. Here is a problem the computer can help us to solve.

Slow beats, a mildly ‘edgy’ and undulating addition to the sound of chords, the hallmark of small degrees of out-of-tuneness are, surprisingly, present in the intervals of correctly tuned pianos. The reason why a piano or any other fixed pitch instrument is tuned with ever-so-slightly out of tune intervals, is as seen in Chapter 2, to allow equality to all tonal centers. That is, to accommodate playing in every possible key, each keyboard note is tuned with a small and equal amount of out-of-tuneness. A democracy of imperfection! For example, the notes of any interval of a fifth, C-G, D-A, E-B, etc. are given the ratio 1:1.4983... rather than the true fifth relationship of 1:1.5 (2:3) and the semitone intervals are equally spaced, with a frequency ratio of 1:1.05947... between each one. By adopting this scheme of equal temperament the small mismatch between measuring by perfect fifths and true octaves – the Pythagorean comma – is hidden or smoothed over by dividing it into twelve equal parts and distributing one part to each note in the chromatic scale. It is a compromise, a conjuring trick, that trades precision in pitch for the utility and flexibility of being able to play in all keys with equal ease. In a sense it is a software sleight of hand over the physical realities, the hardware, of musical sound. Equal temperament creates a *new reality*, a perfectly symmetrical system of keys – a circle rather than spiral of fifths – that doesn't materially exist and yet forms the ‘solid’ context within which tonal music operates. This deliberate compromise or deviation from music’s material foundation involves the *tempering* of intervals.

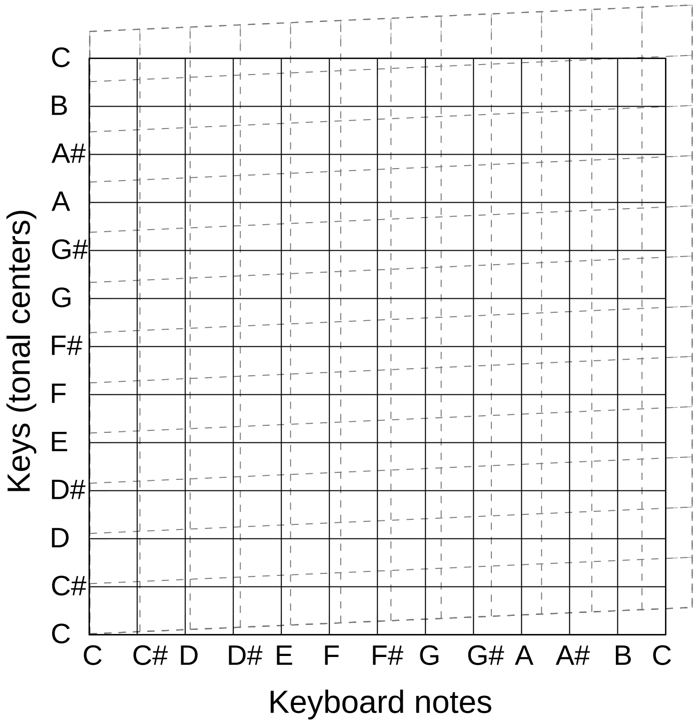


Figure 3.5 The twelve equally tempered keyboard notes and the twelve equal key/tonal centers are represented by the black-line grid. The skewed dotted gray-line grid illustrates (approximately) the relationships as they arise from 'pure' whole number ratios: the difference between computing scales by fifths of 1:4983... and 1:1.5. The success of equal-temperament's slight-of-hand relies on the ear's ability to extract a pure relational 'meaning' out of impure tempered signals.

The root of this not entirely satisfactory compromise lies in fixed pitch instruments, such as the piano or organ, having just twelve 'switches' to express twelve times twelve slightly different relationships – the twelve-note scale viewed from each of the twelve tonal centers: C major/minor, D major/minor, etc. As seen in Chapter 2, the problem is that twelve 2:3 (tonal center) relationships don't quite match seven 1:2 (octave) relationships. However, because the twelve tonal center scales, each of twelve notes, align roughly (but not perfectly) the piano can allow each keyboard note to fulfil twelve very slightly different functions or relationships. The keyboard note C is the tonic (I or i) in its home key of C major/minor but the supertonic (II or ii) in Bflat major/minor and the mediant (III or iii) in Aflat major/minor, and so on through the remaining tonal centers.

In the past, when musicians tended to use fewer key-centers in their music, it was possible to have unequal *temperaments* – unequal spacing of the relationships between the note – and therefore less out-of-tuneness between the intervals of the (fewer) keys in actual use. The cost of this tuning strategy was to make the other keys unpleasantly out-of-tune, and some practically unusable. However, over the years, as musicians explored the artistic resource of modulating to evermore remote regions of the cycle of key relationships, the various unequal temperaments became less and less tenable.

Amongst the democracy of imperfections that equal temperament represents, the interval of an octave has the nominally perfect ratio of 1:2 between the frequencies of its upper and lower notes – the lower note C can be ascribed a notional frequency of '1', and the upper C of the octave interval the notional frequency '2'. It is convenient to write these two notes as C[f=1] and C[f=2] (using the abbreviation 'f' for frequency) and what is really being said about the two notes is that their frequency relationship is the ratio 1:2; any octave interval, whether it is two C's or two G's or two A#'s, shares this particular relationship, the ratio 1:2. (Though nothing is quite as perfect as it seems, as in reality piano tuners add life and brightness to the octaves by stretching them out a little; notwithstanding, here we shall maintain the illusion that all octaves are perfect.)

Now to divide an octave interval into twelve equal semitone steps, to produce all the different notes of the equal tempered scale, we need to find the relationship, the ratio, that will turn C[f=1] into C#, C# into D, D into D#, and so on until after twelve steps the sequence reaches C[f=2]. That is the number which, multiplied by itself twelve times, will give us two, which in abstract mathematics is termed the twelfth root of two. We could proceed by trial and error to find this ratio.

1 x 1 (twelve times) = 1
2 x 2 (twelve times) = 8192

It is clear that the ratio must be between 1:1 and 1:2 and much closer to the former. What about 1:1.1?

1.1 x 1.1 (twelve times) = 3.45 (approx)

So, much nearer but still too large. What about 1:1.01?

1.01 x 1.01 (twelve times) = 1.138 (approx)

Now this number is too small! This approach might take a while and there is an easier way, let the computer do the work for us. Here is a little program, an algorithm or set of instructions, to find the number which, multiplied by itself twelve times, yields the product two (approximately) and thereby the ratio between adjacent semitones in the twelve note scale.

```
number = 1
REPEAT
  accumulator = 1
  count = 0
  REPEAT
    accumulator = accumulator * number
    count = count + 1
  UNTIL count = 12
  number = number + 0.000001
UNTIL accumulator >= 2
PRINT number
END
```

This program is written in the delightful and almost readable BASIC programming language, with the upper case words representing ‘verbs’ or actions and the lower case words ‘nouns’ or names. In fact the names represent address locations in the computer’s memory which contain a ‘value’ – some information, a number. Stepping through this code just as the computer does, executing or carrying out the instructions on each line, is described below and illustrated in the flow chart Figure 3.6. Starting from the number one, the program counts up in 0.000001 increments, until it reaches the number which, multiplied by itself twelve times, is equal to two or just over two.

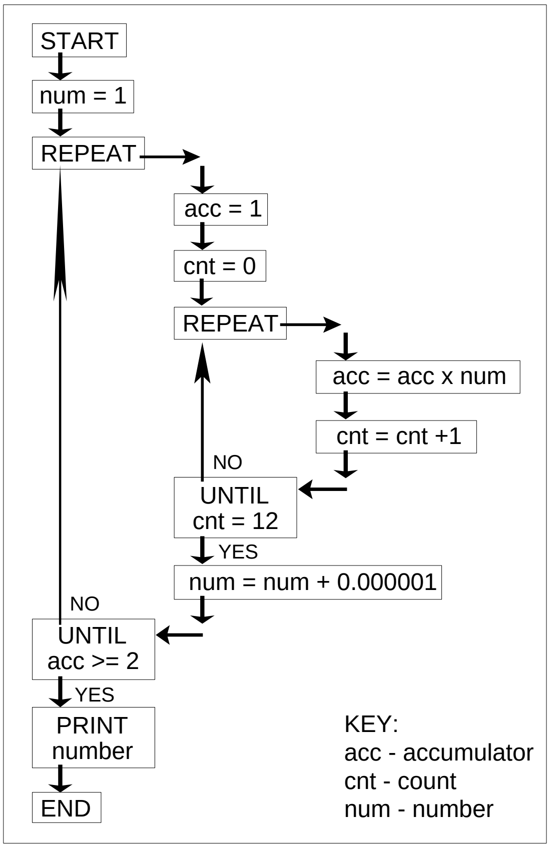


Figure 3.6 Flow chart of the BASIC program to find the ratio for twelve equal divisions within an octave: the semitone scale. Notice how the cycles of the inner REPEAT/UNTIL loop occur within the cycles of the larger outer loop.

1) Set `number` to 1.000000
2) Begin main (outer) loop, which `REPEAT`s the enclosed code `UNTIL number` is equal to or greater than 2, that is C[f=2].
3) `accumulator` is set or reset to 1, the value of C[f=1].
4) Set or reset `count` to 0. (Counting from zero, is not something human beings do instinctively but is second nature to computers, aliens and programmers.)
5) Within each iteration of the outer `REPEAT/UNTIL` loops there is another inner *nested* `REPEAT/UNTIL` loop.
6) Inside this second inner *nested* `REPEAT/UNTIL` loop, twelve `counts` of multiplication '*' are built up in the `accumulator` (which has been set to the value of C[f=1] in line 3.
7) The counter, `count`, is keeping track of the how many *nested* loops have been performed,
8) and when it reaches 12, line 8 releases the loop, otherwise the `UNTIL` directs the flow of the program back into another (nested) inner `REPEAT/UNTIL` cycle.
9) Add 0.000001 to `number`, i.e. `number` grows larger with each successive loop
10) Go back to line 2 and start the process over again, `UNTIL` the `accumulator` is equal to or greater than two '`>= 2`', when it is exit the outer `REPEAT/UNTIL` loop and proceed to line 11.
11) `PRINT` out `number;` this line is executed when the answer has been found.
12) `END` program.

Once the program discovers that `accumulator` is greater than 2 it exits the loop and prints the answer. The number that the program yields is roughly 1.059465, which means that the ratio between the pitch or frequency of any two adjacent equally spaced semitones is 1:1.059465. This will give the sequence of semitone steps of (approximately) C = 1, C# = 1.05946, D = 1.12245, D# = 1.18919, etc. to C >= 2 (with a small margin of error, 0.00003). In fact, it will always be just over two because the particular number which will give exactly two is very, very long. We shall return to the question of its length below.

Nested Structure

But first, the nesting of one `REPEAT/UNTIL` loop inside another must claim attention. This is very much like the nesting of one set of oscillations within another – which we have seen in the previous chapters concerned with music and the harmonic series. Within the *fundamental* loop (i.e. cyclic oscillations) of the first `REPEAT/UNTIL` structure (lines 2–10) a second 'higher frequency' *nested* oscillation, the `REPEAT/UNTIL` structure of lines 5–8, is found; and other levels of nested loops could occur in a program if required. Indeed, many further layers of nested structure would exist underlying this simple program through other unseen levels within the operating system of the computer itself. Programmers use indentation, as in the above little program (and function calls, not used above) to express and keep track of the levels of nested structure within their code. However, the two levels of `REPEAT/UNTIL` indentation present in this small BASIC program illustrate the principle perfectly well. These two levels of nested oscillation can neatly be expressed in terms of meter: the nesting of rhythmic groups – illustrated in Figure 3.7.

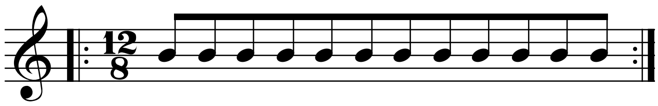
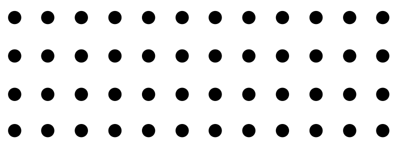


Figure 3.7 In musical terms, twelve cycles or sub-beats of the inner loop nest within each single period or beat of the outer REPEAT/UNTIL loop (indicated by the repeat marks ||: :|| and twelve-eight metrical unit).

The continuous *beam* joining all the note stems declares and expresses the grouping of cycles (notes) within one level of nesting: one `REPEAT/UNTIL` loop. Also, the first note of the group gathers an extra accent by way of being a member of the underlying fundamental (outer) loop. Expressed as a number pattern the inner loop would form a horizontal row of twelve tokens for each iteration of the outer loop – of which there are too many to illustrate in full:



etc...

and as a mutable number the loop could be written: MBN 12_1.

However, twelve is not a prime number, or here not a *prime metrical grouping* and as such could be broken down into smaller sub-groups. In the altered BASIC code below the twelve 'equal' loop cycles have been further subdivided into a loop of three cycles set within a loop of four cycles.

```
 1 number = 1
 2 REPEAT
 3   accumulator = 1
 4   count = 0
 5   REPEAT
 6     counter = 0
 7     REPEAT
 8       accumulator = accumulator * number
 9       counter = counter + 1
10     UNTIL counter = 3
11     count = count + 1
12   UNTIL count = 4
13   number = number + 0.000001
14 UNTIL accumulator >= 2
15 PRINT number
16 END
```

The altered program illustrates the rationalisation of the inner loop: the grouping of three cycles (loops lines 7–10) within the four cycles of lines 5–12. In musical form, this arrangement could again be expressed as a twelve-eight metrical unit. The altered *beaming* of the note-stems is significant, in making clear to musicians the rhythmic precedence: twelve nominally equal notes are played, by accentuation, as four groups of three units – Figure 3.8.

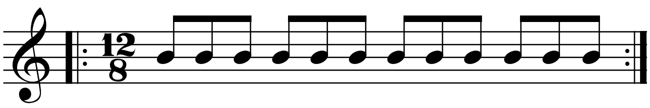


Figure 3.8 A twelve-eight meter within repeat markings expresses the cycles of the outer loop (lines 2–14), the inner loop (lines 5–12) and the innermost loop (lines 7–10). MBN $3_4\ 0_1$ or in factor format 1x4x3.

However, the beaming does not show the subdivision of the four groups of beamed eighthnotes, into two groups of two, which a musician would naturally add to this meter – and which is not present in the altered code either, but could be introduced as another nested loop (not shown). What the musician does 'naturally', is reduce the meter to an efficient and stable arrangement, which amounts to finding the metrical factors or ground state. Here the metrical factors of 12/8 time are found to be (1×2×2×3) = 12, that is MBN $3_2\ 0_2\ 0_1$. And more generally musicians, in exercising their spontaneously creative rhythmic intuitions, are ultimately responding to, playing with, and occasionally opposing, the wide-reaching effects of the second law of thermodynamics. As discussed in the introduction, artists and artisans used and intuitively understood fundamental scientific principles long before their official discovery.

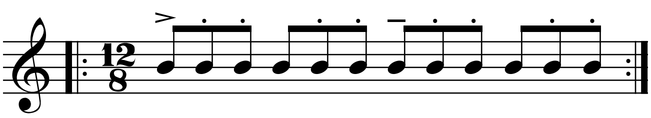


Figure 3.9 The further subdivision of four groups of three (4x3) into two groups of two groups of three (2x2x3): That is a natural twelve-eight meter with a primary dynamic accent '>' and secondary accent '–' explicitly portrayed. Staccato '.' dots articulate lower precedence within the groups of three, the first of each group being of greater weight.

Alternatively, the grouping could be reversed, four loop cycles set within three; metrically this can be expressed as 3/2 time – Figure 3.10a – and the factors are 1×3×4 = 12 with this arrangement, i.e MBN 4_3 0_1. Finally, one could subdivide these inner four cycles into a loop of two within two (Figure 3.10b) and this would reduce the meter to its most efficient and stable state. Metrically this can be expressed as a 3/4 meter and amounts to arranging the factors of twelve in descending order (1×3×2×2 = 12) – the ground state of the mutable number twelve: MBN 2_2 0_3 0_1.

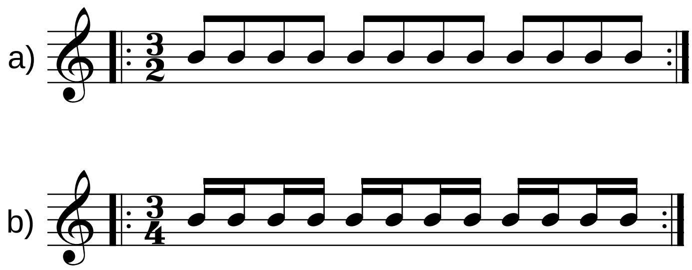


Figure 3.10 Two further, different metrical subdivisions: three groups of four (3x4) and three groups of two groups of two (3x2x2).

Although the connections between the cycles of computer code loops and rhythmic cycles of duration in music might seem rather distant and artificial, by thinking about their nature in an abstract way, one is able to find parallels. Indeed, by viewing the two systems of the digital computer and music, from the perspective of arrangements and aggregations of a fundamental tick or time unit within each system, the character of the structures created out of this elemental unit, whether software or sonatas, takes on many similarities.

Very Long Numbers

Now we return to the question of the exact number which, multiplied by itself twelve times, will yield exactly two – that is the exact ratio between the pitch or frequency of two adjacent semitones, which the BASIC program found to be approximately 1:1.05946... (the three dots meaning that the decimal expansion extends further). This very, very long number is so long that it is probably impossible to tell whether it ever comes to a halt by finding some repeating pattern, or just goes on and on in seemingly random fashion forever! If it does go on forever without finding any stable repeating pattern of digits, with its decimal expansion just getting longer and longer, one could ask: What sort of existence can be ascribed to a number that cannot be written down, however large the sheet of paper is? What if all the (material) structure in the entire universe, used in the most efficient manner, were insufficient to express this number, precisely? Does such an unending fractional number really exist or is it in some way unreal, immaterial? Although it might at first sight seem a theoretical or even trivial point, these are, I think, deep questions and somewhat troubling questions too. Do the very foundations of my livelihood, the equally tempered twelve notes of the scale, rest on a number which cannot be written down exactly, or worse, cannot even be known in principle?

It might be helpful to transpose the question from the realm of numbers in the real world, to the *little world* of a computer, where it could be framed in the following manner. How does a computer manage to handle very large numbers? Most of the present-day computers operate using 32bit binary numbers which look, for example, like: 0100 1100 0001 1110 0000 0100 0000 1111 when written down. With 32 bits or binary digits, of either 0 or 1, it is possible to represent 4,294,967,296 whole numbers. (Under normal circumstances the larger half of these numbers, binary digit sequences with the *top bit set* to one, are treated as *honorary* negative whole numbers.) However, the 32bit computer manages to handle numbers larger than this by using a *floating point* system, the details of which are not important to this argument. The floating point system trades off accuracy against magnitude; essentially, it gives answers just like the BASIC program found for the twelfth root of two. Numbers which are approximate rather than exact. Similarly, a 32bit computer cannot write down or compute exactly, numbers larger than 4,294,967,296, so in a sense, floating point numbers are fantasy numbers. The floating point mechanism in a computer can express roughly what these large numbers might be like, but they don't actually have the *exact* concrete existence within the computer's little world of electrical structures or relationships, which can be ascribed to numbers and ratios within the range 1:4,294,967,296.

Floating point numbers represent a triumph of software over the limitations of hardware. Though slightly inaccurate, they are immensely useful; in fact, without them the computer's functionality would be crippled. And this triumph of ingenious software over limited but brutally powerful hardware, is essential in virtually all areas of computer use. The arts of software enable a magical variety and complexity to arise from just 4,294,967,296 whole numbers (computed at breathtaking velocity) and out of which emerges a myriad wealth of programs, application, utilities, etc. This is a pattern we shall meet again, later on, where a limited number of basic 'states' in a system can be transformed into an abundant cornucopia, through the mediation of (many) layers of nested software: just as the limited, chunky, whole numbers of the 32bit computer are transformed almost magically, into floating point quantities, that possess something approaching the character of the *smooth continuum* of infinitely many magnitudes postulated by abstract mathematics.

MUTABLE NUMBERS, ENERGY AND ENTROPY

Mutable base numbers, or at least the counterpart physical systems they purport to represent, can be thought of as possessing an entropy. That is, a clearly defined set of unique structural 'states', with probabilities of being found in any of these distinguishable internal arrangements. One way of measuring entropy is simply to count the number of distinguishable configurations the system can occupy. Using this method, for example, the primes only have one arrangement[4] as mutable numbers (e.g. twenty-three, MBN 23_1) and so can be considered as possessing a single or maximal level of entropy. In contrast, the number twenty-four can take on many structural forms (e.g. MBN $8_3 0_1$, MBN $6_4 0_1$), twenty individually distinguishable configurations in total – listed in Chapter 1. Thus mutable number twenty-four can be considered to have a relatively low entropy, or put another way, there is a fair probability that it will be found in some state other than its prime arrangement of MBN 24_1. Indeed, given the ever-present operation of the second law of thermodynamics and a cool environment, it is rather unlikely that any physical counterpart of number twenty-four will be found in the highly complex prime state arrangement of h1 through h24, but it is far more likely to exist in a relaxed condition, employing several levels of nested structure closer to its ground state of MBN $2_2\, 0_2\, 0_3\, 0_1$.

Decimal Number 'Formal' Entropy	Number of Unique Arrangements in Mutable Format 'Physical' Entropy	Ratio of Formal Entropy to Physical Entropy
1	1	1.0
2	1 - power of 2,fecund	0.5
3	1 - prime	0.33...
4	2 - power of 2,fecund	0.5
5	1 - prime	0.2
6	3 - fecund	0.5
7	1 - prime	0.142857...
8	4 - power of 2,fecund	0.5
9	2	0.22...
10	3	0.3
11	1 - prime	0.0909...
12	8 - fecund	0.66...
13	1 - prime	0.076923
14	3	0.2142857...
15	3	0.2
16	8 - power of 2	0.5
17	1 - prime	0.0588235...
18	4	0.44...
19	1 - prime	0.0526315...
20	8	0.4
21	3	0.1428571...
22	3	0.13636...
23	1 - prime	0.0434782...
24	20 - fecund	0.833...
27	4	0.148148...
36	26 - fecund	0.722...
48	48 - fecund	1.0
96	112 - fecund	1.1666...

Figure 3.11 A table comparing the 'formal' and 'physical' entropy of the mutable numbers 1 through 24, 36, 48 and 96, and in column three the comparison is expressed as a decimal fraction.

Because Shannon entropy, that is the entropy of information, is measured in numbers – the minimum number of 'bits' (binary digits) required to express the information – and magnitudes expressed as mutable numbers can be considered to possess an entropy in and of themselves; it is possible to compare the 'face value' entropy that a mutable number represents with the actual entropy of the mutable number's range of structural configurations. When this comparison is made, some interesting trends emerge, which are illustrated in Figure 3.11.

In Figure 3.11 the ratio between formal and physical entropy begins at the top at 1.0 and then drops below one until it emerges at the bottom of the table at 1.1666...; within this overall pattern lie a number of trends. Firstly, the ratio between the primes' formal and physical entropy steadily drops away, beginning with the sequence 1.0, 0.5, 0.33..., 0.2, 0.142857..., etc. and continuing unabated in the direction of zero, as all primes have only one mutable number digit sequence – one arrangement. In contrast to the primes, the ratio between the formal and physical entropy of the non-zero positive powers of two remains constant at 0.5 for all values. This constancy is interesting considering the octave transposition role of the power of two frequency relationship – octave transpositions imply no change of harmonic relationships. Thirdly, the set of the most fecund mutable numbers, numbers constructed from the factors two and three initially, perform the opposite role to the primes; their ratios climb steadily from six onward: 0.5, 0.66..., 0.833..., 1.0, and at ninety-six they exceed one with the ratio 112:96 or 1.1666.... These most fecund mutable numbers begin by consisting of many factors of two, and one factor of three, and are similar but not identical to the series of Highly Composite Numbers in traditional mathematics. There are many other lesser fecund numbers in this range, for example those with some factors of two combined with one of five, seven, eleven, etc., or mixed amounts of factors: two with three, with three and five, with three, five and seven, and so on. (In the CHPT19.ZIP directory, in the SCRPTS folder, the Perl script *digseq.pl* will calculate the range of distinguishable digit sequences for any given number.)

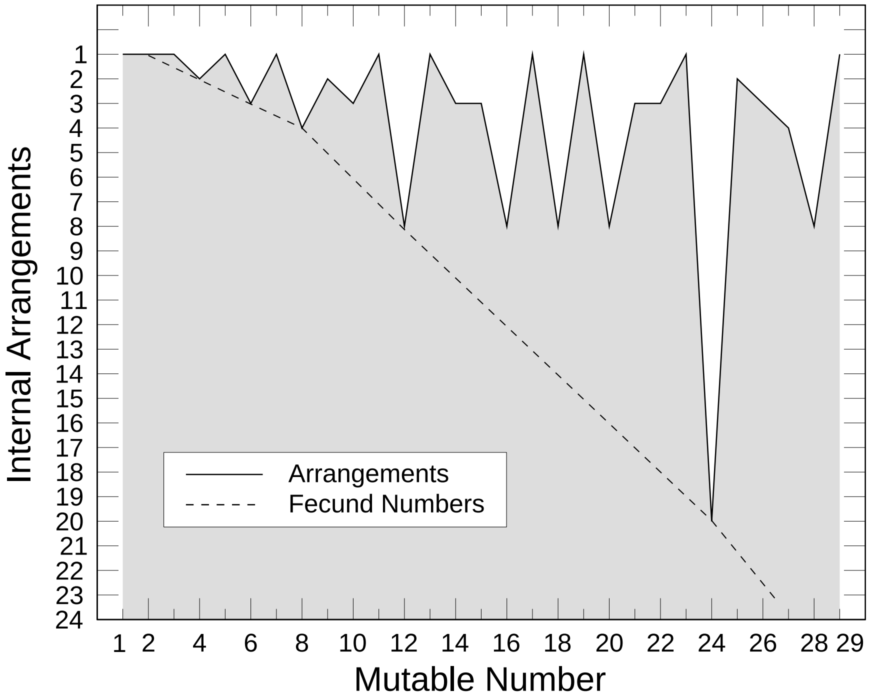


Figure 3.12 Entropy fissures created by the fecund mutable numbers may tend to trap physical systems in patterns which reflect attributes similar to those of a 'harmonic/musical' character.

The deep fissures cut into the graph Figure 3.12 are there because the fecund mutable base numbers possess markedly more internal configurations than their neighbouring integers. While one of these fecund configurations will match the highly ordered additive arrangement of a prime (i.e. h1 through h24), the remainder will consist of less complex (multiplicative) nested arrangements. And these numerous nested configurations represent a range of less energetic structures that a system might adopt. For example, a 'system of twenty-four' could choose MBN 6_4 0_1 rather than 24_1. In comparison the mutable number twenty-three is stuck with one arrangement which makes the probability of finding it in this state absolute. Though the prime state mutable twenty-three, encompassing h1 through h23, is a relatively complex and energetic object in itself, it is stuck forever at a single high level of entropy. It would of course be possible to exchange the positions of oscillators within the number, for example h2 might be boosted to h22 and vice versa, but this would leave the system unchanged when viewed overall. Thus there is zero probability of it being in any other distinguishable configuration, if it were, it would no longer be mutable number twenty-three. Given a very high energy environment, primes such as mutable twenty-three would not look like sore thumbs, as all their neighbours would occupy similarly prime configurations as they inflate to their maximum energy state. In this situation there would be a symmetry amongst all integer mutable numbers. However, should the environment cool, this symmetry would dissolve away with the varied degrees of fecund numbers precipitating out to form less energetic nested arrangements.

Essentially, in a low energy environment, the fecund mutable numbers would form a sequence of energy wells into which a system could fall and through which a system might evolve from one trough to the next, each successively deeper relative to that number's prime state. In addition to the energy-saving attractions of fecund mutable digit sequences, the trend of growth in their range of configurations, exhibited in the table Figure 3.11 and graph Figure 3.13 would also suggest at some point (determined by the prevailing energy level of the environment and the magnitude of the number) that the cumulative total of arrangements would overtake and eventually swamp the configurations of the barren primes and their close relatives. The end result would be an overwhelming probability of finding configurations of fecund mutable numbers in systems consisting of many parts.

Possible examples of fecund mutable number patterns in real physical systems might perhaps be found in the harmonic flavor to the Bode-Titus Law of planetary distance from the Sun or the cumulative total of quarks in the atomic nucleus, taken in 'electron subshell steps' of the periodic table of the elements: 12, 24, 60, 120, ... I would stress that both of these examples, macroscopic and microscopic are offered tentatively, in an exploratory and speculative spirit – and discussed further in Chapter 15.

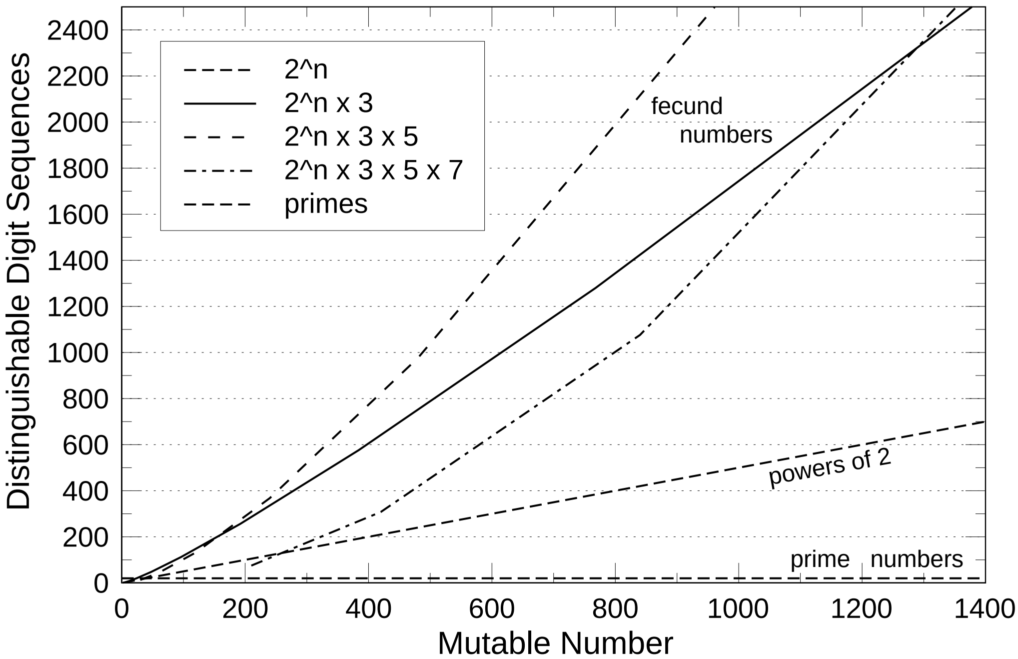


Figure 3.13 A plot of distinguishable digit sequences against value, of some of the more fecund mutable numbers, plus the powers of 2 and primes. In the bottom left-hand corner the dashed line of 2^nx3x5 can be seen to overtake 2^nx3; as does 2^nx3x5x7 at the top right. Far beyond the range of this graph the dot-dash line at 2^8x3x5x7 = 26880, with 204032 distinguishable arrangements, overtakes 2^{11}x3x5 = 30720 which has only 197632 distinguishable arrangements.

Overall, the outstanding feature of the table and graphs, Figures 3.11 through 3.13, is the steady growth trend in the sequence of the mutable numbers most fertile in internal configurations: 2, 4, 6, 8, 12, 24, 36, 48, 72, 96 and their general extrapolation[5], 120, 144, 192, ... Yet, as so often happens, there can be unexpected twists and turns further on down the trail. The graph Figure 3.13 shows that other richer combinations than $2^n \times 3^m$ will eventually steal the mantle of most fecund mutable number from the early leader, probably indicating that no fixed combination of primes will remain the front runner indefinitely. When extended without limit, the series of most fecund numbers gradually enmeshes ever increasing numbers of primes in order to form ever more distinguishable nested structures. And, when translated into the realm of material existence, the efficient, information-rich low-entropy physical structures represented by these most fecund mutable numbers, are the forms most likely to be favored in a mature world ruled by the second law of thermodynamics. Likewise in tonal music, it is these same fecund mutable base numbers, written in musical sound, which underlie and encapsulate the chord progressions that the ear finds so compelling.

Symmetry and Symmetry Breaking

When applying the concept of mutable number entropy to the harmonic aspects of tonal music – that is tonal works considered as quasi-physical systems of related oscillators – a prime state fundamental nesting harmonic series could be extended upward to encompass all the notes of each succeeding chord in a composition, thus enveloping the whole piece in a universal symmetry, each and every chord embedded within the same unchanging extended fundamental series[6].

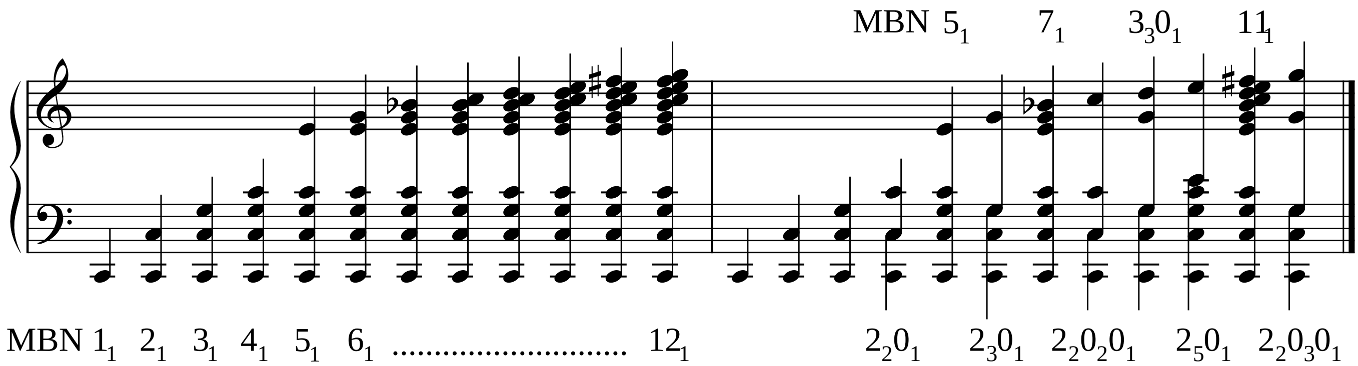


Figure 3.14 On the left the first twelve ‘prime’ state mutable number digit sequences and their equivalent chords; on the right the same values are represented in their lowest energy ‘ground’ states.

However, once such a ‘high energy’ environment is allowed to relax, ‘lower energy’ fecund mutable digit sequences could form out of the prime state arrangements (e.g. MBN 12_1 --> $2_20_20_1$ or 24_1 --> 8_30_1), introducing more structure and differentiation into the system – illustrated in Figure 3.14. Thus out of the symmetrical unity of a single extended fundamental harmonic series, a diverse succession of tonal chords can emerge, in the form of less energetic multi-column mutable number digit sequences – which carry within their leading columns (aggregated series) the outlines of a composition’s harmony. Such a train of thought points toward an understanding of the harmonic physiognomy of the broad corpus of western music, conceived in terms of the breakdown of a prime state symmetry (extended fundamental nesting series) into the many different structural asymmetries exhibited in typical examples of tonal harmony (i.e. multiple, nested, harmonic series). Here some distinct parallels and similarities emerge between the MOS approach and that of Schenkerian analysis. A symmetrical prime state fundamental nesting series looks rather like Heinrich Schenker’s elemental ‘chord of nature’, while the broken symmetry of multi-column digit sequences (a system of nested harmonic series) takes on something of the appearance of hierarchical structural levels – Vordergrund, Mittelgrund and Hintergrund – a central concept in his analyses.

Notes

1. Statistically there is a very, very, very small probability that the entropy of a system will rise of its own accord, as described by Henri Poincaré. The period or cycle time of this occurrence, for a typically complex system, is so vast as to be of little practical significance.

2. Shannon, C.E., and Weaver, W., *The Mathematical Theory of Communication,* (University of Illinois Press, Urbana, 1949).

3. Davies, P., *The Origin of Life,* (Penguin, London, 1998, 2003) p37–42.

4. Theoretically perhaps 1x23 and 23x1 could be counted as two states. One group of twenty-three (MBN 23_1) and twenty-three groups of one (MBN 1_{23} 0_1). Given this approach, logically, it would apply to all prime state configurations (i.e. 1x24 and 24x1) and increase by 1 the values given in the middle column of Figure 3.11. However, the trend exhibited in the right-hand column would not be materially different.

5. A formula or procedure which appears to yield the number of distinguishable arrangements for the early series of most fecund mutable numbers is given by multiplying the number of prime factors plus one (of the fecund number in question) with the product of those same prime factors (the number itself) and dividing by six. For example,

Fecund Number	**Prime Factors**	**Calculation of Distinguishable Arrangements**	
Six	2×3	(2+1) x 6 / 6	= 3 arrangements (0.5)
Twelve	2x2x3	(3+1) x 12 / 6	= 8 arrangements (0.666...)
Twenty-four	2x2x2x3	(4+1) x 24 / 6	= 20 arrangements (0.8333...)
Forty-eight	2x2x2x2x3	(5+1) x 48 / 6	= 48 arrangements (1.0)
Ninety-six	2x2x2x2x2x3	(6+1) x 96 / 6	= 112 arrangements (1.1666...)
One hundred and ninety-two	2x2x2x2x2x2x3	(7+1) x 192 /6	= 256 arrangements (1.333...)
Three thousand & seventy-two	2x2x2x2x2x2x2x2x2x2x3	(11+1) x 3072 / 6	= 6144 arrangements (2.0)

There also appear to be similar, though more complex, procedures for at least some of the other series of fecund numbers such as 2^nx3x5: 30, 60, 120, etc. and 2^nx3x5x7: 210, 420, 840, etc.; and of course the formula for the powers of two is simply the product of the prime factors (the number itself) divided by two.

Lists of the number of arrangements (i.e. digit sequences) of mutable base numbers from one to five thousand and one to twenty thousand can be found in the EXTRAS folder/directory. Files 1-5000.TXT and 1-20K.TXT. The sequence of most fecund numbers is marked in the lists.

6. Fourier analysis, discussed in Chapter 4, predicts that any periodic system is ultimately reducible to a single fundamental harmonic series.