Journey to the Heart of Music

Philip Perry

(Version C)

For my father,
Harold Ernest Perry,
who might well have written a book, had he not devoted so much of his time and energy to the care of his family. With gratitude, love and respect.

"In the snow, flat-topped hillocks and shoulders, outlined with wavy edges, ridge below ridge, very like the grain of wood in line and in projection like relief maps. These the wind makes I think and of course drifts, which are in fact snow waves. The sharp nape of a drift is sometimes broken by slant flutes or channels. I think this must be when the wind after shaping the drift first, has changed and cast waves in the body of the wave itself. All the world is full of inscape, and chance left free to act falls into an order as well as purpose".

Gerard Manley Hopkins – lines describing a snowy landscape, from his Journal: 24th Feb. 1873.

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Preface

Journey to the Heart of Music is a collection of articles and essays outlining the current state of a personal research project, investigating fundamental structures and processes in tonal music – presented in a ‘book-like’ form of numbered chapters plus examples. After a necessarily protracted period of metamorphosis and growth (2003–2009) they are here made available over the internet in PDF format or on compact disc. The present document, while no doubt still leaving much to be desired, marks something of a culmination in proceedings. In time, some extension of the analytical examples may be undertaken, along with a continuing upgrading and updating of the text. All comment and feedback is most welcome, particularly where this leads to a strengthening of the documents through the identification of errors, misconceptions, typological mistakes, etc.; or, the enhancement of different perspectives, new ideas and additional data. A great benefit of internet access and the compact disc format is that it allows far more scope for musical examples to be included – written scores, analyses and some MIDI files. Each CD is individually numbered, while the online documents have dates on their final pages. The latest version of these documents is available from *http://www.pjperry.freeuk.com/index.htm* by following the links. Thank you for your interest and support.

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INTRODUCTION

It is a commonplace observation that there is a close connection between music and numbers, usually everyone sagely agrees... and nothing more is said. While being wholly in accord with the idea that there *is* a great deal of mathematics in music, I would like to go further, and attempt to say precisely what the connection might be. So, for the sake of conciseness and clarity, the hypothesis set forth in this book is: *that western tonal music is one member, of a perhaps broad set of oscillatory systems found in the material world, which are all, at bottom, examples of physical position-value counting structures*. Which might or might not mean something to you the reader. Whichever is the case, I hope you will continue reading so as to understand why and how I have arrived at this conclusion.

Put in straightforward language: Hidden away, below the surface of music, lies a number system (with units: 1, 2, 3, etc. and columns) similar though not identical to the one we use in everyday life. It is, I suspect, this number system which provided the ultimate source of structure and coherence underpinning the great flowering of western tonal music, that began to take shape in Europe in the sixteenth century, reached its zenith during the eighteenth century and faded to a mere shadow in years after 1900. However, though many composers shunned its use in the twentieth century, it has proved to be a vigorous and hardy species: invasive to foreign music cultures and impossible to restrain in popular music genres throughout the world. There is I believe, something rather special and distinctive about the music of the tonal era, we revere the works of masters long after the epochs and societies which gave them life and context have disappeared. Indeed we preserve and cherish them often all the more today, ever reluctant to let go of music which speaks to us still. Like the arabic number system which has come to be used throughout the modern world, tonal music is probably here to stay.

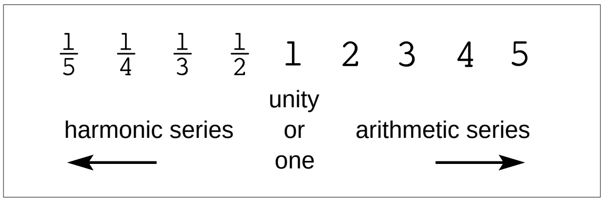
One quiet Sunday morning in 2003, a solution to a small computer programming problem which I had been trying to get around for some time, occurred to me. The problem arose out of a trivial tutorial project devised to help me learn the programming language AWK[1]. Professional computer programming is today the field of highly skilled experts, but in a not too distant past of the home micro-computer, it has been an area where an amateur recreational programmer could happily and harmlessly roam; and apply a little of the miraculous computational power now available, to another great passion: music.

The particular programming language I was attempting to learn is designed specifically for the manipulation of text and therein lay the root of my problem. I wanted to use AWK to manipulate music data, rather than text. In fact, the character of my project was unsuited to AWK's capabilities and it would have been better to change the topic, or the programming language. Indeed, the result of this mismatch was a rather messy sequence of steps involving the conversion of music into a textual description of a score, upon which my tutorial programs could then operate.

The bright idea was that a solution to my problem might lie in expressing the music data – the notes – as parts of harmonic series, and as relationships between these note-laden harmonic series. That is, representing the whole of a composition as interrelated *ratios*. (The harmonic series are the 'notes' of the natural pattern of vibration – the overtones or partials of a base vibration – explored in more detail in Chapters 1 and 7.)

In this new format the chords of a piece of music would be interpreted as incomplete harmonic series and so any collection of notes sounding simultaneously as a chord, could find a place within the overtones of an implied (but often not present) fundamental tone. Thus any chord could be considered a *configuration of partials* – combinations of ratios capable of locking into particular positions within an overtone series: With the ratios or relationships between the implied fundamental tones of the many different chords/series in a composition, providing a structural foundation for the envisioned music data format. A particularly attractive part of this scheme, was that as well as describing the harmonic aspects of music, such a data format could seamlessly encompass the rhythmic and metrical dimension of a score as well, by extending the proportions of the harmonic series up through unity, the number one, to form arithmetic series of whole number relationships.

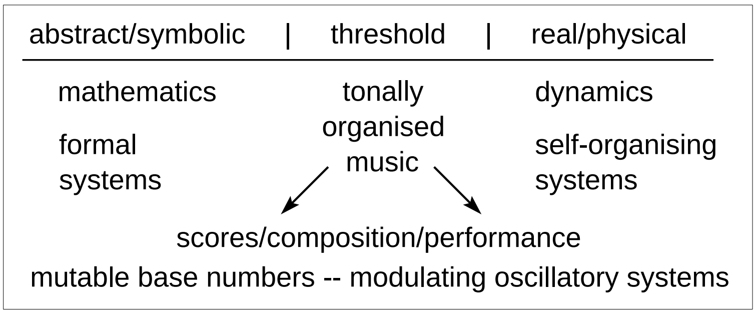
In mathematics arithmetic series are counting sequences: one, two, three, four, etc. and harmonic series the reciprocal form: one, one-half, one-third, one-quarter, etc. From the number one, arithmetic series count upward without limit while harmonic series count down from one, toward zero. From the mathematician's point of view, harmonic and arithmetic series are sequences of abstract relationships, to consider in the mind or manipulate on the page.



In contrast, for the musician, these ratios furnish and govern the very base material – pitch and duration, notes and rhythms – from which compositions are made, and through which, *they acquire physical existence in performance*. When musicians engage with musical sound, they are dealing with a physical phenomenon – expressed as harmonic series of wavelength relationships – emanating from energised material bodies in the form of sound waves and vibration. And for quantities and relationships of time, the musician feels and counts one-and-two-and-three..., with physical movement and gesture, in the motion of an *arithmetic semaphore*. Over many generations musicians have developed a deep intuitive understanding of the nature of ratio, derived literally from 'hands-on' experience of the harmonic series. Musicians found this math in the tonal period, intuitively, but were unable to recognise it as such. The 'ear' understood what the mind could not then grasp – just as, for example, the artisans of the Alhambra discovered all of the seventeen possible two dimensional symmetry patterns in their decoration of the great moorish palace; but didn't have a sufficiently developed mathematics to be able to fully appreciate what they had achieved.

Gradually, out of these ideas of mixing mathematics and music, of treating chords as parts of harmonic series and chord progressions as sequences of interrelated series, came the realisation that my data format might actually encompass, in essence, a description of a physical process of oscillatory computation – a dynamic number system built on a general mechanism of symmetrical exchange – for which I have appropriated the musician's term: *modulation*. By implication, what this data format appeared to be pointing to, is an underlying computational foundation within tonal music itself.

While developing the concept of *oscillatory computation*, the examination of the harmonic structure of tonal music proved particularly helpful: in that music stands on the threshold of material existence, in the sense that musicians as well as composing scores, also give *physical utterance* to their ideas – by playing the music. Music has, so to speak, one foot in each camp, with formal symbolic scores transformed into dynamic sound structures in performance. The rigour of performance forces musicians and audiences to consider which relationships succeed in the material world; and to choose between the former and those which, while looking good on the page, or perhaps fulfilling the logic of some theory or structural idea, don't actually produce satisfactory musical experiences.



Essentially, through the stricture of giving material life to the written encodings of notes, the oscillatory relationships which underlie intelligible music, are found, overall, to be the ratios of simple whole numbers. With the two most basic parameters of traditional western music – pitch and duration – governed by these simple ratios taking on something of a *quantised* character. Glissando and free rhythm aside, for the most part traditional music proceeds by quanta, little steps and jumps of pitch and duration from note to note and chord to chord. However, in contrast to music, where performance validation has limited most composers' effective range of options considerably; 'abstract' mathematics is not so constrained by the imperative of giving material form to its computations, and so mathematicians largely escape these physical disciplines. The 'music of mathematics' has something of the freedom of software about it, whilst the 'computations of music' hint at the more limited characteristics associated with hardware processing. Though composers are free to use any combination of sounds (as a mathematician might perform operations on an unlimited range of real values) generally, throughout the period of common harmonic practice (circa 1500-1900), they were content to work within the closely prescribed limits of relatively few discrete units of pitch and duration.

Mathematics in general and counting in particular, though I didn't realise it at that time, would become the major theme; and the solution to a mundane programming problem was to be the first step in a long and at times tortuous journey. At first, without a map, ill equipped for the task, and with little idea of where each step was leading, instinct, and above all good luck, set the course. Many were the cul-de-sacs stumbled down and circular paths trod. However, little by little, occasionally glimpsed, lost again from sight, then more clearly seen from another angle, gradually, a firmer route and eventual destination emerged. Hazily grasped initially, unexpected, unimagined: *the mutable base position-value number system,* was the awkwardly named yet deeply beautiful goal, I sought. (*Mutable numbers* for short.)

Of course, whether this trail of exploration has actually led into fruitful territory or a barren wasteland of mirages, is a question for others to decide. Essentially, here in these documents, you have the journal of my travels and travails, straightforwardly, though perhaps naively put. Throughout the exploration, time and again, I would find myself walking in the footsteps of other people, following paths often marked out long before. Of the numerous list of guides, there are five in particular: Jean-Philippe Rameau who first articulated the pre-eminent role of harmony and its origin in the *fundamental bass,* Arthur von Oettingen for the theory of *harmonic dualism*, Richard Franko Goldman for his penetrating analysis of *Harmony in Western Music,* Stephen Wolfram for the monumental book *A New Kind of Science* and latterly James Beament's book *How We Hear Music*. Woven through with the ideas and insights of these and many other authors, I have attempted to create a synthesis of musical, mathematical and scientific elements to yield a general relational approach to the structure and evolution of oscillatory systems.

Though the route traced out in the following pages will touch on mathematics, programming and science, it is principally the account of a musical expedition. *A journey to the heart of music*. There is no reason to be daunted or put off by technical terms and references to science or mathematics; topics are explained in the text and terms in the glossary (Chapter 18 – Appendix C). This document is written with no claim to great knowledge or special expertise, but rather, simply to communicate ideas about the fundamental nature of music, which I hope other people interested in such topics, might care to read. Having begun life as a collection of articles[2] and essays there is inevitably some degree of disjunction and repetition in the text; and, by beginning with the core concept – mutable base numbers – there might be a hint of being thrown in at the deep end! (If the text becomes too difficult, a 'quick start' Outline of selected extracts from the main text can be found in the CHPT19 folder/directory, providing a gentle precursor to Chapter 1.) On occasion information presented later within the wider story will also enhance and amplify ideas encountered earlier in the text. There is also a music theory reference section, Chapter 16 - Appendix A, to prepare and remind the reader, if necessary. If the terms *dominant* and *tonic chords* or the symbols V^7 - I are unfamiliar, it might a worthwhile browsing the theory toolkit before Chapter 1.

The 'book' follows a simple plan. After this introduction, Chapter 1 jumps to the end point, presenting an account of the final destination – music as a system of number. This makes for a tough start, but the benefit is that it sets the perspective on the terrain subsequently covered. Chapter 2 then returns to the beginning, from where the text leads the reader, step by step along the path originally followed, from music through to numbers. In passing, some of the landmark ideas and individuals from the past are incorporated in the story, alongside more recent developments. However, while it is perfectly possible to read 'cover to cover', as some chapters are more or less for reference (e.g. Chapter 10) and others are quite technical and/or tangential (e.g. Chapter 14), the first reading of the text might well involve some skipping over or skimming through the remoter stretches. Of the major features, Chapter 1 outlines the core concept – music as a number system, Chapter 6 introduces the central mechanism – the algorithm of symmetrical exchange, Chapter 8 provides the historical background, Chapter 9 presents the dynamics of the model and Chapter 15 places the overall idea in a broader context beyond music. With these footholds secured the more technical and esoteric chapters will fall more easily into place. Finally a 'spin-off' from the model of music presented, is the concept of *reflection* – the harmonic inversion of a composition – introduced in Chapter 14 and illustrated in Examples A to F.

Although the central concept of *music as a number system* provides the overall conclusion presented in *Journey to the Heart of Music*, a significant portion of the text is framed in terms of a parallel and equivalent view of music (in principle) as a self-organising dynamical system – a *pseudo-physical relational scheme* with characteristics not entirely dissimilar to wholly natural systems. This twin track approach is pursued by interleaving chapters principally concerned with tonal compositions viewed as dynamic pseudo-physical systems, with others focusing more on matters of number and computation. Being able to approach the subject from these two complementary perspectives allows *a connection to be drawn between what is, in principle, a dynamic system of the material world and a formal (number) system, deriving significance from the context of the human intellect.* As to the question of whether any real (rather than 'in principle') self-organising dynamical systems of the type proposed actually exist in the physical world, I do not know the answer. I keep an open mind, about what is no more than an intuition, a guess; and, in the closing Chapter 15 explore some interesting patterns in a range of physical systems. My intention in reaching out to other fields of knowledge are principally illustrative and exploratory – to highlight parallels and suggest connections; additionally, as the early development of the model was in part based on the examination of the structure of a number of physical systems for which there was good objective data, telling the full story of mutable numbers requires the inclusion of this material. However, I realise that such an approach might be viewed as little better than fanciful, and so I would encourage the sceptical reader to simply ignore this highly speculative aspect, if they prefer, and treat the model solely as a theoretical construct capable of providing *an interpretation* of the structure of tonal music devoid of any linkage with systems found in the physical world.

Notwithstanding that an interest in music and some knowledge and experience of its techniques and technicalities will be an asset, and allowing that some sections are for technical reference rather than discursive reading; there is nothing in these documents which is inherently difficult to grasp. And it is my express intention that, overall, the general reader as well those of a musical background, should come to understanding what is at heart a simple idea, which I believe to be beautiful, and hope, might also prove to be useful.

Synopsis

Working on the supposition that western music evolved over the heads and beyond the time scales of individual composers and musicians, to find the most effective organisational model for a relational oscillatory system (in the form of tonally organised music circa 1500–1900) and that through a close examination of this music the nature of the underlying scheme can be divined; a model of *modulating oscillatory systems* (MOS) is proposed as the ultimate structural device underpinning tonal music. Though primarily directed toward the harmonic analysis of traditional western music, it is a *general* relational model, which might also prove useful in the interpretation of other musical genres and cultures; and more widely, in the generic study of oscillatory/periodic systems: by virtue of its high level of abstraction and computational focus. Essentially, a piece of music is viewed as being a set of evolving frequency relationships ranging from the period of the entire composition, through durational periods of meter/rhythm and pitch periods of notes/chords, up to the highest frequencies of timbre. Indeed, by generalising the principle of oscillation over music's full frequency range, the grouping of audible frequencies into 'columns' (of a positional number system) becomes no less unnatural, or unexpected, than the familiar grouping of rhythmic pulses into meters. Through this metrical approach, the MOS model reduces a 'dynamical oscillatory structure' – in principle a tonal composition – to a succession of numbers in a *mutable base position-value number system*; a counting scheme similar to fixed base systems like decimal or binary, though richer, in that most numbers can be accessed by a variety of different digit sequences. And it must be stressed, this number system is organic and integral to the 'ear's natural understanding' of tonal music and not some arbitrary projection of mathematics onto music. Thus by identifying individual chords (harmonies) as representing particular digit sequences, the feature of multiple digit sequences describing one and the same value may then be used to elucidate the nature of harmonic progression in traditional western music: Which by implication, perhaps, also offers an insight into the mechanism through which the processes of aural cognition turn objective musical sound into a meaningful language – *a number system*.

Once grasped, the conception of harmony as digit sequences (written in musical sound) in the mutable base number system, and harmonic progression as exchanges between different digit sequences representing the same value, a fresh perspective can be taken on a wide range of historical, musicological, physiological and theoretical topics.

Acknowledgements

I am most grateful for the generous help provided by Gerald Fitton in the preparation of the first article in the precursor series *Elements of Music?* and to Paul Beverley, the editor of *Archive* magazine for his long-suffering care and attention throughout the series. With great generosity, some years later, Paul Beverley additionally edited the text of this document: A substantial task for which I am deeply grateful.

Also I would like to acknowledge my debt to the contributors to the online encyclopedia *Wikipedia*, for the provision of much information and, in particular, many of the biographical facts and photographs scattered through the text of *Journey to the Heart of Music*.

Meter

Ideas related to metrical structure in music play a significant role in the story and I introduce a short example here to establish clearly in the mind a central principle of the argument pursued in *Journey to the Heart of Music*.

The above extract is drawn from a Courante in the *English Suites* by J.S. Bach, a piece renowned for an unstable metrical structure. The music veers between measures of three metrical units and measures of two metrical units, measures of time signature 3/2 and 6/4, without settling down to one or the other meter. The durational period common to both meters is the quarternote – counted out above the staff. There are a total of twenty-four quarternote periods in the entire four bar extract. Below the staff, the two competing metrical structures of three groups of two quarternotes per bar and two groups of three quarternotes per bar, are delineated. *Over the four measure extract both meters, though of differing internal structure, span twenty-four quarternotes and are equivalent in this regard.*

I suspect it would be near impossible to hear twenty-four quarternotes as perfectly even metrical sub-units. Normally the contours of melody and dynamic accent coax the listener into perceiving or imposing some real or imagined metrical pattern upon the notes. In combination the ear and processes of aural cognition, seeking to make sense of the incoming aural stimuli, discover and/or create metrical patterns and hierarchies. A simple recurring metrical pattern provides a most useful general algorithmic framework to aid perception, where no two measures are precisely identical in accent and stress: Order is found or imposed, understanding enlarged and amorphous complexity rendered intelligible.

In the above example, while two different metrical entities, 6/4 meter and 3/2 meter, vie for precedence, both meters have a point of equivalence or conjunction over the period of four measures and twenty-four quarternotes. Indeed, the meters conjoin at one, two and three bars also, however, the four measure sum of twenty-four sub-units has a particular connection with the foundational harmonic exchange of the dominant-to-tonic full cadence, a chord progression which occupies a central position in the story of this book. *In the following chapters the principle of metrical ordering and equivalence is extended from the realm of pulse and duration to the domain of audible sound, where it will be applied to the ordering and equivalence of oscillations and periods of pitch, providing the basis for a new approach to understanding the nature of harmony in western music.*

Briefly moving beyond the sphere of music to mathematics and physics, meter remains a potent concept: In that the prime numbers may be identified with simple meters (e.g. 2/4, 3/8, 5/4 time), which have no symmetric division except one; and similarly the composite numbers find an echo in regular compound metrical forms such as 4/4, 6/8, 9/8 time; while the oscillations and periods of physical systems in the material world, considered in terms of meter, combine elements both musical and mathematical. [30/08/09]

Notes

1. AWK originally written in 1977, and much enhanced in later years, is named after the initials of its three creators, A.V.Aho, P.J.Weinberger and B.W.Kernighan, from AT&T Bell Laboratories. The Free Software Foundation package GNU AWK (gawk) is the most commonly available version of this classic text manipulation utility of the UNIX computer operating system (http://www.gnu.org/software/gawk/gawk.html). AWK, I have found, is a most useful tool, with many hidden depths. Easier to get into and use than its big brother Perl and much more compact, AWK is an interpreted language, implemented on almost all operating systems and so its programs (scripts) are entirely transportable between computers.

2. *Elements of Music?,* Archive Nov.03 Vol.17.2 - June.05 Vol.18.9, a speculative and exploratory set of articles and files examining structure in oscillatory systems.

Conventions: A pitch standard of middle C = 256Hz (a little lower than Concert Pitch, A = 440Hz) sometimes called scientific or philosophical pitch, is used throughout – for the convenience of its concordance with the octave-doubling powers of two frequency sequence, e.g. $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, etc., $2^8 = 256$. Linked in with this choice is the fact that the vast majority of examples and discussions focus on the ‘prototype’ key of C-major. Any major key could have been used – what is true of one is true of all. The minor mode is viewed as an ‘inflected’ or altered form of the major key. Usage is International/American English and in matters of typography generally only standard fonts and text characters have been employed; thus some signs are written out (e.g. Aflat) and others substituted. Finally, a nexus is drawn between this proto-key or *tonal center of C* and the ratios of the harmonic series, such that each ratio is associated with a note letter (and generally sharp sign if required, e.g. Bflat = A#). Implicit in this convention is the view of the scale tones as a system of *dynamically varying* ‘pure’ ratios and relationships, rather than a fixed (and ‘impure’) grid of equally tempered pitches. This I suspect, is what the tolerance of the ear allows the processes of aural cognition to extract from objective musical sounds.