JOURNEY TO THE HEART OF MUSIC

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For my father,
Harold Ernest Perry,
who might well have written a book, had he not devoted so much of his time and energy to the care of his family: with gratitude, love and respect.

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Preface

Journey to the Heart of Music is formed from a collection of articles, essays and examples outlining a personal research project investigating fundamental structures and processes in tonal music. After a protracted period of metamorphosis and growth (2003–2009) the text reached something approaching maturity in the third edition (2009). The present document, the fourth revised edition, while no doubt still leaving much to be desired, marks the end of this endeavour. It remains unclear to me whether my approach to understanding the ultimate nature of music has borne fruit or not: Whether there is something of interest and value expressed within these pages or merely a mistaken illusion. Naturally, I hope it is the former rather than the latter. Indeed, if I did not believe there was some validity to the ideas expressed here, I would not have continued to pursue the project. Yet still, I am content that you, the reader, will make your own judgement. Thank you for your interest and support.

Philip Perry, 2015.

INTRODUCTION

It is a commonplace observation that there is a close connection between music and numbers, usually everyone sagely agrees... and nothing more is said. While being wholly in accord with the idea that there is a great deal of mathematics in music, I would like to go further, and attempt to say precisely what that connection might be. Indeed in this document, I hope to present a reasonably complete mathematical theory of tonal music – whilst also, upon the sidelines, musing rather speculatively about further possible applications of the model. So, for the sake of conciseness and clarity, the hypothesis set forth in this book is: *that western tonal music is one member, of a perhaps broad set of oscillatory systems found in the material world, which are all, fundamentally, physical position-value counting structures*.

Put in straightforward language: hidden away, below the surface of music, lies an implicit number system (with units: 1, 2, 3, etc. and columns) similar though not identical to the one we use in everyday life. It is, I suspect, this number system which provided the ultimate source of structure and coherence underpinning the great flowering of western tonal music, that began to take shape in Europe in the sixteenth century, reached its zenith during the eighteenth century and faded to a mere shadow in years after 1900. However, though many composers shunned its use in the twentieth century, it has proved to be a vigorous and hardy species: invasive to foreign music cultures and impossible to restrain in popular music genres throughout the world. There is I believe, something rather special and distinctive about the music of the tonal era, we revere the works of masters long after the epochs and societies which gave them life and context have disappeared. Indeed we preserve and cherish them often all the more today, ever reluctant to let go of music which speaks to us still – perhaps more cogently than the music of our own era. Not unlike the Indo-Arabic number system which has come to be used throughout the modern world, tonal music and the number system that it harbors, is probably here to stay.

One quiet Sunday morning in 2003, a solution to a small computer programming problem which I had been trying to get around for some time, occurred to me. The problem arose out of a trivial tutorial project devised to help me learn the programming language AWK[1]. Professional computer programming is today the field of highly skilled experts, but in a not too distant past of the home micro-computer, it has been an area where an amateur recreational programmer could happily and harmlessly roam; and apply a little of the miraculous computational power now available, to another great passion: music.

The particular programming language I was attempting to learn is designed specifically for the manipulation of text and therein lay the root of my problem. I wanted to use AWK to manipulate music data, rather than text. In fact, the character of my project was unsuited to AWK's capabilities and it would have been better to change the topic, or the programming language. Indeed, the result of this mismatch was a rather messy sequence of steps involving the conversion of music into a textual description of a score, upon which my tutorial programs would then operate.

The bright idea was that a solution to my problem might lie in expressing the music data – the notes – as parts of harmonic series, and, as relationships between these note-laden harmonic series. That is, representing the whole of a composition as interrelated *ratios*. (The harmonic series are the 'notes' of the natural pattern of vibration – the overtones or partials of a base vibration – explored in more detail in Chapters 1 and 7.) Relationship proved to be the foundation stone of this system: the elemental stuff of music. In music the dimensions of pitch and duration have no intrinsic meaning, it is only through the relationships between units of these two elemental qualities that any specific significance arises. The frequency 256 Hz or 261.6 Hz, middle C, has no intrinsic 'C-ness', it only takes on this meaning when the notes around it such as the dominant note G and subdominant note F cast the note C in the role of tonic and make it the tonal center. Thus I found by focusing on the connections between musical objects, rather than relying upon fixed or assigned elements, a fully relational approach to music could be developed.

In this new format the chords of a piece of music would be interpreted as incomplete harmonic series and so any collection of notes sounding simultaneously as a chord, could find a place within the overtones of an implied (but often not present) fundamental tone. Thus any chord could be considered a *configuration of partials* – combinations of ratios capable of locking into particular positions within an overtone series: with the ratios or relationships between the implied fundamental tones of the many different chords/series in a composition, providing a structural foundation for the envisioned music data format. A particularly attractive part of this scheme, was that as well as describing the harmonic aspects of music, such a data format could seamlessly encompass the rhythmic and metrical dimension of a score as well, by extending the proportions of the harmonic series up through unity, the number one, to form arithmetic series of whole number relationships; thus, at least notionally, encompassing both the harmonic and metrical dimensions of music in a single format.

In mathematics arithmetic series are counting sequences: one, two, three, four, etc. and harmonic series the reciprocal form: one, one-half, one-third, one-quarter, etc. From the number one, arithmetic series count upward without limit while harmonic series count down from one, toward zero. From the mathematician's point of view, harmonic and arithmetic series are sequences of abstract relationships, to consider in the mind or manipulate on the page.

$$\frac{1}{5} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{2} \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$

harmonic series ← unity → arithmetic series

In contrast, for the musician, these ratios furnish and govern the very base material – pitch and duration, notes and rhythms – from which compositions are made, and by which *encoded notes acquire physical existence in performance*. When musicians engage with musical sound, they are dealing with a physical phenomenon emanating from energised material bodies in the form of sound waves and vibration: Notes consisting of harmonic series of wavelength relationships; and for quantities and relationships of time, the musician feels and counts one-and-two-and-three..., with physical gestures describing a arithmetic semaphore. Over many generations musicians have developed a deep intuitive understanding of the nature of ratio, derived literally from 'hands-on' experience of the harmonic series.

Essentially, through the stricture of giving material life to the written encodings contained in scores, the oscillatory relationships that underlie intelligible music, are found, overall, to be the ratios of simple whole numbers. With the two most basic parameters of traditional western music governed by these simple ratios (i.e. pitch and duration) taking on something of a *quantised* character. Glissando and free rhythm aside, for the most part traditional music proceeds by quanta, little steps and jumps of pitch and duration from note to note and chord to chord. However in contrast to music, where what might be termed 'performance validation' limited tonal composers' effective range of options considerably, 'abstract' mathematics is not so constrained by the imperative of giving material form to its computations, and so mathematicians largely escape these physical disciplines. The 'music of mathematics' has something of the freedom of software about it, whilst the 'computations of music' hint at the more limited characteristics associated with hardware processing. Though composers are free to use any combination of sounds (as a mathematician might perform operations on an unlimited range of real values) generally, throughout the tonal era (circa 1500–1900), musicians were largely content to work within the closely prescribed limits of relatively few discrete units of pitch and duration.

Ultimately, harmony and meter in traditional western tonal music reveal themselves to be essentially the constructs of lower order whole number relationships – simple ratios. Ratios for example, such as 1:4 of the quarternote to the measure or 3:2 of the dominant to the tonic. Musicians found this math in the tonal period, *intuitively*, but were unable to recognise it as such. *The 'ear' understood what the reasoning mind could not then fully grasp.* The development of western music proceeded largely by trial, error and inspiration: composers and musicians, over time, gradually explored the habitable range of sound ratios, with each generation seeking out new territories to annex to the tonal system. Interestingly, in a similar manner, arguably, the artisans of the Alhambra discovered all, or almost all, of the seventeen possible two dimensional symmetry patterns in their decoration of the great moorish palace,[2] but didn't have a sufficiently developed mathematics to be able to fully appreciate what they had achieved. But significantly, both in western music and Moorish decoration, the systems within which they were working contained firm physical and mathematical constraints which helped guided their evolution. Not until the nineteenth century did mathematicians develop suitable concepts of symmetry to understand what the old artisans' eye and ear knew from the beginning. Thus I shall argue and explain, that rather as geometry might be broadly conceived as 'visual' form of mathematics, equally, tonal music should be understood as an 'aural' expression of arithmetic: The computation of numbers in sound.

Gradually, out of these ideas of mixing mathematics and music, of treating chords as parts of harmonic series and chord progressions as sequences of interrelated series, came the realisation that my data format might actually encompass a description of the physical progress through a composition expressed in terms of *oscillatory computation*. Indeed, what this data format appears to point to is an underlying computational foundation within music itself, where commensurable chord progressions and changes in meter *compute equal sums* (accessible to the ear) which negotiate a smooth succession between adjacent musical events: A mechanism for which I have appropriated the musician's term *modulation*. This process of oscillatory computation, when taken in aggregate, allows the performance of a tonal compositions to be interpreted and explained in terms of number processing – arithmetic.

While developing the concept of *oscillatory computation*, the examination of the harmonic structure of tonal music proved particularly helpful: in that music stands on the threshold of material existence, in the sense that musicians as well as composing scores, also give *physical utterance* to their ideas – by playing the music. Music has, so to speak, one foot in each camp, with formal symbolic scores transformed into dynamic sound structures in performance. The rigour of performance forces musicians and audiences to consider which relationships succeed in the material world; and to choose between the former and those which, while looking good on the page, or perhaps fulfilling the logic of some arbitrary theory or structural idea, don't actually produce satisfactory musical experiences.

abstract/symbolic	threshold	real/physical
mathematics	tonally organised music	dynamics
formal systems		self-organising systems
scores	↙ ↘	performance
mutable base numbers		modulating oscillatory systems

Mathematics in general and counting in particular, though I didn't realise it at that time, would become the major theme, and the solution to a mundane programming problem was to be the first step in a long and at times tortuous journey. At first, without a map, ill equipped for the task, and with little idea of where each step was leading, instinct, and above all good luck, set the course. Many were the cul-de-sacs stumbled down and circular paths trod. However, little by little, occasionally glimpsed, lost again from sight, then more clearly seen from another angle, gradually, a firmer route and eventual destination emerged. At first hazily grasped, unexpected, unimagined; *the mutable base (position-value) number system*, was the awkwardly named yet deeply beautiful goal, I sought (abbr. MBN or *mutable numbers*).

Of course, whether this trail of exploration has actually found fruitful territory or a barren wasteland of mirages, is a question for others to decide. Essentially, here in these documents, you have the journal of my travels and travails, straightforwardly, though perhaps naively put. Throughout the exploration, time and again, I would find myself walking in the footsteps of giants, following paths often marked out long before. Of the numerous list of guides, there are five in particular: Jean-Philippe Rameau who first articulated the pre-eminent role of harmony and its origin in the *fundamental bass*, Arthur von Oettingen for the theory of *harmonic dualism*, Richard Franko Goldman for his penetrating analysis of *Harmony in Western Music*, Stephen Wolfram for the monumental book *A New Kind of Science* and latterly James Beament's book *How We Hear Music*. Woven through with the ideas and insights of these and many other authors, I have attempted to create a synthesis of musical, mathematical and scientific elements to yield a general relational analysis of the structure and evolution of tonal music (and other oscillatory systems).

Though the route traced out in the following pages will touch on mathematics, computer programming and science, it is principally the account of a musical expedition. *A journey to the heart of music*. There is no reason to be daunted or put off by technical terms and references to science or mathematics; topics are explained in the text and terms in the glossary (Appendix C). This document is written with no claim to great knowledge or special expertise, but rather, simply to communicate ideas about the fundamental nature of music, which I hope other people interested in such topics, might care to read. Having begun life as a collection of articles[3] and essays there is inevitably some degree of disjunction and repetition in the text; and, by beginning with the core concept – mutable base numbers – there might be a hint of being thrown in at the deep end! (If the text becomes too impenetrable, a 'quick start' outline of selected extracts from the main text can be found in Appendix E, providing a gentle precursor to Chapter 1.) On occasion information presented later within the wider story will also enhance and amplify ideas encountered earlier in the text. There is also a music theory reference section, Appendix A, to prepare and remind the reader, if necessary. If the terms *dominant* and *tonic chords* or the symbols V^7–I are unfamiliar, it might a worthwhile browsing the theory toolkit first.

The 'book' follows a simple plan. After this introduction, Chapter 1 jumps to the end point, presenting an account of the final destination – music as a system of number. This makes for a tough start, but the benefit is that it sets the perspective on the terrain subsequently covered. Chapter 2 then returns to the beginning, from where the text leads the reader, step by step along the path originally followed, from music through to numbers. In passing, some of the landmark ideas and individuals from the past are incorporated in the story, alongside more recent developments. However, while it is perfectly possible to read 'cover to cover', as some chapters are more or less for reference (e.g. Chapter 10) and others are quite technical and/or tangential (e.g. Chapter 14), the first reading of the text might well involve some skipping over or skimming through the remoter stretches. Of the major features, Chapter 1 outlines the core concept – music as a number system, Chapter 6 introduces the central mechanism – the algorithm of symmetrical exchange, Chapter 8 provides the historical background, Chapter 9 presents the dynamics of the model and Chapter 15 places the overall idea in a broader context beyond music. With these footholds secured the more technical and esoteric chapters will fall more easily into place. Finally a ‘spin-off’ from the theoretical model of music presented here, is the concept of *reflection* – the symmetric inversion of a composition – introduced in Chapter 14 and illustrated in Examples A to F.

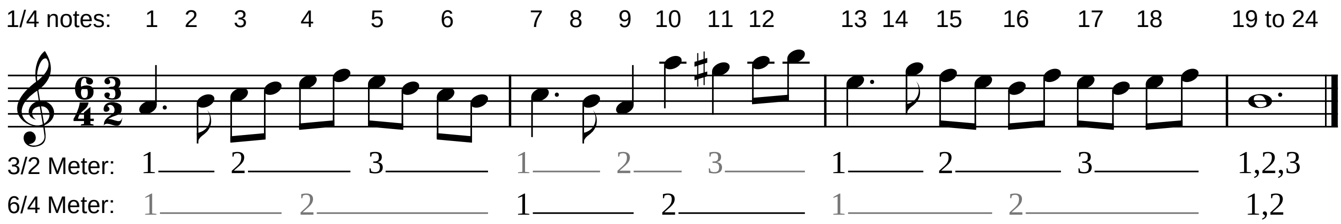
Although the central concept of *music as a number system* provides the overall conclusion presented in *Journey to the Heart of Music*, a significant portion of the text is framed in terms of a parallel and equivalent view of music functioning as (in principle) a self-organising dynamical system – a *quasi-physical relational scheme* with characteristics not entirely dissimilar to those found in wholly natural systems. Such pseudo-dynamical, and perhaps real entities – almost self-contained relational machines – are termed *Modulating Oscillatory Systems*. Abbreviated to MOS for short. This twin track approach is pursued by interleaving chapters principally concerned with tonal compositions viewed as dynamic pseudo-physical systems, with others focusing more on matters of number and computation. Being able to approach the subject from these two complementary perspectives allows *a connection to be drawn between what is, in principle, a dynamic system of the material world yet also a formal (number) system, deriving significance from the context of the human intellect.* (In practical music analysis these two threads – of the physical MOS and the formal MBN – may be combined; with the former embodying the perceived musical sound and the later providing a underlying mathematical framework.)

All of this ultimately rests upon a simple observation: that at the elemental level whole number relationships underlie and inform the architecture of both the wide material world and the little world of tonal music; and, that these common properties of waves are likely to be manifest, to a greater or lesser extent, in many structures throughout the universe, just as they are in music. Thus a deep understanding of music might provide clues to unravelling other systems, and vice versa. However, as to the question of identifying genuine (rather than ‘in principle’) examples of self-organising dynamical systems of the type proposed, I keep an open mind about what is no more than an intuition, a guess; and, in the closing Chapter 15 some interesting patterns in a range of physical systems are explored. Though I must frankly admit that my expertise in these regions is limited and it is only in the field of music that I have been able to attempt a reasonably comprehensive application of the concept of modulating oscillatory systems and mutable numbers – that is, to offer *a fundamental theory of tonal music in the form of the MOS model*. My intentions in reaching out to other fields of knowledge are principally illustrative and exploratory: to highlight parallels, suggest connections and not least of all, to seek inspiration. Additionally, as the early development of the model was in part based on the examination of the structure of a number of physical systems for which there was good objective data, telling the full story of mutable numbers requires the inclusion of this material. Yet, I realise that such an approach might well be viewed as little better than fanciful, so I would encourage the sceptical reader to simply ignore this highly speculative aspect, if they prefer, and treat the model solely as a theoretical construct capable of providing *an interpretation of the structure of tonal music* devoid of any linkage with systems found in the natural world.

Notwithstanding that an interest in music and some knowledge and experience of its techniques and technicalities will be an asset, and allowing that some sections are for technical reference rather than discursive reading; there is nothing in these documents that is inherently difficult to grasp. And it is my express intention that, overall, the general reader as well those of a musical and scientific background should come to understanding what is at heart a simple idea, which I believe to be beautiful, and hope, might also prove to be fruitful.

Meter

Ideas related to metrical structure in music play a significant role in the story told in this book and I introduce a short example here to establish a central principle of the argument pursued in *Journey to the Heart of Music*: that a description of harmonic progression framed in essentially metrical terms reveals tonally organised music to be, ultimately, *a positional number system written in sound*. And, it is perhaps more than coincidental that in their passage from the environment to the human mind, the frequency relationships of musical sound are imbued with something of a metrical quality by their conversion into nerve impulses in the auditory pathway of the ear.



The above extract is drawn from a Courante in the *English Suites* by J.S. Bach, a piece renowned for an unstable metrical structure. The music veers between measures of three metrical units and measures of two metrical units, measures of time signature 3/2 and 6/4, without settling down to one or the other meter. The durational period common to both meters is the quarternote – counted out above the staff. There are a total of twenty-four quarternote periods in the entire four bar extract. Below the staff, the two competing metrical structures of three groups of two quarternotes per bar and two groups of three quarternotes per bar, are delineated. *Over the four measure extract both meters, though of differing internal structure, span twenty-four quarternotes and are equivalent in this regard.*

I suspect it would be near impossible to hear twenty-four quarternotes as perfectly even metrical sub-units. Normally the contours of melody and dynamic accent coax the listener into perceiving, or imposing, some real or imagined metrical pattern upon the notes. In combination the ear and processes of aural cognition, seeking to make sense of the incoming aural stimuli, discover and/or create metrical patterns and hierarchies. A simple recurring metrical pattern provides a most useful general algorithmic framework to aid perception where no two measures are precisely identical in accent and stress: order is found or imposed, understanding enlarged and amorphous complexity rendered intelligible.

In the above example, while two different metrical entities, 6/4 meter and 3/2 meter, vie for precedence, both meters have a point of equivalence or conjunction over the period of four measures and twenty-four quarternotes. Indeed, the meters conjoin at one and two measures as well as four, however, the four measure sum of twenty-four sub-units has a particular connection with the foundational harmonic exchange of the dominant-tonic full cadence, a chord progression which occupies a central position in this book. *In the following chapters the principle of metrical ordering and equivalence is extended from the realm of pulse and duration to the domain of audible sound, where it will be applied to the ordering and equivalence of oscillations and periods of pitch, providing the basis for a new approach to understanding the nature of harmony in western music.*

Briefly moving beyond the sphere of music to mathematics and physics, meter remains a potent concept: in that the prime numbers may be identified with simple meters (e.g. 2/4, 3/8, 5/4 time), which have no symmetric division except by single pulses (i.e. division by 1); and similarly the composite numbers find an echo in regular compound metrical forms such as 4/4, 6/8, 9/8 time which are divisible into smaller rhythmic sub-units. Though an obvious example, the numeric character of music is patently manifest. As also are the connections with physics: where the multitudinous regularities, oscillations and periods found amongst physical systems throughout the material world – stretching from particles to planets – when cast in terms of meter, so often display characteristics reminiscent of musical organisation.

Synopsis

Underlying the structure of harmony and meter in tonal music there is a positional number system,
and thereby it follows that
a tonal composition can be interpreted as a sequence of numerical operations – a computation.

Working on the supposition that western music evolved over the heads and beyond the time scales of individual composers and musicians to find the most effective aural model for a relational oscillatory system (in the form of tonally organised music circa 1500–1900); and, that through a close examination of this music the nature of the underlying organisational scheme can be divined; a model of *modulating oscillatory systems* (MOS) is proposed as the ultimate structural device underpinning tonal music. Though primarily directed toward the harmonic and metrical analysis of traditional western music, it is a *general* relational model, which might also prove useful in the interpretation of other musical genres and cultures; and, more widely, in the generic study of oscillatory/periodic systems: by virtue of its high level of abstraction and computational focus.

Essentially, a piece of music is viewed as being a set of evolving frequency relationships ranging from the period of the entire composition, through durational periods of meter/rhythm and pitch periods of notes/chords, up to the highest frequencies of timbre. Indeed, by generalising the principle of oscillation over music's full frequency range, the grouping of audible frequencies into 'columns' (of a positional number system) becomes no less unnatural, or unexpected, than the familiar grouping of rhythmic pulses into meters. Through this metrical approach, the MOS model reduces a 'dynamical oscillatory structure' – in principle a tonal composition – to a succession of numbers in a *mutable base position-value number system* (MBN): A counting scheme similar to fixed base systems like decimal or binary, though richer, in that most numbers can be accessed by a variety of different digit sequences. And it must be stressed, this number system is organic and integral to the 'ear's natural understanding' of tonal music and not some arbitrary projection of mathematics onto music. Thus by identifying individual chords (harmonies) as representing particular digit sequences, the feature of multiple (different) digit sequences describing one and the same value may then be used to delineate and elucidate the nature of harmonic progression in traditional western music: which by implication, perhaps, also offers an insight into the mechanism through which the processes of aural cognition turn objective musical sound into a meaningful language – *a number system*.

Once grasped, the conception of harmony as digit sequences (written in musical sound) in the mutable base number system, and harmonic progression as exchanges between different digit sequences representing the same value, a fresh perspective can be taken on a wide range of historical, musicological, physiological and theoretical topics.

Acknowledgements

I am most grateful for the generous help provided by Gerald Fitton in the preparation of the first article in the precursor series *Elements of Music?* and to Paul Beverley, the editor of *Archive* magazine for his long-suffering care and attention throughout the series. With great generosity, some years later, Paul Beverley additionally edited the text of this document: a substantial task for which I am deeply grateful.

Also I would like to acknowledge my debt to the contributors to the online encyclopedia *Wikipedia*, for the provision of much information and, in particular, many of the biographical facts and photographs scattered through the text of *Journey to the Heart of Music*.

Notes

1. AWK originally written in 1977, and much enhanced in later years, is named after the initials of its three creators, A.V.Aho, P.J.Weinberger and B.W.Kernighan, from AT&T Bell Laboratories. The Free Software Foundation package GNU AWK (gawk) is the most commonly available version of this classic text manipulation utility of the UNIX computer operating system (http://www.gnu.org/software/gawk/gawk.html). AWK, I have found, is a most useful tool, with many hidden depths. Easier to get into and use than its big brother Perl and much more compact, AWK is an interpreted language, implemented on almost all operating systems and so its programs (scripts) are entirely transportable between computers.

2. I am grateful to Chad Twedt for pointing out that there is a degree of interpretive disagreement amongst scholars as to the precise number of symmetry patterns represented in the decorations of the Alhambra palace.

3. *Elements of Music?,* Archive Nov.03 Vol.17.2 – June.05 Vol.18.9, a speculative and exploratory set of articles and files examining structure in oscillatory systems.

Conventions: A pitch standard of middle C = 256Hz (a little lower than Concert Pitch, A = 440Hz) sometimes called scientific or philosophical pitch, is used throughout – for the convenience of its concordance with the octave-doubling powers of two frequency sequence, e.g. $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, etc., $2^8 = 256$. Linked in with this choice is the fact that the vast majority of examples and discussions focus on the ‘prototype’ key of C-major. Any major key could have been used, given the symmetrical relationships of the key system – what is true of one is true of all. The minor mode is viewed as an ‘inflected’ or altered form of the major key. Usage is International/American English and in matters of typography generally only standard fonts and text characters have been employed; thus some signs are written out (e.g. Aflat) and others substituted. Finally, a nexus is drawn between this proto-key or *tonal center of C* and the ratios of the harmonic series, such that each ratio is associated with a note letter (and generally sharp sign if required, e.g. Bflat = A#). Implicit in this convention is the view of the scale tones as a system of *dynamically varying* ‘pure’ ratios and relationships, rather than a fixed (and ‘impure’) grid of equally tempered pitches. This I suspect, is what the tolerance of the ear allows the processes of aural cognition to extract from objective musical sounds.