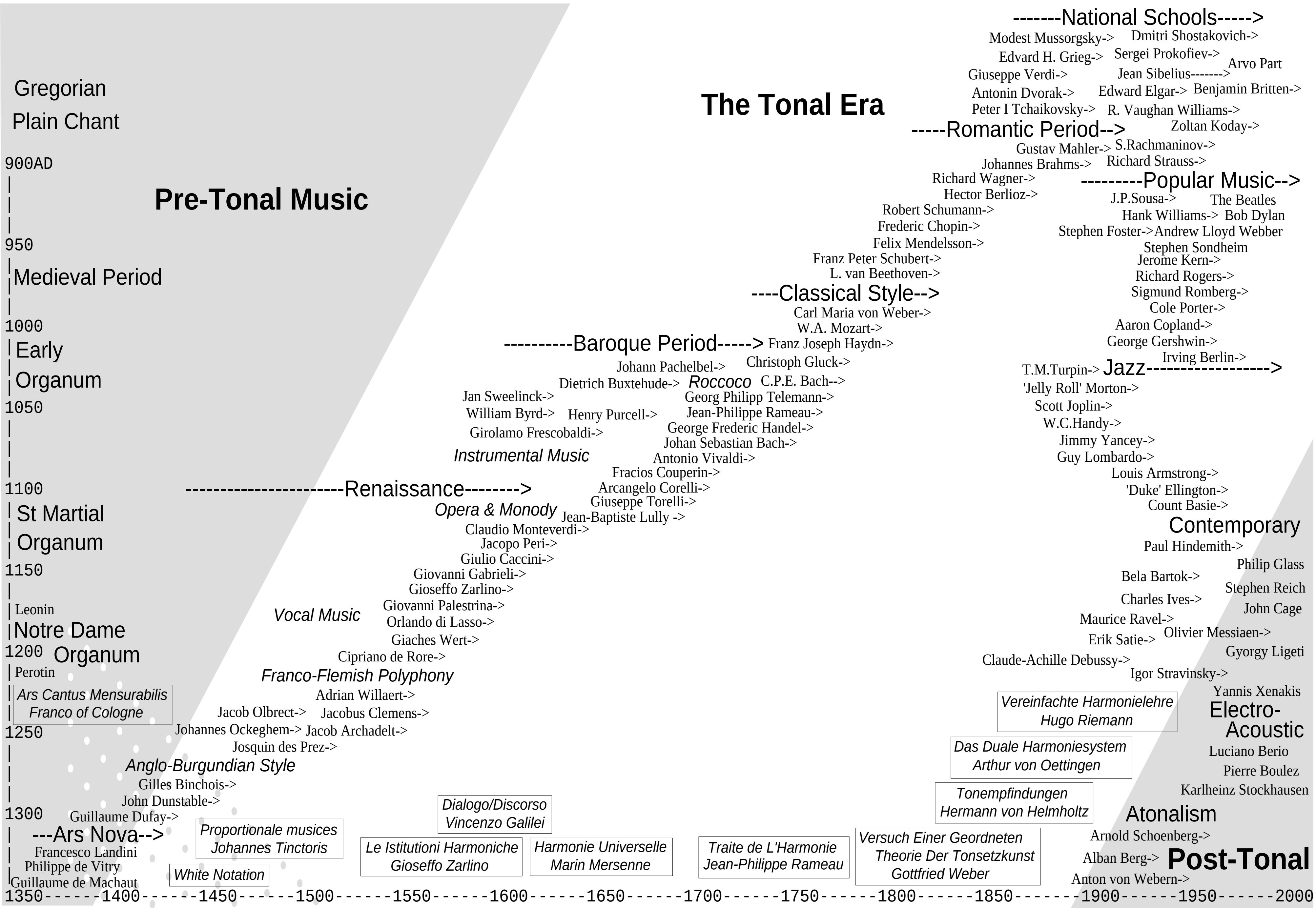
A SURVEY OF WESTERN MUSIC

VIEWED FROM THE PERSPECTIVE OF TONAL COMPUTATION

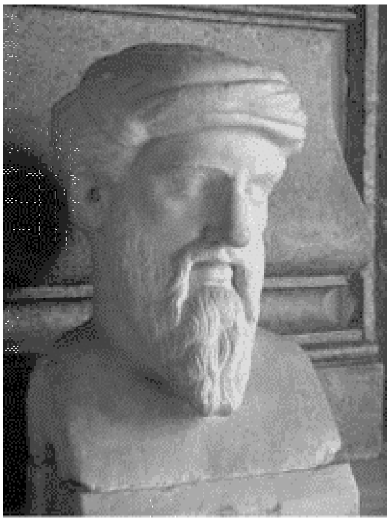
This document traverses the broad landscape of western music with the object of providing an interpretation of music's historical development cast in terms of mutable base numbers and the agency of 'tonal computation' as described in the other appendices; thus hopefully demonstrating how these ideas unify and make intelligible the long arc of the tonal era, circa 1500–1900 AD. Along the way some notice is taken of the theorists who sought to explain the nature of western music, and in particular, their struggle to explicate the mystery of the minor third and triad.

In a manner rather like viewing an impressionist painting, coming too close to the canvas can actually inhibit an observer's ability to grasp the overall work. Similarly, some trends in western music best reveal themselves when viewed at a large scale and from a high-level perspective. Though the *styles* – the external clothes of form and idiom – changed markedly from period to period, from masses and motets to concertos and symphonies, from unaccompanied voices to mighty orchestras, an underlying unity pervades the music of western Europe from the sixteenth century through into the twentieth century. Indeed this epoch could be referred to as the period of common *harmonic* practice – to extend a little the focus of a frequently used phrase. Apparent discontinuities cut across the surface: the change from vocal to instrumental expression around 1600, the demise of counterpoint around 1750, the emergence of a mass audience in the nineteenth century, the resurrection of 'forgotten' styles in the twentieth century by the early music movement, etc. Yet underlying the varied musical topography of so many styles, fashions and periods the irresistible tectonic force of tonal computation in music was gradually asserting itself. At first unnoticed, yet ever inching forward over generations of musicians and composers. And though new 'contemporary' styles would arise in the twentieth century in reaction to its dominance, none has as yet been able to dislodge it from 'the mind' of western civilization. Moreover the practice of computation by means of mutable numbers is spreading out into other world music cultures where it is proving to be equally irresistible, and adaptable, to local musical preferences and practices. And so it is to this broad sweep of the inner commonality of tonal harmony – the lingua franca of mutable numbers expressed in sound – that we turn our attention.

To aid this high level approach the chart below provides an overview of the stylistic development of western music, with a time-line beginning in the medieval epoch tracing the centuries from 900 AD at the top left-hand corner of the chart down the left-hand margin and then across the bottom of the chart. Gray shaded areas mark the *'Pre-Tonal Era'* to the left and the *'Post-Tonal Era'* to the bottom right. Between these two 'bookends' lies the broad white slash of the *Tonal Era*. These somewhat arbitrary boundaries should be considered porous, thus the pre-tonal era boundary is gray-white dotted and the gray post-tonal section does not fill the whole right-hand margin – allowing the white tonal era slash to continue on passed the year 2000. This sharing of the right margin acknowledges the continuing, and indeed somewhat refreshed tonal musical practices and traditions, up to and including, the present day.


Gregorian
Plain Chant
900AD
950
Medieval Period
1000
Early
Organum
1050
1100
St Martial
Organum
1150
Leonin
Notre Dame
1200
Organum
Perotin
Ars Cantus Mensurabilis
Franco of Cologne
1250
1300
---Ars Nova-->
Francesco Landini
Philippe de Vitry
Guillaume de Machaut
Pre-Tonal Music
----------------------Renaissance-------->
Anglo-Burgundian Style
Gilles Binchois->
John Dunstable->
Guillaume Dufay->
Jacob Olbrect->
Johannes Ockeghem->
Josquin des Prez->
Jacobus Clemens->
Jacob Archadelt->
Adrian Willaert->
Franco-Flemish Polyphony
Cipriano de Rore->
Giaches Wert->
Orlando di Lasso->
Giovanni Palestrina->
Vocal Music
Gioseffo Zarlino->
Giovanni Gabrieli->
Giulio Caccini->
Jacopo Peri->
Claudio Monteverdi->
Opera & Monody
Instrumental Music
Girolamo Frescobaldi->
William Byrd->
Jan Sweelinck->
Henry Purcell->
Dietrich Buxtehude->
Johann Pachelbel->
----------Baroque Period----->
Jean-Baptiste Lully ->
Giuseppe Torelli->
Arcangelo Corelli->
Fracios Couperin->
Antonio Vivaldi->
Johan Sebastian Bach->
George Frederic Handel->
Jean-Philippe Rameau->
Georg Philipp Telemann->
Roccoco
C.P.E. Bach-->
Christoph Gluck->
Franz Joseph Haydn->
W.A. Mozart->
Carl Maria von Weber->
----Classical Style-->
L. van Beethoven->
Franz Peter Schubert->
Felix Mendelsson->
Frederic Chopin->
Robert Schumann->
Hector Berlioz->
Richard Wagner->
Johannes Brahms->
Gustav Mahler->
The Tonal Era
-----Romantic Period-->
-------National Schools----->
Modest Mussorgsky->
Edvard H. Grieg->
Giuseppe Verdi->
Antonin Dvorak->
Peter I Tchaikovsky->
Dmitri Shostakovich->
Sergei Prokofiev->
Jean Sibelius------->
Arvo Part
Edward Elgar->
Benjamin Britten->
R. Vaughan Williams->
Zoltan Koday->
S.Rachmaninov->
Richard Strauss->
---------Popular Music-->
J.P.Sousa->
The Beatles
Hank Williams->
Bob Dylan
Stephen Foster->Andrew Lloyd Webber
Stephen Sondheim
Jerome Kern->
Richard Rogers->
Sigmund Romberg->
Cole Porter->
Aaron Copland->
George Gershwin->
Irving Berlin->
T.M.Turpin->
Jazz------------------>
'Jelly Roll' Morton->
Scott Joplin->
W.C.Handy->
Jimmy Yancey->
Guy Lombardo->
Louis Armstrong->
'Duke' Ellington->
Count Basie->
Contemporary
Paul Hindemith->
Philip Glass
Bela Bartok->
Stephen Reich
Charles Ives->
John Cage
Maurice Ravel->
Olivier Messiaen->
Erik Satie->
Gyorgy Ligeti
Claude-Achille Debussy->
Igor Stravinsky->
Yannis Xenakis
Electro-
Acoustic
Luciano Berio
Pierre Boulez
Karlheinz Stockhausen
Atonalism
Arnold Schoenberg->
Alban Berg->
Post-Tonal
Anton von Webern->
Vereinfachte Harmonielehre
Hugo Riemann
Das Duale Harmoniesystem
Arthur von Oettingen
Tonempfindungen
Hermann von Helmholtz
Versuch Einer Geordneten
Theorie Der Tonsetzkunst
Gottfried Weber
Traite de L'Harmonie
Jean-Philippe Rameau
Harmonie Universelle
Marin Mersenne
Dialogo/Discorso
Vincenzo Galilei
Le Istitutioni Harmoniche
Gioseffo Zarlino
Proportionale musices
Johannes Tinctoris
White Notation
1350------1400------1450------1500------1550------1600------1650------1700------1750------1800------1850------1900------1950------2000

THE ANCIENT BACKGROUND



Picture courtesy Wikipedia

Pythagoras of Samos, circa 575–495 BC, was born on the Aegean island of Samos to prosperous parents (as the story is told). Pythagoras used his inheritance to travel widely and learn of the accumulated knowledge of other civilizations, the Babylonians and Egyptians, as well as visiting Thales of Miletus the father of Greek science and natural philosophy. Over many years of travel and adventure he acquired a broad base of learning, which he distilled into a credo, part philosophical part religious, centered on the concept that *mathematics represents the ultimate level of reality* and that all phenomena in the world can be understood in terms of number. This idea has remained a central guiding insight of natural philosophy and science down to the present day. Traditionally, Pythagoras' discovery of the link between music and numbers – the ratios of musically pleasing intervals of pitch – is credited to the ringing sounds emanating from a blacksmith's workshop as the workmen employed different-sized hammers and anvils. Pythagoras realised that the relative size and weights bore a proportional relationship to the ringing tone they produced. In later life Pythagoras established a brotherhood of like-minded 'philosophers' at Croton, a Greek settlement in southern Italy. After a peaceful start, the secretive brotherhood of Pythagoreans became embroiled in local political arguments and their community was attacked. The precise time and place of Pythagoras' death is unknown, but, the Pythagorean brotherhood continued in various forms for centuries afterwards, and Pythagoras' influence has remained an enduring and fruitful legacy in western philosophy and science.

Incredibly, the ancient theorists grasped the essential nature of music, overall, at the first. *Music is number expressed in sound.* Numbers are ratios: values expressed as ratios of a unit. The frequency of notes in music are related by ratio – as are values of duration. Pythagorean thinking reverenced the numbers 1, 2, 3 and 4 – the tetrad – thus the musical interval ratios of the octave 1:2, the fifth 2:3 and the fourth 3:4 were considered foundational. With these intervals a scale may be constructed.

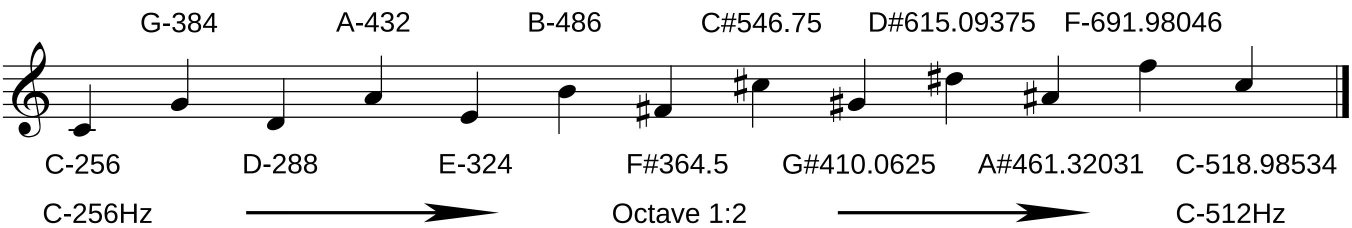


Figure 1. Beginning at middle C (at philosphical pitch appropriately) multiplying by a fifth 2:3 and dividing by a fourth 3:4 in turns, produces a scale of twelve notes. However, there is a discrepancy between middle C at the start and the finally derived treble C. [Values truncated at five decimal places]

Precise Pythagorean relationships deliver the uncomfortable sound of an out of tune octave 1:2.0272864 ! The difference between this awkward ratio and the beautiful 1:2 has been named the Pythagorean comma. (Perhaps trauma would have been more appropriate.) Adjusting each interval by one twelfth of the comma produces the modern equal-tempered scale we use today. The slightly impure intervals that result from this process are thankfully accommodated by the tolerance of the ear. The human hearing mechanism will accept a considerable degree of variance in order to classifying sound stimuli into simple relational categories. The near conjunction of the Pythagorian comma is a *happenstance* of number and nature; which both provides scales redolent with approximations to simple ratio intervals (1:2, 2:3, 3:4, 4:5 major third, 5:6 minor third, 5:7, 8:9 whole tone, 15:16 semitone) and the possibility of forming a closed cycle of twelve tonal centres (the major and minor keys) by tempering the intervals. Yet as soon as the theorists had leapt to an apparently overarching solution to the nature of music, dogmatic thinking, the pursuit of numerically perfect intervals, allowed the mystery of music to escape from under their claws. It would be 2000 years before the theorists got close enough for a second try! For monophonic music consisting of melody alone or melody plus drum and/or other heterophonic instrument the Pythagorian scale is serviceable but as polyphonic music developed in Europe cracks in the Pythagorian system appeared, driven by its less than beautiful major and minor third interval ratios: 64:81 and 27:32 respectively.

Effectively there is a discontinuity between ancient musical practice and music in the modern period, –i.e. western music from circa 500 AD. The living tradition of ancient music-making was more or less completely extinguished in the west during the long eclipse of the dark ages. Other traditions of more primitive music came with the invading armies and settlers, and established traditions were lost or withered in the disruption. Later, as the Church established a leading cultural position, it preferred to expunge memories of ancient pagan practices, or sometimes, remould them to suit the Christian ethic. Little is really known of what ancient western music – that is for the most part ancient Greek music – actually sounded like. Perhaps some inkling of the flavor of this lost tradition can be found in living monophonic music cultures, but the precise character of its soundscapes are probably gone forever. However, what is reasonably clear is that it was melodic/monophonic, perhaps heterophonic at times, but not polyphonic as in the modern period, and, with a close nexus between words and melodies, probably employing expressive microtonal inflections of its scales. As with almost all things Greek, it spawned a complex theory, and it is hard today to credit the importance that the great philosophers attached to the ‘proper’ use of music and the influence they felt it possessed for good or ill. Perhaps some explanation of this power lay in the seamless union with which ancient music and ancient poetry appear to be have been joined, each amplifying the potency of the other.

Picture courtesy of the SMT

Anicius Manlius Severinus Boethius, circa 477–524 AD. Born probably in Rome into an old aristocratic line that claimed Roman Emperors in both his mother’s and father’s families, Boethius’ social position ensured that he received a high quality education and a thorough grounding in the classics. At a critical moment in western history, poised between the ancient and medieval periods, and with the knowledge, inclination, wealth and leisure necessary for the task, he set about translating and commenting upon many major works of scholarship from the Greek world still extant and acceptable to the church – Aristotle and the Neoplatonists. Though the project was left unfinished at his untimely death (executed on the orders of King Theoderic of the Ostrogoths) the body of translated work that was accomplished by Boethius would become the major source of ancient learning throughout the medieval period. With regard to music theory, Boethius' treatise *De institutione musica* represented almost the sum total of the medieval world's understanding of ancient theory and musical practice. And when the universities of Europe began to develop, his work on music, as well as in many other areas, was incorporated into the quadrivium course of arithmetic, geometry, astronomy and music – the study of number in all its then understood forms. Boethius' talents and learning had attracted the attention of Theoderic in Ravenna, and he occupied a number of high government offices in the king's service. In time he fell foul of political and religious opponents and was accused of treason and conspiracy – in the cause of the Byzantine Emperor Justin. While languishing in prison, awaiting execution, he wrote perhaps his greatest and most personal work, the *Consolation of Philosophy,* a dialogue between himself and 'Philosophy' cast in the form of a compassionate woman.

Though the sounds and performance practice of ancient music died away under the combined onslaught of barbarism and Christianity, much ancient music theory survived and became established within medieval thought, principally through the influential works of Boethius. Working at the watershed between the ancient and medieval periods (circa 500 AD), Boethius' translations and commentaries on a selection of the Greek texts then available – arithmetic, logic, music, philosophy and of course theology – managed to salvage something from the wreck of the ancient world. Later, when the revival of ancient learning began in earnest in the Renaissance, the surviving texts were scoured for all their ancient knowledge; inspiring a new input of theory and speculation, into the vibrant western polyphonic tradition of the fifteenth and sixteenth centuries. Although ancient music theory, as it was understood – the Pythagorean scale and the Greek modes for example – exerted immense influence during the one thousand years that separated Boethius from the tonal era (–i.e. 500 to 1500 AD), this influence remained always tangential, almost a parallel universe, to the world of practical music-making in the medieval period and beyond. It was only after the middle of the sixteenth century, in the work of Gioseffo Zarlino (and other like-minded scholars) that the old knowledge began to interact meaningfully with current musical practice: As this influential theorist and practising composer/musician struggled to reconcile the music of his revered master, Adrian Willaert, with ancient music theory. At last, progress would begin again. Open rational minds once more asked penetrating questions about the nature of real living music, and were prepared to accept new answers in return.

MEDIEVAL PERIOD

Tonal harmony did not spring into being suddenly, or through some once-and-for-all revolutionary change of style in the western tradition. Indeed, to a varying extent elements of tonal organisation are to be found in all music and every tradition. It is a fairly natural and inevitable consequence of music-making, and requires considerable effort to eradicate, as the atonal music of Arnold Schoenberg and his followers testifies. Indeed, the roots of tonal organization probably lie deep within the processes and mechanisms of the human hearing system, and that redoubt would not become accessible until the twentieth century.

Western music has enjoyed a rather atypical development in comparison to other music cultures, a journey that took it down the path of polyphonic sound spread over a wide frequency range, and it is through this 'accident' of artistic evolution that the full possibilities of tonal harmony as an organisational principle were to be uncovered. Certainly there wasn't anything preordained in this development; other musical traditions have taken to similar paths to some degree: employing drone accompaniments, parallel melody and all sorts of heterophonic effects; but none were to go as far down this rather narrow avenue of development as the western tradition, and there have been costs as well as benefits.

"Polyphony as such, therefore, is not exclusively Western or European; what is distinctive about Western music is that Western composers have specialized in writing polyphony. What in other musical systems is an incidental factor, in Western music is an essential one. We have developed polyphony to a unique degree, and, it must be admitted, at the expense of certain melodic and rhythmic subtleties that are characteristic of the music of other civilized nations, India and China, for example."

Donald Jay Grout – A History of Western Music[1]

The medieval epoch in western Europe was to serve as a long and slow gestation period before the birth of fully fledged, harmonically governed, tonal music. Through the medieval period a number of related elements came together to enhance the likelihood of this outcome. First and foremost, the development of organum: the introduction of polyphonic vocal and/or instrumental 'partnering' melodies to the hitherto solo Gregorian plain chants. This practice, probably begun as a form of improvised parallel melody, went on to become established and codified, with many additional elements, such as freer motion between and within parts, the addition of a third voice, the development of stylistic refinements to cadences, etc.

Over the years from its inception somewhat before 1000 AD, the organum style underwent many changes and developments, with the first named composers known to history being Leonin and Perotin of the twelfth century Notre Dame School, centered on Paris. It is interesting that in Europe the quite natural impulse to introduce some form of accompaniment to the chanting of religious texts should take a polyphonic-melodic path, rather than the more common form of a drone or rhythmic drumming, though of course it may just have been chance pure and simple. However a possible explanation for this atypical choice might be found in the acoustical environment produced by the architecture of Gothic churches, abbeys and cathedrals. The resonant qualities of these great buildings literally creating something of an organum effect by combining the melodic intervals of plainsong into occasional 'intervals' through their long echoing resonances. Or perhaps singing in parallel fifth and/or octaves to suit differring vocal ranges eventually developed into a style of chanting – improvised organum. Given the absence of written records of how the practice began, this part of the story will probably always remain a mystery. However, what can certainly still be heard today, is how appropriate the gloriously empty sonorities of the organum style are to their native acoustic.

Another related element of medieval music-making is the long tradition of building organs in cathedrals and abbeys during the period. Medieval organs were uncontrolled monsters (operated by levers played with fists and feet) in comparison to the refined instruments of the tonal era. However, despite their wild and thunderous character, with all ranks sounding simultaneously, they significantly covered a remarkably broad spectrum of pitches. For example, in central Germany the Halberstadt organ built in 1361, and described by Michael Praetoris (1571–1621), possessed ranks of pipes sounding across a six-octave range; and significantly, this broad pitch range was carried over into the 'modern' organs which were to develop and rapidly mature from the fifteenth century onward. The extension of music over such wide pitch ranges, which is equivalent to the sounding of many frequencies of the harmonic series simultaneously, was a crucial foundation upon which the tonal revolution, pursued by succeeding generations, would be built.

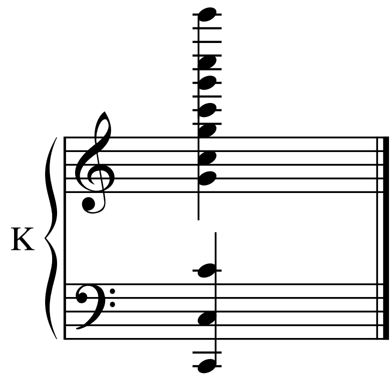
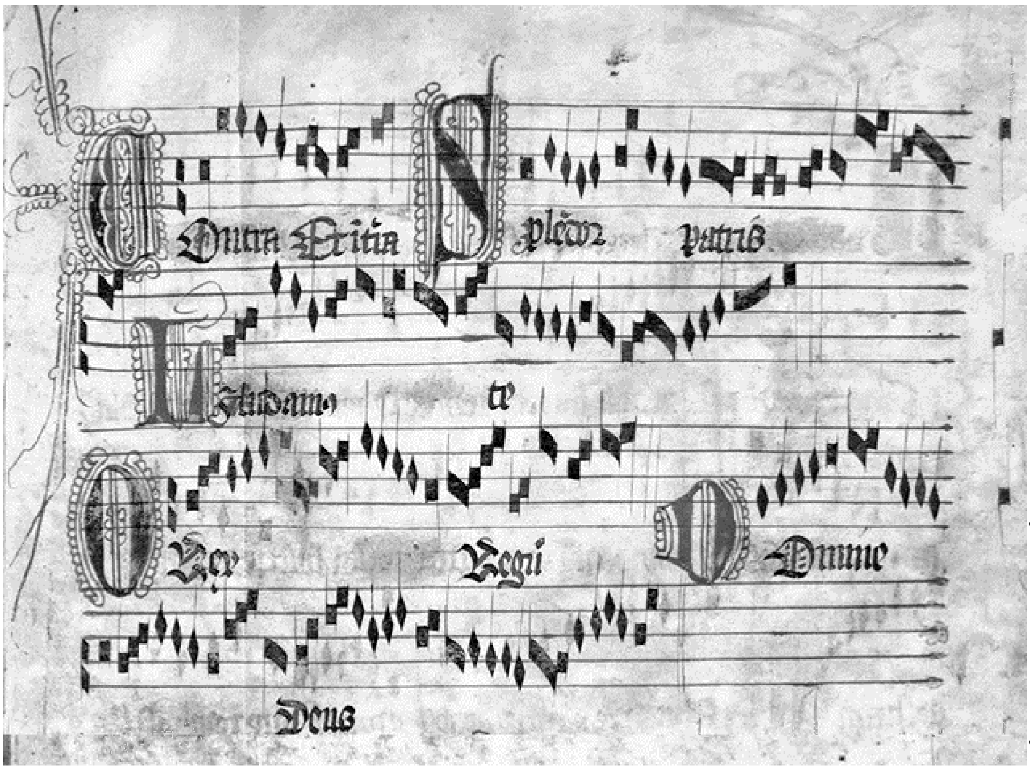


Figure 2. The six octave range of pitches produce by playing 'middle C' on the Halberstadt organ, built in 1361.

At the close of the medieval period, during the fourteenth century, a new musical style came to prominence, the *Ars Nova*, taking its name from a treatise by Phillipe de Vitry (1291–1361), where amongst other things, the equal merit of duple meters was recognised, alongside the triple meters predominantly used in organum. Hitherto the development of the organum style had been dominated by musicians working in Paris, but at the end of the middle ages the musical avantgarde spread also to Italy. In France, under the leadership of the poet–composer Guillaume de Machaut (c1300–1377) some of the older practices and something of the sternness of the old school, the *Ars Antiqua,* remained. However, in the warmth and light of Italy, far less emphasis was placed on the scholastic disciplines of the earlier style and a freer more directly harmonic music emerged in the work of Francesco Landini (1325–1397). Overall, the *Ars Nova* can be typified as one of those moments of relaxation (like for example the Rococo period) that often follow long and intense periods of activity, where the stylistic developments had become increasingly complex and overwrought. In the secular works of this time, the beginnings of tonal harmony were particularly evident in the fourteenth century practice of *musica ficta*: The alteration of modal harmony, particularly at cadences, which brought the music significantly closer to the dominant-tonic formula of the period of common harmonic practice. The development of musica ficta might be taken as one of the first overt signs of an underlying change in the way music was being aurally processed and understood in minds of European musicians. A revolution was under way, though long and slow in gestation, now the first tremors were being felt.

Although music theory lay at the center of academic life in the medieval period, as a member of the quadrivium of subjects: arithmetic, geometry, astronomy and music (the study of ratio in all its manifestations) it rarely made any direct or meaningful connection with practical music-making. The medieval view of music theory saw practical music largely as an imperfect reflection of the heavenly order, and theorists principally dwelt on, and attempted to articulate, these eternal verities. This produced a highly abstract and mathematical approach which, in aspects of the *mensural system* – the notational theory developed in the late middle ages – penetrated remarkably far into the core process of computation by mutable numbers.



Picture courtesy Biblioteca de Catalunya

Franco of Cologne, circa 1225–1290. Little is recorded of Franco's life: his place and time of birth and death are unknown. However, by his own accounts, he was Papal Chaplain and the Preceptor of the Knights Hospitallers of St John at Cologne, and other sources suggest he was by birth German. Franco of Cologne's position as the most influential music theorist of the late thirteenth century rests on his principal published work *Ars Cantus Mensurabilis (c.1250),* a treatise which introduced, in essence, the modern system of music notation, where duration is indicated by the note's written form. As is the case with other innovative music theorists, there is strong evidence that Franco was also a practical composer, with links to the Notre Dame School of Paris, though no surviving compositions can be surely attributed to his hand.

Franco of Cologne codified the existing ideas in the second half of the thirteenth century in *The Art of Measurable Music* (Ars Cantus Mensurabillis, c.1275) and over the latter years of the medieval period and into the Renaissance, notational developments largely followed the logic of his forward-looking work. Musicians at first used dark symbols, then later, between 1450 and 1600, developed a *white notation* from which many of the note shapes and other music symbols used today originated. A little of this system of proportions has survived to the present day in the alla breve time signature – C with a vertical slash. One particular area of the system of mensural notation, the theory and practice of durational proportions – a shifting scheme of relative note lengths – e.g. dupla 1:2, sesquialtera 2:3, sesquitertia 3:4, etc. – essentially encapsulates the *algorithm of symmetrical exchange* which lies at the heart of the concept of tonal computation and mutable numbers as presented in these appendices.

Cogently, the medieval theorists classified the relationships between notes of different duration and the meters that they expressed, incorporating them into a consistent and comprehensive theory – *The Mensural System.* Yet remarkably working before the era of harmonic common practice, this scheme, which they applied to the durational and metrical dimension of music alone, also essentially embodies the process of symmetrical harmonic exchange underlying the concept of tonal computation by mutable base numbers. *However, for the medieval scholars working at a time before the tonal era had begun, in spite of having explored so much of the theory and mathematics encapsulated within tonality, their work came too early for its full significance to be understood. The discovery of tonal computation, when it did come in western music, was to be an intuitive and visceral encounter made by practical composers and musicians, and ironically by then the medieval theory, which could have made sense of their great discovery, had faded from view.*

Throughout the middle ages musical thought was predominantly linear. The original plain chants were elongated into tenor *cantus firmi* over which were lain semi-autonomous vocal parts of a very different character, and sometimes even set in a different language. The two or three parts proceeded with scant regard for the vertical 'harmonic' implications of their motion. Soft or sharp second dissonances (8:9, 15:16) and major or minor thirds 64:81 and 27:32 (4:5, 5:6) inevitably occurred between the moving parts scattered amongst the theroretically sanctioned intervals of fourth, fifth and octave. Though such dissonances were rigidly excluded from cadences. Thus the harmonic intervals of a third, both minor and major crept into western musical practice as a form of accidental, unavoidable, dissonance. Practicing polyphony has consequences. Theorist furrowed their brows.

Here I should note that a license to poke a little fun at music theorists and tickle their angsts comes from being of them myself, ever smitten by beautiful ratios and schemes of perfection. What fools we are to think this world so simple. For theorists, the minor third and minor triad in particular, would prove obstinately resistant to their taxonomic idealism. And like so many theorists I too have a predilection for counterpoint. It is the meat in the burger. Something for the ear to track and the conscious mind to follow. So it was with a sense of loss, that gradually as I worked out the system of mutable number processing, it became clear that counterpoint was a secondary component, almost a complex textural feature, and that harmony was the primary structural element in western music. Much music exists, beautiful and satisfying, without any trace counterpoint. The opposite is perfectly feasible, as demonstrated by the twentieth century atonal school.

THE RENAISSANCE

To start with, it took some time for the system of ‘physical’ number processing – tonal harmony – to become established in the western musical tradition. In the music of the fifteenth century, for example, the masses and motets written by Johannes Ockeghem (circa 1425–1495) still lack the essential directionality that the most urgent tonal computation brings to music – that forward motion engendered by dominant-tonic harmony. However, in the secular works of the time, for example the chansons of Guillaume Dufay (c.1400–1474) and Gilles Binchois (1400–1460), the flame of tonal computation can be decerned. This dichotomy between coexisting conservative and radical trends would often be repeated in later periods, a dichotomy encapsulated in Claudio Montiverdi’s (1567–1643) terms *prima* and *seconda prattica* – the old contrapuntal and new harmonic styles.

Picture courtesy Wikipedia

Johannes Tinctoris, 1435–1511. A native of the Low Countries, Tinctoris spent his early years studying and working at Orleans in northern France, where he eventually held the cathedral position of director of music. Tinctoris is also known to have worked at Cambrai in 1460 where it is speculated that he would have come into contact with Guillaume Dufay. Like many Franco-Flemish composers of the 'Netherlands' school he was drawn to Italy by the many opportunities that country offered, and in 1472 he travelled to Naples. Most of his mature years were to be spent south of the Alps. Johannes Tinctoris was both a practical composer – though very few works have survived – and an ardent music theorist with at least six publications to his name, covering a wide range of topics. In *Proportionale musices* he demonstrates the taste for counterpoint typical of his age – a predilection which came to overlay some of the sweetness of harmony found in the older Burgundian style. A man of many parts, Tinctoris was in addition to his career as musician, theorist and cleric, also a poet, mathematician and lawyer.

The development of tonal harmony was somewhat like a self-ignited forest fire: at first there is little to see, a patchwork of trends which might or might not lead on to a fire. In the heat of the day, many areas of woodland lie on the verge of ignition, one or two perhaps achieve a little combustion but then fizzle out in the cool of the evening. As the summer continues, eventually, a lightning strike, or spontaneous combustion in favourable conditions, go on to develop into a fully fledged wildfire. Once the fire gains a hold, and without external intervention, it burns until its fuel is exhausted. This encapsulates the computational view of the history of tonality. At the close of the middle ages the western music culture drew ever closer to the edge of computational conflagration as it wandered down the path of harmonic extension – through composers writing music in many simultaneously sounding parts, polyphony, producing successions of chords, which for all intents and purposes were successions of harmonic series, more or less complete. From the solo melodies of plainchant, western music developed through the many styles and stages of organum, until by the fifteenth century, and particularly in the secular genres of the time, the western musical tradition reached the point of computational ignition – the intuitive discovery of number processing in musically organised sound – tonal harmony.

At this point, it appears that an English composer played a crucial role in pushing the western tradition over the edge, onto a track that would eventually lead to Bach, Beethoven, and the Beetles. John Dunstable (c.1385–1453) more that any other (though it is perhaps a little unfair to single out one name), is responsible for taking the stylistic tradition of the late middle ages in the direction of the sweet harmonies of the major third. There is some evidence for a long parochial tradition in England of organum in thirds – a practice called gymel. The significance of the legitimization of both the major and minor third (at least in practice if not in theory) leading to their more frequent use as regular harmonic intervals, lies in the crucial positions they have in the harmonic series – between h4 and h7. By filling out the chords with these two intervals, particularly at cadences, the full force of the dominant to tonic sesquitertia 3:4 full cadence is felt.

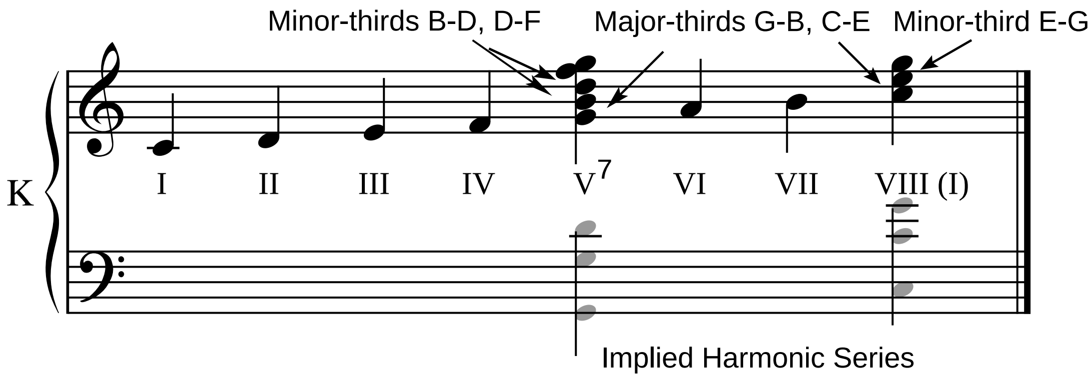


Figure 3. A dominant-seventh chord on fifth degree of the C-major scale resolving to the common major chord on the first degree (tonic), a full or ascending perfect cadence.

Tinctoris the Flemish music theorist wrote, in 1475, of music being, “a new art, the source of which was among the English with Dunstable at their head, and contemporary with him in France, Dufay and Binchois”. This was at the time of King Henry V’s dominion in France, which gave the innovation a chance of catching the mainstream of European musical culture. The trend wasn’t entirely initiated by the English discant style; already in the Ars Nova the rather sensuous appeal of thirds had been recognised and exploited. However, the English style accelerated a trend, that went on to be further developed by the composers of the Burgundian school – the vanguard of European stylistic development in the early and middle fifteenth century. The rest, as they say, is history. The Burgundian style evolved into the dominant Franco-Flemish school of the Renaissance, which in time led on to the Early and Later Baroque schools, Rococo, Classical and Romantic styles of the period of common (harmonic) practice.

Yet the emergence of the Franco-Flemish or Netherlanders style, in one regard marked a backward step, in that contrapuntal thinking, long dominant in medieval organum, again came to the fore. Indeed, throughout the history of western polyphonic music, a cyclical pattern of periodic harmonic advance, followed by a creeping return of counterpoint, can be discerned. However, over all the cycles, a trend toward greater and more thorough degrees of harmonic organisation is also exhibited. Over the long term, the grip of harmony strengthens perceptibly. Interestingly, one significant route by which counterpoint counter-attacked in the early Renaissance involved the manipulation of proportional features of the mensural notation. Composers of the time, for example, Johannes Ockeghem (1425–1495), Jacob Obrecht (1452–1505) and the great Josquin des Prez (1450–1521) employed proportional relationships, often hidden behind cryptic performance directions, to play intellectual games, set puzzles and generally show off their compositional skills. The inevitable result of introducing durational proportions between parts was to strengthen the contrapuntal element. The early Renaissance proved to be a time in the stylistic ‘harmony versus counterpoint’ cycle that favored such deepening.

However, by this stage in the story of western music the Pythagorean scale was creaking under the weight of the harmonic usage of the day. The addition of a fourth part and then a fifth to the standard vocal disposition could only increase the tendency toward writing harmony that included major 4:5 and minor 5:6 thirds. Music was evolving and composers wrote music that sounded right or good to them and their patrons' ears. Pythagorean dogma was being overruled by practice and the preferences of human hearing.

To sing a melody or listen with understanding to a piece of tonal music requires that short-term acoutic memory holds on to a reference pitch, a tonal center. This can normally only be done if the notes of the melody or piece bear a simple quantitative relationship to that reference pitch. The ratios 1:2. 2:3, 3:4 lie close to the reference point of 1:1; but 4:5 and 5:6 are not much further removed or much less supportive of the tonal center. Pythagorean theory debarred these newfangled intervals of a third, yet they were harmonious to the ear and emerged naturally out of the four and five part writing of the fifteenth century.

Picture courtesy Wikipedia

Gioseffo Zarlino, 1517–1590. Was born in Chioggia, on the south side of the Venetian lagoon, Italy, Zarlino spent his whole life in and around Venice. His early education was undertaken by the Franciscans, with which his family had strong links, and, in due course, he was himself to join the order, rising through the ranks to become in later life a canon at Chioggia cathedral where as a young man he had occupied the positions of lay clerk and organist. As befits a Renaissance scholar Zarlino was a universalist, studying grammar, logic, philosophy, arithmetic, geometry, music and languages under many leading figures in these fields. In music his teacher was the influential Netherlander Adrian Willaert, maestro di cappella of St Mark's, Venice. And to a significant degree, Zarlino, in this theoretical writing, was attempting to reconcile Willaert's 'modern' techniques and style with the then current theory of music, by extending its compass beyond that inherited from the ancient world – the Pythagorean view of consonance. His great treatise, *Le Istitutioni Harmoniche,* 1558 sought to 'legitimatise' the intervals of the major and minor third (and their inversions) in the eyes of theorists by way of extending the Pythagorean view of consonant numbers from 1, 2, 3, 4 up to 5 and 6 – *the senario.* Thus the major third ratio 4:5 and minor third ratio 5:6 were made acceptable. In time, Zarlino was to occupy his master's position as maestro at St Mark's (1565–90), becoming a teacher of great influence, through the distillation of Willaert's principles of composition in his widely circulated published works – both in the form of music theory and practical compositions. Amongst Zarlino's own pupils was Vincenzo Galilei, the father of Galileo the founder of modern science.

In Zarlino's great treatise *Le Istitutioni Harmoniche* of 1558, a foundational work of modern music theory, are found amongst other advances, the observation that while the major triad arises from the harmonic division of a string or monochord, that is $^1/_2$ $^1/_3$ $^1/_4$ $^1/_5$ $^1/_6$ segments, the minor triad is formed through arithmetic extension in equal segments of 2, 3, 4, 5, 6 lengths: A mirror image of the major, formed from descending frequencies. He considered the minor triad to be less perfect than the major because the frequency relationship of the minor-third did not conform to the ascending harmonic mean, as does the major third in the major triad. In the nineteenth century music theorists would attempt to develop Zarlino’s observation into a symmetrical minor principal – Dualism. An endeavour that would ultimately be shipwrecked on the hardest of rocks, scientific fact.

Soon enough the pendulum was to swing the other way, toward the harmonic principle of organisation, which manifest itself particularly in the secular forms of the late Renaissance: in France the chansons of Claudin Sermisy (1490–1562), Clement Janequin (c1485–1560) and Pierre Certon (c1510–1572); while in Italy accompanied songs of a simple chordal character, called frottole, were much in vogue in the latter fifteenth and early sixteenth centuries. Alongside, and influenced by these simpler harmonic forms, the more conservative contrapuntal tradition of the Netherlanders took on a lighter, more transparent and balanced character, to reach a peak of polyphonic achievement in the late Renaissance, exemplified by the works of Jacob Archadelt (c.1505–c.1560), Adrian Willaert (c.1490–1562), Giovanni da Palestrina (c.1526–1594) and Orlando di Lasso (1532–1594).

For a moment, harmony and counterpoint stand face to face in a balanced equality, but however strong the scholarly and learned predilection for the subtleties and complexities of counterpoint, the *harmonic principle would not be denied*, though time and again, the stylistic course was set towards a theoretically driven, overwrought contrapuntal coast, the ineluctable attraction of harmonic organisation, acting like a steady undercurrent, would draw the ship of musicians off course, and to itself. Though from now on until well into the twentieth century the structure of music would be ruled by principles of harmony (that is mutable number computation in sound) counterpoint survived, indeed prospered for a time in subservience. The ear’s capacity to track multiple sound sources, simultaneously, is remarkable; and this facility is much exercised, entertained and delighted by the complexities of counterpoint set within a harmonic framework.

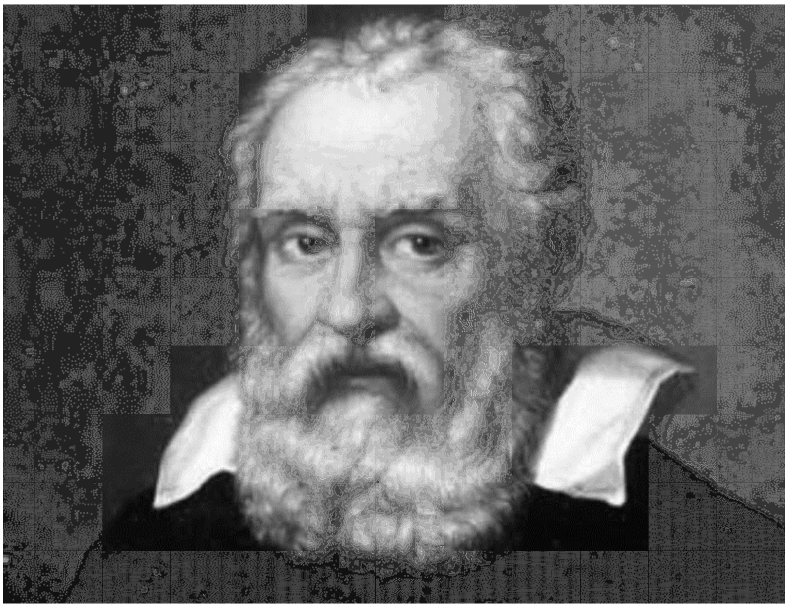
Up to this point the development of polyphony had been an almost totally vocal undertaking. A good, free, cultivated human voice can produce sound with a wide range of well in tune overtones. This is necessary for polyphonic music because one singer’s overtones are another singer’s notes and it is desirable that all should be well in tune together. During the medieval period, with the exception of the organ, musical instruments possessed less than ideal harmonicity – their overtones were out of tune. Only contemplate the shawm! However, through the renaissance period musical instrument harmonicity was improving. At first bowed gut-strung viols and consorts of recorders led the way, soon to be followed by other instruments at varying rates of development. It was a continuous process through to the end of the nineteenth century. Even so, by the opening of the baroque period musical instrument makers had made sufficient progress for concerted instrumental practice to become the dominant form of musical expression henceforth.

THE BAROQUE PERIOD

With the advent of the Baroque style, harmony finally and unequivocally became the predominant organisational principle in western tonal music. From about 1600 to 1900 AD it would reign and rule supreme; and even Johann Sebastian Bach (1685–1750), probably the greatest and most skilled contrapuntist of the western tradition, subordinated the thread of his melodic thought to the logic of harmonic progression. These three hundred wonderful years of fully harmonic governed music, fall roughly into four sub-spans of seventy-five years each: the Early Baroque (1600–1675), the Mature Baroque (1675–1750), the Rococo/Classical (1750–1825) and the Romantic (1825–1900) periods. Of course, a time frame is only helpful as a rule thumb; many are the exceptions to such hard and fast divisions. However, in tracing the stylistic development of western music through this framework, a further simple rule of thumb might also be helpful: an analogy with literary forms. One could characterise the Baroque style as that of a monologue or lecture, a single linear harmonic argument, pursued with logical rigor (and often considerable vigor) from start to finish, frequently imbuing the music with a strongly directional, and at times mechanistic, character. In contrast the Rococo/Classical Viennese school might be viewed as the age of dialogue while the Romantic period could be characterised as full fledged drama.

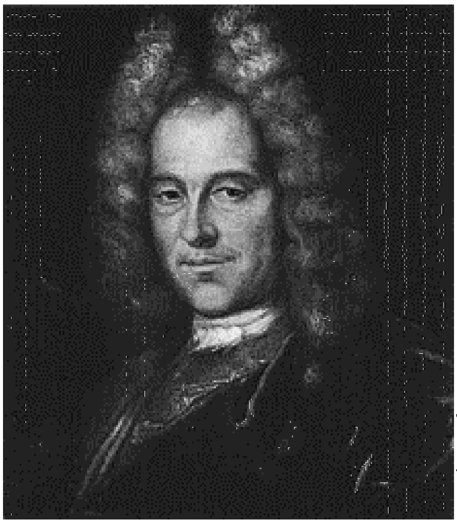
Early Baroque

Ironically the Baroque style grew out of attempts to revive the music of ancient Greece, undertaken by musicians and scholars in Italy, at the close of the sixteenth century. Although little really was known about ancient music, these men of the Renaissance, guided principally by literary scholarship and theory, sought to reconstruct the ancient art in all its imagined purity. The result of their theorising and experiment was *monody*, a highly mannered form of solo song, with an unobtrusive harmonic accompaniment. Though monody was probably not particularly close to their goal of reviving ancient Greek song, it was an expressive new art form. Ironically the theorists by trying to turn the clock back made a great leap forward! Effectively they had invented *operatic recitative*, and the central technique of the Baroque period, the *basso continuo* – the encapsulation of harmony as the dominant organisational principle expressed through a bass line with numeric figures referencing the intended or inferred chord progressions. Their next move would be the invention of opera itself, and this task fell to Jacapo Peri (1561–1633), Guilio Caccini (c1546–1618) and Claudio Monteverdi (1567–1643) amongst others. At the center of this movement for the reformation of music (along what was believed to be the lines of the ancient and original art) were a group of scholars and musicians in Florence – the Florentine Camerata. One of the Camerata’s leading members was the composer and scholar Vincenzo Galilei (c.1520–1591) a pupil (and later critic) of the great Renaissance theorist Gioseffo Zarlino (1517–1590) and father of Galileo.



Vincenzo Galilei (1520–1591) was born at Santa Maria a Monte, Italy; and as a young man he moved to Pisa where he married the well born Giulia Ammannati. His first born son, Galileo, who would go on to establish the beginnings of modern science. Vincenzo was a lute virtuoso and a man of many interests. He studied with Zarlino in Venice and was drawn into the forward looking intellectual circle of the Florentine Camerata, as well as corresponding with Girolamo Mei a prominent scholar of ancient Greek texts. He played a prominent role amongst the group of musicians and thinkers attempting to resurrect ancient Greek musical practice and wrote an influential treatise, Della Musica Antica et della Moderna, in 1581. Activities that laid the foundations for the coming Baroque style. Significantly, he approached some problems in music theory by way of experimental research and his basement in Pisa was no less than the first ever acoustics laboratory, and perhaps the place where his son Galileo first cut his scientific teeth.

Vincenzo's major published work is the *Dialogue on Music Ancient and Modern* of 1581. He, like Zarlino, was much interested in uniting the theory and practice of music in a way medieval scholars had not been. He conducted musical experiments to ascertain or demonstrate points of theory concerning scale temperaments and the nature of physical oscillation. There have been suggestions that his son, Galileo, might have been influenced by this empirical approach. In 1588 Vincenzo conducted a number of experiments, perhaps aided by his son, which led to the discovery of the exact mathematical formula relating the frequency and tension of a vibrating string. Galileo was at this time living back at the family home in Florence, having left off this mathematical studies at the University in Pisa, and if not directly involved, he must have been aware of, and influenced by, his father's empirical approach and mathematical description of physical phenomena. Indeed, here is a remarkable conjunction: the father of modern music theory Zarlino, the birth of fully fledged harmonically governed music in the western tradition and the father of physical theory in European science, Galileo, so closely intertwined. This is not to say that there was some direct causal linkage, but rather that the tonal harmonic system and the scientific system of mathematical physics grew upon a common foundation – that modern western music and modern western science are siblings.



Picture courtesy Wikipedia

Johann Fux, c1660–1741. Came from humble background in Styria, Austria and was educated by the Jesuits. He made a number of trips to Italy to further his musical development and was eventually taken into the Imperial service of Leopold I as court composer. He would serve the two following emperors also. Although distinguished in his lifetime for the music he wrote, it was his textbook on contrapuntal technique *Gradus ad Parnassum,* 1725 (Steps to Mount Parnassus) that would influence generations of composers almost to the present day. The book was in Bach's library, Haydn studied it diligently and recommended it to Beethoven, Mozart had a copy. Western music might well have had a different destination if the theorists had ruled, however composers took what they needed of counterpoint and bent it to the laws of harmony.

Mature Baroque

As so often happens after a revolutionary change, something of the older tradition finds its way into the new system: that is an accommodation formed between the old contrapuntal thinking and the new harmonic style that had emerged in the early baroque period. A compromise between Monteverdi's prima and seconda prattica was reached on the strict understanding that counterpoint was henceforward to be subservient to harmony: the new seconda prattica. The harmonic principle now finally entered into its inheritance. Tonal computation, number processing by means of harmonic progression, henceforth would imbue European music with a characteristic logic and unity which only mathematics possesses. Though of course music remained much and many things besides, in its essential heart it was now *mathematics in sound.* And in the first vigor of youth the arithmetic was conducted with a baroque intensity, whether fast or slow, as for example, is found in the works of Antonio Vivaldi.

Through the early years of the seventeenth century, it was principally in Italy that the compact (capitulation?) between melody and harmony was forged, with the re-entry of counterpoint into the new harmonic style being achieved largely in the medium of instrumental music. Notably melody often took on a new harmonic garb of arpeggio, spelling out the notes of chords in joyful and loyal submission. In keyboard compositions Girolamo Frescobaldi (1583–1643), his pupil Johann Jakob Froberger (1616–1667) and Tarquinio Merula (c.1694–1665) figure in the development of the Baroque idiom and forms, while in northern Europe the techniques of the English late Renaissance keyboard school, exemplified by William Byrd (1543–1623) and John Bull (c.1562–1628), reached the Dutch composer Jan Sweelinck (1562–1621) through the close contacts between England and the Low Countries, and from there passed into the German organ school led by Dietrich Buxtehude (1637–1707) and Johann Pachelbel (1653–1706).

However, it was in ensemble music that the Italians of the seventeenth and early eighteenth centuries made their greatest achievements. Foremost amongst many names, are those of Arcangelo Corelli (1653–1713) and Antonio Vivaldi (1678–1741). Corelli, working in Rome, a bastion of the prima prattica, developed an assured synthesis of the older contrapuntal style and the new harmonic writing in his many trio sonatas and concertos. Corelli's larger-scale concerti grossi were only published after his death but were written and performed well before 1714, and, with contributions from composers of the Bologna school, for example Giuseppe Torelli (1658–1709), the concerto form passed into the versatile and adept hands of Vivaldi, working in Venice. The mature Baroque style, of which Vivaldi was the first great master, had arrived.

The influence of the Italian innovations and the prestige of their achievements at this time was felt in all corners of Europe. The convention of using the Italian language for written directions in scores stems from this long period of pre-eminence. From the late Renaissance through the early Baroque, Italy was the cockpit of music, drawing musicians and composers from across Europe to its enriching fount, and radiating the new baroque idiom to other lands, as many Italians found employment abroad. Quite early in the period, Jean-Baptiste Lully (1632–1687), Italian by birth, took the new ways to France, where he skilfully adapted them to the language and tastes of the French. Later Jean-Philippe Rameau (1683–1764) and George Frederic Handel (1685–1759) studied and/or worked in Italy early in their careers, the latter particularly assimilating the ways of the Italian Baroque, which he took on to an England denuded by civil war and the sadly premature death of Henry Purcell (c 1659–1695).

Picture courtesy Wikipedia

Jean-Philippe Rameau, 1684–1764 was born in Dijon, eastern France, the son of the local organist. The record of Rameau's early life has significant gaps, and, until he attained some measure of success in Paris in mid life, the story is incomplete. It is believed that his only formal musical education came at his father's hand, and indeed, that his general education was rather meagre. It is known that he briefly travelled to Italy in 1701 and around this time held the post of organist at Clermont-Ferrand in central France. In 1706 Rameau moved to Paris where he attempted to establish himself as a composer and teacher, publishing a set of harpsichord pieces. However, success at this time eluded him. Little is known of the years from 1706 except that at some time after 1715 he returned to the post of organist at Clermont-Ferrand. In 1722 the *Traite de l'harmonie reduite a ses principes naturels* (Treatise on harmony reduced to its natural principles), the first of his many publications appeared, gradually making a name for Rameau as a music theorist. In 1723 he returned to Paris determined to try again to build a career as a composer. It was to take a further eight years of effort before the breakthrough came, by way of the patronage of the noble and extremely wealthy 'La Poupliniere' – Alexandre la Riche de la Poupliniere. From 1731, with his patron's support and at the late age of forty-eight, Rameau's career as a composer at last began to make progress, with the 1733 production of *Hippolyte et Aricie,* his first large-scale success. Further operas and opera-ballets followed. However, success in composition neither deflected nor diminished Rameau's striving to explain, in rational terms, the fundamental nature of music. Despite success in composition and theory, and a reasonable prosperity, detractors and critics abounded. In later life considerable amounts of Rameau's time were taken up embroiled in the arguments and controversies of various intellectual and artistic factions. He was never an easy man to know or befriend, and few if any then (or many since) could see beyond his awkwardness of manner, both in his life and his writings, to recognise and acknowledge the full profundity of his achievement.

Rameau had a dual career, first as a music theorist and later as a highly successful composer, and, as his compositions were written in maturity, they owe somewhat more to his French instincts than might the works of a younger man freshly returned from the Italian sun. As the foremost French composer of the late Baroque period he is justly esteemed, but this achievement has overshadowed an equal or greater achievement in music theory. Throughout a long life spanning the height of the French Enlightenment, Rameau strove to construct a theoretical synthesis for music, comparable to that which Newton had provided for physics. In many publications, of which the *Traite de Harmonie* of 1722 is the best known, Rameau set his theories in the broader context of general musical practice, –i.e. composition and the art of figured bass accompaniment (derived from monody). It is perhaps significant that, like Zarlino, he was a practical composer, as well as theorist seeking the underlying principles which guided his artistic choices.

Implicitly Rameau believed there was a rational basis to empirical musical practice – the intuitive choices of composers and musicians – and he devoted this life to discovering what that ground might be. Though he was ultimately unable to bring his ideas to a fully satisfactory conclusion, Rameau made large strides toward a complete theory of tonal music. Often relying on the work of others, as well as his own thoughts and 'experiments', he ascertained 1) the foundational role of the *corps sonore*, the default vibrational pattern of an oscillating body (–i.e. the harmonic series) as the ultimate source of musical phenomena, 2) the nature of the rootedness of chords in the *basse fondamentale*, 3) the common identity (through the fundamental bass) of chords whether in root position or inversion and 4) that melody is essentially an expression of harmony in linear form. All, except perhaps the last proposition, have been generally accepted since his day as major elements of modern harmonic theory. Statements 2), 3) and 4) above, logically flow from statement 1) regarding the corps sonore: chords are configurations of harmonic frequencies (partials) implying, and sometimes also supplying, a fundamental frequency, a *root.*

Where Rameau, as others, experienced difficulty (for example, Jean le Rond d'Alembert, a mathematician and sometime collaborator with Rameau) was in accounting for the nature of the minor chord with a similar degree of elegance as the major triad emerges from C-h4, E-h5 and G-h6 of the harmonic series. To achieve an explanation Rameau was forced to resort to a somewhat arbitrary assumption about an arithmetic series being 'suggested' by the harmonic relationships of the corps sonore, which then could be used to generate a descending sequence from the fundamental: C-h1, F-h$^1/_3$ and Aflat-h$^1/_5$.

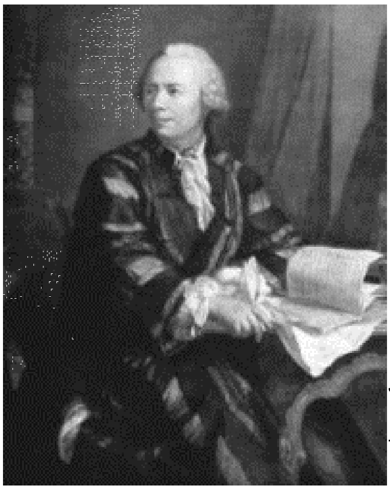
Frequency: A♭h1/5 <--- Fh1/3 <--- Ch1 ---> Gh3 ---> Eh5

Wavelength: A♭λ5 <--- Fλ3 <--- Cλ1 --> Gλ1/3 --> Eλ1/5

Figure 4. The major (ascending) and minor (descending) triads emanating from a single fundamental Ch1, illustrated in ratios of frequency and wavelength.

Rameau had marvellous theoretical instinct, perhaps in part stemming from his compositional gifts and experience, which helped him to make such remarkable progress overall. However, although his instinct had essentially led him correctly to the door of the conundrum, he lacked the key to unlock the secret of the minor chord. Like Zarlino before him, the arithmetic relationship associated with the minor chord hung tantalisingly close to a convincing explanation, yet still proved to be beyond reach. Zarlino had delineated the relationship in 1558, now two hundred years later, Rameau set the descending arithmetic series within the context of the ascending corps sonore, but could find no convincing explanation for so doing.

The next major step towards understanding the minor chord would not come for another hundred years, but two mathematicians were to make contributions in the meantime: Leonard Euler in relating consonance and dissonance to the frequency ratios of intervals and Joseph Fourier in discovering that any complex periodic wave pattern, including musical sound, can be reduced to a set of simple whole numbered sine waves of varying intensity, –i.e. parts of a harmonic series. Fourier's work demonstrated that each harmonic 'frame' of a composition, each chord, ultimately reduced to a single, though often extensive, harmonic series. While Euler developed a mathematical function that ranked the 'suavity' or sweetness of intervals in a more comprehensive and systematic manner thus providing some basis for the conception of a consonance-dissonance continuum.



Picture courtesy Wikipedia

Leonhard Euler (1707–1783), was born the son of a protestant pastor and a pastor's daughter: Paul Euler and Marguerite Bruckner, at Basel, Switzerland. Along with his two sisters he received an early education firmly founded upon his parents' Christian convictions, which remained a part of his character throughout his life. Although the family had moved away from Basel, Euler returned there, furthering his studies at the University of Basel and receiving private tuition from the renowned mathematician and family friend Johann Bernoulli. Though originally destined to follow his father into the church, Euler's self-evident mathematical talent and Bernoulli's influence swung him towards a mathematical vocation. In 1726 Euler obtained a position at the Imperial Russian Academy through his connection with Bernoulli. From 1727 to 1741 Euler lived in St. Petersburg, gradually advancing his position to become head of the mathematics section in 1733. In 1734 he married Katharina Gsell, the daughter of an art teacher, bought a house in St. Petersburg and began what was to become a large family. Many different areas of mathematics were enlarged and enriched by Euler; amongst his many contributions, he advanced the understanding of the relationship between primes and integers, and one piece of work, the Euler beta function, was used by Gabriele Veneziano in the twentieth century in the initial development of string theory. Another area of interest to Euler was the long-standing connection between mathematics and music, about which he wrote in *Tentamen Novae Theoriae Musicae* of 1739. In this work he set forth a mathematical approach to consonance and dissonance, and, in the final chapter, began to investigate the process of modulation. By this time, Euler's remarkable mathematical ability and productivity had been noticed abroad and in 1741 Frederick the Great of Prussia lured him to Berlin with the offer of a position. Euler was to spend twenty-five extremely productive years in Berlin, although, his relationship with the king was eventually to sour. Euler's unflinching religious convictions sat uneasily within one of the most sophisticated and cosmopolitan of European courts. Luckily, in 1766 Euler was able to return to St Petersburg under the patronage of Catherine the Great, where he remained for the rest of his life. His last years were overshadowed by the loss of his wife in 1773 and the general deterioration in his eyesight; however, he continued to work on mathematical problems to the end of his life.

Handel made the pilgrimage to Italy in 1706, furthering his already formidable compositional technique, which he applied to opera and sacred music while there. He meet Corelli and the Scarlattis, both father and son, absorbed all he heard while travelling widely, and in 1709 managed to have his opera *Agrippina* performed in Venice. J.S. Bach never made the journey to Italy (though as a young man he walked 250 miles to hear Buxtehude play in Lubeck), but he is known to have closely studied many of the works of Vivaldi, and also the great French keyboard composer Francios Couperin (1668–1733), thoroughly assimilating the techniques of both countries. The great achievements of J.S. Bach, too numerous and well known to rehearse in full here, were to mark the close of the Baroque period. Almost every form Bach touched, he enriched, often fulfilling and rounding out the lines of development begun earlier in the Baroque period, for example in his *Chorale Preludes* and *The Art of Fugue*.

In the two volume collection of forty-eight preludes and fugues, *The Well-tempered Clavier*, filled with pieces written in each of the twenty-four major and minor keys, Bach demonstrated the utility of the complete cycle of key relationships. That is, the logical extension of harmonic computation to the level of key relationship, producing a closed system of twelve tonally related sub-systems: the twelve major and minor tonal centers. A scheme of relationships engineered via the most musically powerful of the modulations – the sesquialtera 3:2 or the sesquitertia 3:4 exchanges. The achievement in the mature Baroque of not only a comprehensive form of harmonic structuring founded on a single fundamental frequency (–i.e. a key or tonal center), but also the extension of that organisational principle into a cyclic system of harmonically related structures (twelve tonal centers) probably marked out the approaching zenith of mutable base position-value number processing in western music.

It was in 1714 that Giuseppe Tartini (1692–1770) the virtuoso violinist, and composer of music which foreshadowed the coming Rococo style, made a remarkable discovery – difference tones: The impression of a lower frequency, a bass note, created in the ear and aural processes by two higher sounding notes, now called Tartini tones or difference tones. Tartini tones are most clearly provoked by loud third intervals in a high register. As a virtuoso violinist and teacher he found these 'virtual' bass notes useful in regulating the intonation of double stops – pairs of notes played simultaneously on the violin. The amazing point is that the Tartini tone does not objectively exist, it is created within the human hearing system. The significance of Tartini's discovery of difference tones is that it was the first inkling of the fact that the ear and processes of aural cognition are constructed so as to sift out a pitch of more fundamental period from an often complex collection of frequencies; and that the note 'heard' may not exist beyond being a subjective construct of the ear.

"Recent research has brought about the startling result that certain well-established musical sounds, e.g., that of the G-string of the violin, are physically non-existent, being produced only aurally as the differential tones of their upper partials."

Willi Apel - Harvard Dictionary of Music[1]

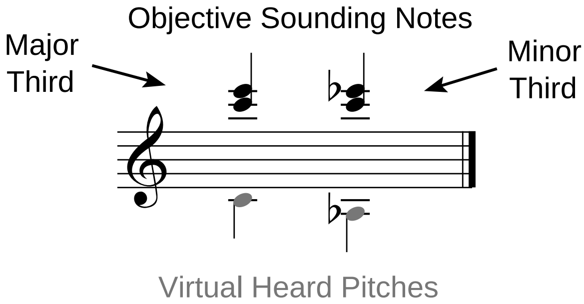


Figure 5. The Tartini tones generated by a major and minor third. While the major third doubles its lowest objective note two octaves below, the minor third produces an 'original' virtual pitch two octaves plus a major third below.

Organ buiders also use the difference effect to give the impression of a deep bass stop by employing two smaller pipes sounding in the position of overtones h2 and h3, and in the cornet registration separate flue stops masquerading as h1, h2, h3, h4 and h5 of the harmonic series give a convincing impression of a rich and refinded reed instrument.

Notwithstanding, more often in musical performance (but not always) the note heard does objectively exist but it is a frequency of little strength or amplitude. For the most part, and particularly in the lower registers of most instruments, this weakly present (and sometimes non-existent) frequency is bolstered and enriched by the ear's processing of its associated higher frequencies. The human ear sweeps in a multi-frequency harmonic spectrum of sounds and delivers single frequency 'note' pitch sensations to the mind.

There is a complex relationship between the notes we perceive and the actual physical sound stimuli which gives rise to them. Even today the workings of the ear and aural cognition are not fully understood. However it is reasonably clear that note sensations are produced from an amalgam of many frequencies and that these note or pitch sensations carry information derived from the whole spectrum of the sound, for example in the tone color of a perceived pitch and awareness of the direction from which it comes. Tone color is encoded in the character of the pitch sensation and is familiar to us all but directional awareness arises in the mind opaquely.

Another attribute of the hearing mechanism, whether learned or innate, is the visceral appreciation of the pitch relationships between succeeding chords. Chord sensations are an amalgam of simultaneous individual pitches. The resources of the human hearing system are deep. Although the individual notes within a chord may be picked out by an experienced listener, the ear also presents something analogous to directional information in the form of the perception of a *root* pitch: A sense of what the fundamental period of the complete pitch sensation is – should the individual notes, taken together, form a recognisable part of a harmonic series. The 'rooted' nature of most chords would appear to be something of a meta-pitch sensation derived from the multiple spectrums of simultaneous notes. However, this rootedness of chords is not perceived overtly for the most part (as in the special circumstances of the difference tone sensation) but rather like the directional attribute of sound stimuli, is sensed obscurely. Thus lending the root perception a somewhat abstract quality though one positioned below the sensation that give rise to it.

It is then only a small step further from these mutely perceived chord roots to the mathematics of proportion which arise from a sequence of chord sound-spectra in succession and the ratios described by the motion of their successive roots. This is visceral harmonic progression, a human experience, and largely an emotive experience. The mathematics presented in the accompanying appendices, the mutable number mathematics that wraps around this human sensibility is cerebral, a construct not so directly accessible or experiential; forming an analysis that extends the inherent logic in the musical sound to its natural conclusion. That the abstract mathematics of mutable numbers and the physical experience of music should be so closely entwined is perhaps remarkable.

Tartini's discovery lent additional credence to Rameau's theory of the fundamental bass, which was itself an outgrowth of the earlier inventions of monody and the figured bass. Here lies two pervasive strands of thought: Rameau's identification of harmony and chord progression at the center of tonal music, while other more conservative theorists preferred to maintain and develop a traditional approach based on scales. The former, deductive in character, focusing on the vertical relations of harmony, and the latter rather more descriptive, melodic and 'horizontal' in outlook. Rameau, and his theoretical heirs, for example Hugo Riemann, aspired to understanding the processes of harmony, the functions of chords, rather than contenting themselves with taxonomic enumeration.

However, it was from among the scale theorists, that the immensely versatile Roman numeral (I, II, III, IV, V, VI, VII) scale-step approach emerged, first systematically developed by Gottfried Weber (1779–1839), and soon universally recognised and adopted. Notwithstanding the utility of Weber's labelling scheme, it unfortunately doesn't enlighten or inform the theorist-musician as to why the chords behave as they do. The challenge for the more ambitious functionalists is to find a rigorous theory of harmony that can both explain the nature of chord progression, and simultaneously subsume within its contours the patently successful Roman numeral scale approach. Though Weber himself strongly rejected any idea that his scale degree labels could, or should, convey any prescriptive information concerning the functions of chords, over time, a functional aspect has gradually attached itself to his system.

Picture courtesy de.Wikipedia

Jacob Gottfried Weber 1779–1839, was born at Freinsheim, Rhineland-Palatinate, Germany. From a young age he showed an interest in music and played the flute with skill. After early schooling, Weber when on to study law at Heidelberg and Gottingen, graduating in 1802. Throughout his life, while pursuing a demanding career in the civil service and the law, at Mannheim, Mainz and later Darmstadt, Weber also indulged an interest in music teaching, composition and music theory. Longstanding friend of his name sake, the composer Carl Maria von Weber, J.G. Weber was a man of many parts: concert promoter, overseer of local theatres, church music reformer and judge. In 1824 he founded the influential music periodical *Cacilia* (published by Schott at nearby Mainz), of which he was the editor until his death in 1839. Through this rather scholarly and conservative journal, he promulgated both his musical tastes and his trenchant views on music theory. Also published by Schott was Weber's principal theoretical treatise *The Theory of Musical Composition* (Versuch Einer Geordneten Theorie Der Tonsetzkunst), in three and four volume editions, where he introduced the systematic use of Roman numeral labelling of chords, by reference to their roots, and confronted its accompanying corollary of multiple meanings (–i.e. different functions) for each label. For example, chord V in the key of C major, is chord I in G major and III in E minor. As with Rameau, Weber's direct experience as pedagogue and composer perhaps exerted a significant influence on his theoretical thinking. Gottfried Weber died at Bad Kreuznach on 21st September, 1839.

THE CLASSICAL STYLE

After 150 years of near continuous development culminating in the masterful, highly wrought works of J.S. Bach, there was a short period of relaxation – the Rococo era – ushered in as a reaction against such complexities, this a new ‘naive’ style was imbued to a large degree with the Enlightenment’s fascination with the pastoral idyl. The Rococo or ‘gallant style’ proved a brief but decisive interlude, in that the grip of contrapuntal thinking, which had lasted throughout the Baroque period, though in a subservient role to harmonic organisation, was finally shaken off completely. Quite quickly, the Rococo ideals of simplicity and elegance, expressed almost entirely in purely harmonic form – chord progressions and expressive melody, harmonically governed – deepened and matured into the Classical style. In the analogy made with literary form, the change that occurred from Baroque to Classical styles is that of monologue to dialogue.

What has been characterised as a Baroque lecture, metamorphosed into a Classical discussion or debate. The extended linear harmonic logic of the Baroque monologue was chopped up into little pieces and stuck back together to form a multi-faceted Classical collage, with a sense of balance and contrast holding the ring between competing ideas. The dialogue principle provided a highly flexible and adaptable formula with a tendency to favor shorter themes and motives, which could fill out the underlying harmonic progressions with a necessary level of interest and ‘activity’ – much as the longer lines of contrapuntal writing had done in the Baroque style. This basic formal invention, Classical sonata form, could span both the small and large scales, from a short discussion between strangers on a street corner or even a conversation with oneself (–i.e. the unaccompanied solo sonata), to a town meeting or great parliamentary debate. The essence of sonata form was simplicity itself: character one makes a statement or proposition, character two disagrees and makes a more or less contrasting declaration, they proceed to discuss and debate the relative merits of the two ideas, finally coming to a resolution or accommodation of some sort, the conclusion. The musical driving force to impel the two characters’ arguments along lay readily to hand in the dominant-tonic chord progression – the most powerful of the mutable number exchanges. And with the Baroque legacy of the fully developed cycle of key relationships, the contrasting propositions of the sonata-debate form could be articulated as conflict, tension and resolution between different keys. The Classical symphony represents the ultimate large-scale form, based on the principle of tonal dialogue, but such a scalable method could be used throughout the range, e.g. piano sonatas, string quartets and concertos. Symphonies would continue to dominate the musical landscape well into the twentieth century, though composers would on occasion feel the need to supplement the form with narrative features, leading in the later Romantic period to the symphonic poem and music drama.

The changeover from Baroque to Classical occurred in the time of J.S. Bach’s many musical children: Carl Philipp Emanuel (1714–1788), Wilhelm Friedemann (1710–1784) and Johann Christian (1735–1782), to name but a few! And in the Germanic lands, the Rococo took the different and rather more intense character of the *empfindsamer stil*, a somewhat more ‘romantic’ version of the style. A fusion of the lighter, essentially French strain of the gallant, with the expressive German style, yielded the Classical amalgam of expression bounded by an elegant restraint, the instincts and ideals of the eighteenth century enlightenment expressed in music.

The core works of the Classical period are to be found in the compositions of Franz Joseph Haydn (1732–1809) and Wolfgang Amadeus Mozart (1756–1791), the former experiencing a gradual and extended development of the possibilities of sonata form while the latter, comet-like, illuminated the creative firmament with his astounding gifts, all too briefly. They were personal friends, each regarding the others work with high respect, and at different periods of their careers they influenced one another – as also did the music of Haydn’s brother, Michael Haydn (1737–1806) and of course others to a lesser degree. Joseph Haydn is generally regarded as the father of the symphony, and in his long years of service on the isolated

Esterhazy estates he gradually developed the new form, as he simultaneously honed his own compositional skills. In all he was to write more than 100 symphonies while Mozart's last, the Jupiter, stands at forty-one.

The works of Ludwig van Beethoven (1770–1827) also fall within the Classical period, and though influenced to a degree by the work of his contemporaries, his musical thought tended to be much more individual. In music as in life, Beethoven was his own man. Though he only produced nine symphonies, they are of a scale and stature which took the symphony beyond its purely classical form. Yet it would not be true to say that Beethoven had no regard for the music of others; often he would seek out and study the scores of his fellow composers. Indeed, right at the zenith of the tonal era, in the Classical period, the principal composers of the time, Haydn, Mozart and Beethoven, all began to look back to the previous generation, valuing what came before, and this trend was only to intensify in later years. For example, Mendelssohn's role in the promotion and re-publication of Bach's music, or Beethoven writing to F.A. Hoffmeister[2] in 1801:

"Your desire to publish the works of Sebastian Bach is something that really warms my heart which beats sincerely for the sublime and magnificent art of that first father of harmony."

This change of attitude to the music of past ages is significant. In earlier periods, that is until the full emergence of tonal harmony in the Classical period, music was very much an art of the moment. Up to this time, the musicians of the day made and performed music of the current time; hardly anyone one considered playing the old music of former generations. (There are a few exceptions from earlier times but the general trend is what is important.) The current 'modern' style, whatever that style happened to be at that moment, was more or less, all of music. Innovation and fashion could keep the current style fresh by 'moving on', thus constantly relegating the music of earlier times to oblivion. For example,

"Tinctoris [the Flemish music theorist] in 1475 thought that no music worth hearing had been written before 1440; the Swiss theorist Glarean in 1547 was persuaded that no one could ever surpass the music of Josquin des Prez (d. 1521); for Zarlino [...] writing in 1558, the acme of musical art was reached in the works of his contemporary, Willaert..."

Donald Jay Grout – A History of Western Music[3]

However, at this peak in the story of tonal music, and perhaps more generally throughout the eighteenth century, something very different was beginning to emerge in western music: tonally organised harmony was showing itself to be something more powerful, wide ranging, and altogether rather different from a 'mere' style or fashion of the moment. What had apparently grown up almost unnoticed, was an aural mode of thought and communication – *the language of numbers expressed in sound.* Once the language became established in the minds of musicians and the general populous, two effects were to become discernible. First, they could if they wished, apply the same mode of aural processing to the tonally organised music of former generations with equal satisfaction, once the small obstacle of the cliches and mannerisms of the older style had been assimilated. And, second, although this would not become apparent until the twentieth century, the majority of members of the western tradition would show themselves extremely reluctant, not to say highly resistant, to abandoning their tonal language for any other mode of aural expression and communication. Western music was no longer simply a succession of *styles*, it had become a *continuum* of tonally organised music, clothed in a succession of outwardly differing stylistic expressions. Today, after more than 100 years of hard labor, expended by countless idealistic composers and musicians, advancing almost every conceivable variation of contemporary non-tonal style (or even language), the vast majority in the western tradition, plus many new recruits from other music cultures, have hardly budged an inch in their predilection for tonally organised music: *aural, mutable base number processing.*

Picture courtesy Wikipedia

Jean Baptiste Joseph Fourier (1768–1830), was the son of a tailor, born at Auxerre, France. Orphaned at the age of eight, he was brought up at St. Mark's convent in Auxerre, where he also received his early education. With few prospects for advancement due to his lowly birth, Fourier entered the army as a teacher of mathematics. He was an ardent supporter of the Revolution, and his participation in the overthrow of the ancien regime opened up prospects of promotion. Fourier advanced to take a position at the Acole Polytechnique and soon came to the attention of Napoleon who supported his academic work and made use of his organisational talents by appointing him Governor of Lower Egypt during his abortive eastern expedition. In the years following the French expulsion from Egypt, Napoleon continued to make use of Fourier in administrative posts in France. Alongside his political duties Fourier carried out his scientific and mathematical investigations into the nature of waves and the propagation of heat. After his patron's final fall from power in 1815, Fourier moved to England where he continued his researches; returning eventually to Paris, where he died in 1830. The theorem which bears his name and the Fourier analysis which flows from it is probably the most used and perhaps the most useful mathematical procedure ever discovered or devised.

THE ROMANTIC PERIOD

The change of style wrought by the Romantic movement in the nineteenth allowed something of a *new face* to be placed over the mutable number system, but the heat from the fire was dying, the fuel supply – novel extensions to the system of harmonic computation – were rapidly being depleted. The change from Classical to Romantic style, to continue the analogy used above, was rather like changing the mode of expression from prose dialogue to impassioned poetic recitation. The basic grammar and vocabulary of the language of tonal computation remained much the same, but the way these elements were used, declaimed, changed considerably. As in poetry, not only what is said, but how it is said, is important in ascertaining and understanding the message – as Franz Schubert exemplifies so well.

Great dynamic contrasts were introduced which might imbue simple chord progressions with intense meaning, sudden changes in tempo perhaps illuminate the troubles of the artist's soul! The cool and measured detachment of Classicism was exchanged for the luxuries and licence of self-expression, a limitless yearning for the unknowable and unattainable. The trend began with Beethoven, towering over the Classical achievements of his contemporaries in Vienna, whilst also foreshadowing the expressive objectives of the generation to come. However, for all the intensity of Beethoven's music, he rarely lost sight of the need to maintain a disciplined approach to form. While exerting a huge influence on the Romantic composers, he was not one of them.

In the years after Beethoven's death, Robert Schumann (1810–1856), Franz Liszt (1811–1886), Hector Berlioz (1803–1869) and Peter Ilyich Tchaikovsky (1840–1893) carried the vanguard of emotion forward, while Felix Mendelssohn (1809–1847), Frederic Chopin (1810–1849) and Johannes Brahms (1833–1897) tempered their work with a degree of Classical restraint, as had Franz Schubert (1797–1828) earlier. Perhaps Richard Wagner (1813–1883), Gustav Mahler (1860–1911) and Richard Strauss (1864–1949) were the last to extract a little new heat out of the dying embers, with exchanges involving extended chords and unexpected resolutions, before the fire of novel extensions to the language of tonal computation were to be fully exploited and exhausted in western art music.

The central engine of tonal harmony, the simple powerful dominant-seventh/tonic relationship, by this time had been overlaid by many extra intervals, dominant-nineths, elevenths, thirteenths, as each generation strove to extend the language and exploit new relationships. However, not unlike the process of nuclear fusion that powers the stars, this approach is one which yields declining returns. These ever more complex chords become increasingly difficult for the ear to unravel, giving progressively less sense of directionality to the music. Mirroring the gradual loss of tonal direction through the nineteenth century, composers, again taking a hint from Beethoven (the Pastoral and Choral symphonies), began to compensate for the weakening of the absolute element of firmly perceived mutable number processing, by introducing programmatic and narrative features into their works. This trend was to culminate in the forms known as the symphonic poem (Strauss) and music drama (Wagner). These gigantic compositions written at the end of the Romantic period, scored for huge orchestras, perhaps convey something of a sated melancholy and nihilism often associated with the end of an era – *the end of the tonal era.* To continue the physical metaphor, one might likening these huge works to the dying red giant stage in a star's evolution.

The nineteenth century brought great advances in acoustics and music theory; two outstanding individuals, one associated particularly with the former was Hermann L.F. von Helmholtz[4] (*On the Sensations of Tone, 1863*) and with the latter Arthur von Oettingen[5] (*The Dual Harmony System 1866, 1913*). Both were scientists and talented musicians, but not composers. Helmholtz (1821–1894) exemplified the dispassionate scientific approach of careful experiment, leading to theoretical conclusions; it was he who discovered the (still disputed) existence of summation tones. On the conundrum presented by the minor triad Hellmholtz, echoing Zarlino and Rameau, concluded that it must in some way be a secondary and synthetic phenomenon, derived from the major principle embodied in the lower ratios of the harmonic series.

Picture courtesy Wikipedia

Hermann Ludwig Ferdinand von Helmholtz, 1821–1894. Born at Potsdam, the son of a school headmaster, the young Helmholtz showed a marked inclination for the natural sciences. Originally intended for a career in medicine by his father, Helmholtz's devotion to research drew him toward his undoubted scientific vocation. Helmholtz contributed original research in many areas – thermodynamics, sensory physiology, ophthalmics and electromagnetism – and in 1863 he published a penetrating and influential book on acoustics and auditory perception: *On the Sensations of Tone as a Physiological Basis for the Theory of Music.* As befits a keen student of the senses, he was a good amateur pianist and sensitive musician who found the compromise of equal-temperament difficult, indeed painful to bear. Through his experimental skills, designing and improving instruments for detection and measurement of sound, Helmholtz discovered the existence of summation tones: ghost tones created within the mechanisms of the ear, similar to the stronger difference tones first noticed by Tartini. He designed a harmonium for the study of overtones and a detector which bears his name – the Helmholtz resonator – which he used in his research into the nature of consonance and dissonance: the key to which he believed lay in the interaction of the fainter, higher harmonics. Helmholtz concluded from his research that the ear was more complex in its workings than had previously been thought, and put forward a hypothesis of the ear as a non-linear mechanism. That is to say that the connection between objective aural stimuli and what we perceive is not a simple or directly linked relationship; there are discontinuities and complex relationships between input and output. One area of criticism levelled at his work was that it was rather atomist in its approach, focusing on sustained and isolated sounds, taking little account of music as a dynamic sequence of stimuli. In 1871 Helmholtz was appointed Professor of Physics at the Humboldt University in Berlin, where he remained, increasingly laden with honours in later years, until his death in 1894. While working in Berlin, he came into contact with Max Planck the father of quantum theory and A.A. Michelson whose experiments measuring the speed of light were to be crucial to twentieth-century physics.

Helmholtz's conclusion that the human hearing mechanism is non-linear was a marked advance in understanding. For millennia it had been assumed that the sensation of heard sound was a direct representation of objective music in performance. In contrast to Helmholtz, and building on the earlier philosophical approach of Moritz Hauptmann[6] (*The Nature of Harmony and Metre, 1853, 1873*) and Helmholtz's own work, Arthur von Oettingen (1836–1920) developed Rameau's intuition concerning the descending structure of the minor triad forming an arithmetic mirror image of the ascending major triad. Oettingen noticed that the overtone series of the constituent notes in the minor triad had a common partial two octaves above the fifth (e.g. h10-E, h12-G and h15-B all share the common partial h60-B) and that this shared overtone structure is matched, in reverse, by the harmonics below the major triad (e.g. h4-C, h5-E and h6-G all share the common fundamental h1-C). Although Rameau had derived the minor triad from below the fundamental tone of the major triad he then placed it by 'suggestion' alongside the ascending series; now Oettingen, developing Hauptmann's views, proposed that the minor triad should be considered as having an equal and independent status with regard to the major triad, with its own downward operating, 'phonic', generative principle.

Picture courtesy Wikipedia

Moritz Hauptmann, 1792–1868. Born at Dresden, Hauptmann received a broad education and was intended to follow his father's profession of architecture. However his great gifts drew him into a musical career, after a short time as a practising architect. A talented violinist, he was something of a polymath interested in a wide variety of subjects both scientific and artistic. His musical career was mentored by Louis Spohr and later, on Felix Mendelssohn's recommendation, he was appointed Kantor of the Thomasschule – a position once occupied by Bach. He was a joint founder of the Bach Gesellschaft in 1850 and professor of counterpoint and composition at Leipzig. Influential as a teacher, his principle theoretical publication *Die Nutur der Homonik und Metrik* was characteristically elegant and philosophically sophisticated, deeply thoughtful though somewhat detached from practical music making. Using Hegelian methodology, Hauptmann reinforced Zarlino's thoughts concerning the 'dualistic' or opposite natures of the major and the minor third interval. An idea further enlarged on by Arthur von Oettingen. Moritz Hauptmann died at Leipzig on 3rd January, 1868.

Descending Undertone Series	h60-B	h60-B	h60-B
	h50	h48	h45
	h40	h36	h30
	h30	h24	**h15-B**
	h20	**h12-G**	
	h10-E		Minor Triad
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Ascending Overtone Series	**h6-G**		Major Triad
	h5	**h5-E**	
	h4	h4	**h4-C**
	h3	h3	h3
	h2	h2	h2
	h1-C	h1-C	h1-C

Figure 6. A major CEG triad derived as overtones of h1-C and and the minor EGB triad from 'undertones' of h60-B.

Oettingen never expressed his ideas in a way contrary to the accepted principles of acoustics – though some of his more ardent supporters did at times, Hugo Riemann for example. However, the basic stumbling block to the acceptance of this elegant theory of mirror triads of ascending major 'tonic' structures and descending minor 'phonic' structures, is the plain fact that the ear cannot recover wavelength information in this way, and so does not recognise descending phonic structures as such. Although harmonic dualism, to use Oettingen's chosen term, experienced a considerable vogue in Germany in the late nineteenth and early twentieth centuries, it eventually became discredited, as no way around its fatal flaw could be found. Again intuitive insight had led up to the door of the minor paradox without the key to unlock it being found.

Picture courtesy Wikipedia

Arthur (Joachim) von Oettingen, 1836–1920. Born into a noble family with scholarly traditions, originating from southern Germany but long resident at Tartu, Estonia (then Dorplat, Livonia), v. Oettingen studied sciences first at the University of Dorplat and later in Berlin and Paris. Returning to Dorplat in 1863, he completed his teaching qualifications and took the post of Professor of Physics at the university in 1866. (Two brothers Georg and Alexander were concurrently Professors of Medicine and Theology.) The same year saw the first publication of his theory of harmonic dualism – *Harmoniesystem in dualer Entwickelung.* Oettingen continued to refine his radical ideas on harmony and published two further accounts in 1902–6 and 1913, as well as numerous articles in the fields of physiology and physics. He held the chair of physics at Dorplat until his retirement in 1894, and throughout his tenure was active in local scientific and musical societies. A keen musician, Oettingen was president of the Dorplat musical association and director of an amateur orchestra. His retirement years were spent in Leipzig, where he held an honorary professorship; during these years the two later statements of the theory were published.

Hugo Riemann was probably the most influential and versatile music theorist of these times, he wrote widely on almost every musical topic, and in his writings he attempted to incorporate Hauptmann's and v. Oettingen's dualistic harmony within a more or less complete music theory. Riemann adopted Oettingen's notation, and further developed the implicit harmonic relationships that emanate from a dualist approach into a fully fledged account of harmonic progression in tonal music. This took the form of a theory of chordal *function* expressed in relation to the tonal center – the tonic chord.

Picture courtesy Internet

Karl Wilhelm Julius Hugo Riemann, 1849–1919, the son of a prosperous landowner and music lover, was born at Grossmehlra near Sondershausen in Thuringia, Germany. He was educated at local schools and his talent as a pianist was nurtured with lessons from distinguished music teachers. Riemann travelled to Berlin and Tubingen universities, where he first studied law, and later, literature, history and philosophy. As a young man he served in the army during the Franco-Prussian war of 1870–71. Attracted by the creative arts, Riemann had hoped to make his way as a poet but failed to find a publisher. He also tried his hand at composition, but where Riemann found he could make a mark, was in his writings on musical topics. His first publications appeared under a pseudonym in the *Neue Zeitschrift fur Musik.* After leaving the army he set his sights on a musical career and chose to study at the Leipzig Conservatory. In the years up to the turn of the century Riemann held a number of teaching posts around Germany while also producing a stream of voluminous writings on a wide range of musical topics, e.g. *Musiklexikon, Harmony Simplified, Great Composition Tutor,* etc. He married Elizabeth Bertelsmann in 1876, with whom he had five children. Gradually his reputation and influence as a music critic, theorist and musicologist grew, at first within Germany and later throughout Europe and North America. In 1901 he returned to Leipzig as a professor. In 1908 he was appointed as the founding director of the Collegium Musicum musicology institute and, in 1914, founding director of the State-Saxon Research Institute for Musicology. Hugo Riemann died on the 10th July, 1919 in Leipzig.

The general notions of Riemann's harmonic functionalism would establish themselves, almost universally, as the standard method of harmonic analysis in the twentieth century, through the attachment of his 'functional dynamics' to Weber's Roman numerals. Thus, in the minds of music theorists and analysts, a descriptive Weberian chord sequence like, for example I–vi–ii–V–I, would take on a dynamic directional connotation, impelling the phrase towards the cadence. Also to be of great influence, in the second half of the twentieth century, was Hugo Riemann's development of dualism's implicit creation of a relational harmonic 'space' – first articulated by Oettingen. Riemann's representation of a tonal network (tonnetz) of nodes and transformations based on the functional relationships of tonic, dominant and subdominant (extended to include secondary chords as well) would prove a fruitful starting point for the mathematically minded 'neo-Riemannian' theorists of more recent times.

Riemann struggled throughout his career to overcome dualism's central weakness, but in the end he had to admit defeat and settle for theoretical and psychological arguments – unbending science would not allow the existence of an objective generative undertone series. In this regard, his conclusions mirrored his great predecessor, Rameau; in addition to which, the character of Riemann's career-long refining and revising of his theoretical precepts, expressed in numerous books, also finds an echo. The range and volume of Hugo Riemann's thought and writings are encyclopaedic, covering almost every aspect of music: harmony, counterpoint, meter, rhythm, notation, performance, historical topics, music psychology, etc., to name only a small portion!

Of particular significance was Riemann's foundational work in the field of musicology, where he pioneered with others (e.g. F. Chrysander and G. Adler) research into, and publication of, the historic canon of tonal music. From its beginnings earlier in the nineteenth century, the growth of interest in the music of previous generations of composers was accelerating, as musicians and the wider musical public found that their common musical language – tonality – gave them access to a historic archive of untold riches. In 1850, the founding of the Bach-Gesellschaft society, dedicated to unearthing and publishing the then almost unknown works of J.S. Bach (a task that was to take half a century or more), was a milestone and signpost to the future.

Right up to the present day musicians, music scholars and historians have been toiling to salvage, research and perform almost every last remnant of our musical heritage. Gradually the mists of oblivion have been beaten back, as the forgotten music, the forgotten instruments and the forgotten performance practice of previous centuries have been excavated or reconstructed. Of first concern were accurate scores, but later, concern for authenticity extended to the types of instruments and the styles of performance that might be appropriate for historic works. Over time, the horizon has been pushed back ever farther; for example, Monteverdi, whose music was unknown before 1950 is now seen as a pivotal figure, and today one might reasonably expect to hear performances of Landini or Leonin. All this raises the question: Why are we, unlike earlier generations in the western tradition, so interested in the music of the past?

THE TWENTIETH CENTURY

As with the author of *Huckleberry Finn*, reports of the death of tonal music around the turn of the twentieth century had been greatly exaggerated. Though composers had finally extracted and exploited every peripheral chord progression from the system by the end of the nineteenth century, and generally felt that the time to 'move on' had arrived, the western musical culture en masse saw it differently. Over the preceding centuries a language had grown up, been acquired and thoroughly assimilated by the tradition; any attempt to usurp its role or arbitrarily impose another language was as likely to succeed as an act of Congress substituting Esperanto for English as the national language of the USA. What some composers of the twentieth century failed to grasp or simply ignored, was that tonality is (or had developed into) a language and was not simply a (superficial) style. The rules of the game had changed fundamentally: in regard to the choice of the language of musical communication, they, the composers, were no longer in the driving seat.

Composers are of course entirely at liberty to use other languages, invent new languages or make whatever innovations they please, and some of the bravest have done just that, but the inevitable consequence of such a course of action is that their music will only be intelligible to a tiny minority, their efforts condemned to the sidelines – ivory towers and musicology departments. Historically, when art music was the preserve of a small elite such prescriptive, 'top-down' developments might possibly succeed, but not in a modern democratic mass society. Most twentieth century composers took the more practical middle way, blending greater or lesser amounts of the common tonal tongue with new features or borrowed elements from other or earlier traditions. And this approach, generally produced outcomes somewhat along the same lines as that of the late Romantic period, in that the music naturally lacks the clear tonal focus of earlier periods, though of course expressed in a wide variety of new, often energetic or even abrasive, styles.

Richard Franko Goldman 1910–1980, was born in New York, the son of the prominent bandmaster Edwin Franko Goldman, leader of the Goldman Concert Band. Goldman's father was a gifted cornetist who had attended the National Conservatory and studied composition under Dvorak. Richard Goldman went to Columbia University where he studied music under the direction of Nadia Boulanger and Wallingford Riegger. From 1937 to the death of his father in 1956 Goldman was assistant conductor of the Goldman Band, possibly providing the copious program notes which accompanied the summer concerts at Columbia and New York universities and in Central Park. During the years 1947 to 1960 R.F. Goldman was on the faculty of the Julliard School and in 1968 became director of the Peabody Institute in Baltimore. Richard F. Goldman was a prolific writer and music critic, from 1948 to 1968 served as New York critic for the Musical Quarterly, championing the works of Henry Cowell and Elliott Carter amongst others. In 1965 Goldman published his insightful book *Harmony in Western Music*. Significantly he was throughout his life closely involved with the 'nuts and bolts' of composition and the arranging of other composers' music, which perhaps (as with other music theorists) informed his intuitive grasp of the underlying processes and mechanisms of music, impelling his analyses unerringly toward the central logic of the tonal system. Richard Franko Goldman died on the 19th January 1980.

(Essentially, it was the collision of Goldman's analysis of tonal harmony read many years ago, with more recently acquired ideas derived from an interest in computer science which interacted and combined to form the basis upon which *Mutable Numbers* and the *Model of Modulating Oscillatory Systems* is constructed. I offer my grateful thanks to the author.)

Overall, by the end of the nineteenth century, after more than 300 years of growth, the tonal language of mutable numbers had reached a fulsome maturity. All the workable chord progressions, both from around the core dominant-tonic exchange, and the less powerful peripheral chordal and key progressions and successions, had been thoroughly exploited. Given that the western tradition is a written one, with a highly developed notational system, the possibility of simply cycling around forgotten older styles as a solution to the problem of what to do next, rather in the way the fashion industry produces an unending succession of 'new' styles inspired by previous eras, was not viable – as unlike clothes, the works of the historic masters don't wear out and get thrown away. It could be argued that essentially, both the historic canon and the vast bulk of popular music fall into the same broad category of *classic tonal music*. Most high art composers in the twentieth century (as opposed to those writing in more popular genres) were not content with 'classic' tonal expression and felt the need to produce something more 'radical'. Most of the high art music of the twentieth century falls into this category, *radical tonal music*.

During the twentieth century, the more traditionally minded (often nationalist) composers writing radical tonal music, e.g. Edvard Grieg (1843–1907), Charles Ives (1874–1954), Ralph Vaughan Williams (1872–1958), Jean Sibelius (1865–1957), tended to 'look through' the window of tonal computation to earlier epochs or peripheral genres, finding inspiration in pre-tonal modal styles, folk music and, in particular, the landscapes and character of their national homelands. Others took inspiration from exotic music cultures or ideas from other branches of the arts. For example, the 'impressionistic' style adopted by Claude-Achille Debussy (1862–1918) was influenced by the music of South East Asia, as well as the great school of French painting of the time. Also a significant group experimented with a variety of different avenues: Bela Bartok (1881–1945), Kurt Weill (1900–1950), Dmitri Shostakovich (1906–1975), Igor Stravinsky (1882–1971), Paul Hindemith (1895–1963), for example. Many were influenced by jazz.

All these various approaches essentially continued using the underlying computational system of tonality, to a greater or lesser degree (while generally avoiding the overt use of anything too closely resembling the harmonic practice of previous generations) with additional elements taken from their chosen source mixed in to leaven the offering. One might say that they were living off the combination of 'residual heat' left behind after the tonal fire had burnt itself out, while bringing other non-tonal material to lay amongst the embers. Similarly, the various strands of jazz have woven elements taken from the livelier rhythmic traditions of Africa into what is basically a tonal fabric – though often using progressions of a somewhat opaque character, thereby facilitating the free flow of improvisation. Meanwhile, in the western popular genres, the language of mutable numbers rages on entirely unabated, seemingly able to recycle classic tonality endlessly, aided perhaps by the limited shelf-life of the majority of its products. Amongst the base load, there are of course many gems to be found, and, like so much twentieth century music, the influence of jazz is pervasive.

Other more revolutionary-minded composers of the twentieth century chose to reject the established language and start afresh: Arnold Schoenberg (1874–1951), Alban Berg (1885–1935), Anton von Webern (1883–1945), Pierre Boulez (b.1925) Karlheinz Stockhausen (b.1928), John Cage (b.1912), etc. Turning their backs on tonality, or perhaps moving on beyond it, opened the door for the return of counterpoint with a vengence. The compass tilted back to that contrapuntal coast, so desired by theorist and pundit alike. However, the language of tonal computation has proved to be almost completely impervious to all attempts to shift to a new paradigm. So far, every attempt at forging a new language appears to have failed to reach, let alone engage, a wide audience. Perhaps there are no viable new (artificial) languages comparable to the one that nature has provided in the form of tonal computation – mutable numbers expressed in sound. At the beginning of the twenty-first century, there are some signs of an acceptance of this, with a discernible movement among composers back to the language of tonality as the foundation for coherent communication in western music.

From the standpoint of mutable base numbers, atonal music can be accommodated within its framework as the 'limit' of tonality: the point at which every chord is described by a single 'prime state' harmonic series stretching up from a suitably chosen fundamental, without nesting – that is without tonal chord relationships. Most probably all these extended series would be beyond the ear's tonal comprehension, as is intended. Indeed, from the tonal perspective of the mutable number model, all atonal compositions appear essentially the same, somewhat analogous to samples of gases in equilibrium states of maximal entropy: Where each example may be described by one piece of information, the temperature. Thus a single average measurement, of all the prime state mutable numbers found in an atonal composition, would be the extent of the model's penetration into this alien territory. Though any atonal composition would present a complex array of configurations, these would all go unnoticed from this point of view as they contain little or no tonal information or structure – they have no accessible nested series for the processes of aural cognition to grasp and decipher. Though, of course, it must be emphasised that this is not to say that, taken on their own terms, these compositions are not highly structured works of art music.

The twentieth century has seen music theory blossoming into a multi-faceted disciple with many influences coming from diverse fields, for example developments in psychology, analytical approaches in other branches of the arts and latterly perhaps, computers and computer science, plus recent progress made in the field of neuroscience. Also the development of a highly mathematical atonal theory based on the integer model of pitch, (–i.e. the twelve tone chromatic scale modelled as whole numbers 0 1 2 3 4 5 6 7 8 9 10 11) has developed into a significant area of study. This spreading of wings has to some extent come at the cost of a loss of focus on the central concern of previous centuries: *determining the fundamental nature of western music, as it exists*. Indeed, some might argue that today it is difficult to talk of 'western music' as a single entity, given the diversity of contemporary genres which abound. I would disagree: one single genre dominates overwhelmingly, that of tonal music in all its many incarnations – art music both old and new, popular music, jazz, world music, etc.

The diversity of contemporary non-tonal genres perhaps should be considered a somewhat more peripheral phenomenon, when viewed from the perspective of the relatively small numbers of participants involved. And perhaps to some extent the emphasis placed on the integer model in atonal theory has tended to direct attention toward the study of the twelve-note scale as the fundamental 'object' in western music and away from Rameau's conviction, that for tonal music, the fundamental object is the corps sonore – the natural harmonic oscillations of a sounding body. Ultimately music theory's principal aim must be to understand and explain the nature of music as it is. On any reasonable measure, that places tonally organised music at the head of the queue. However, notwithstanding tonal music's premier position, it should always be borne in mind that out of all the lands which constitute the continent of sonic art, many perhaps as yet undiscovered, the principality of tonal music occupies but a tiny part. For atonal music the integer model is a natural choice which has yielded many deep and valuable insights, but for the study of the predominant mode of musical expression and communication in western culture, the harmonic series probably provides a more apt foundation. Ideally, the two approaches should be linked (–i.e. the mathematics of Fourier analysis and set/group theory) so that each may contribute to a greater whole.

THE MINOR THIRD

As to the long-running mystery of the minor chord, music theorists in the twentieth century were so busy with their many new concerns, that rather less attention was devoted to the problem. However acoustics, physiology and psychology came to their aid; and perhaps also the realisation that the form western tonal music has assumed is largely an outward manifestation of the workings of the human ear and aural system[7].

The sensation generated by a single note with well in tune partials is clean and clear, and familiar to all. Two simultaneous notes with well in tune partials produce more complex sensations in the mind. For the prime or unison interval 1:1 the harmonic series of the two notes will align but the relative strengths of their partials may vary. Thus two distinct instruments may or may not be perceived by the listener. The octave interval 1:2 is similar, though the even numbered harmonics of the lower note will be reinforced by the upper note. The octave sensation is 'open' and 'firm'. Assuming there are two separate sources or instruments, no matter what the phases of vibration are, both combinations of notes, the prime and the octave, will produce a pattern of vibration that repeats at the same rate as the lowest note in the pair. Thus the these two intervals share many characteristics and are sometimes difficult to disentangle aurally because of their unified character.

The interval of a fifth 2:3 goes further, in that the sensation involves a more distinctly perceived co-mingled of its constituents. As with all simultaneous notes an experienced listener can identify individual pitches, yet it would appear that intervals and chords produce categorical sensations in which the constituent pitch ratios greatly influence the overall perception of tone color. This is somewhat analogous to the mutation or mixture stops made by organ builders, where staggered ranks of individual pipe frequencies set at intervals of fifths and octaves are subsumed into the overall tone of a registration. When listening to a bare fifth interval an overriding essence of austerity is the impression received. Regardless of phase the fifth combination has a oscillatory repetition rate $^{1}/_{2}$ that of the frequency of the lowest note.

The fourth 3:4 interval is superficially similar to the fifth in tone color but is subtly very different in that it has a oscillatory repetition rate $^{1}/_{3}$ that of the lowest note. Beament[7] points out that it is this repetition rate relative to the ratio of an interval that the ear grasps as being an interval's distinctive characteristic. These repetition rates also match the pitch of difference tones discussed earlier. Thus if a fourth interval was composed of middle C and middle F, the underlying repartition rate would match bottom F on the bass staff. Significantly, when inverted the fourth becomes a fifth and so the repartition rate also changes. Thus the inversion of a fourth or fifth changes the intervals identity.

The major third 4:5 interval is often perceived to be a bright and vigorous sound, an impression enhanced by it being wider than true in equal-temperament (4:5.18...). The interval's tone color shares something of the character of the brass family's prominent 5th harmonic profile. Its oscillatory repetition rate is $^{1}/_{4}$ that of the intervals lowest note and thus it combines naturally with a fifth interval to form a well rooted major triad with repartition rates matching one octave and two octaves below. Equally, its inversion the minor sixth 5:8 interval shares the major third's $^{1}/_{4}$ repetition rate and thus maintains its compatibility with the fifth interval. The tone quality of the major third interval pervades western music.

So finally the conundrum, the minor third interval – ratio 5:6. The tone color of the minor third is perhaps perceived to be solemn and veiled. A character it shares with church bells which also feature a minor third interval above their 'strike' note. The oscillatory repetition rate of the interval is $^{1}/_{5}$ that of its lowest note, which means that the frequency of its repetition rate does not match any note present in the interval. The perception of the minor third is somewhat unstable and dependant on context. Sounded alone, the interval could be heard as the bottom half of a minor triad, eg. A-C-(E) or the top half of a major triad, eg. (F)-A-C. The repartition rate of the interval matches the latter interpretation, though the (F) frequency lies two octaves lower. The interval's inversion, the major sixth 6:10 is perhaps even more susceptible to such interpretation by the ear (F)-C-A, particularly if either is sounded in a high register. However, there is almost always going to be forceful harmonics present in the sound counteracting any 'Tartini effect'. Indeed in practice the potentially ambiguous sensation generated by the minor third interval is stabilised by the presence of an overt fifth in the minor triad, ratios 10:12:15. And even without an overt fifth, it is principally the harmonics of a minor triad that will be incorporated into the tone color of the minor third sensation.

Minor Third **C-h1**.......C-h2....G-h3....C-h4...**E-h5**
A-h1...A-h2....**E-h3**....A-h4..C#h5.**E-h6**

Dissonance arises from the interference and irritation caused by closely spaced sound stimuli within the mechanism of the ear and degrees of consonance from the relative absence of such turbulence. The spacing of sound stimuli at the interval of a minor third falls just within the limits of perceived consonance, while a major second is mildly dissonant and a minor second sharply so. It is something of a continuum from semitone to octave. The prime and intervals of a minor third and larger, up to the octave, are inherently consonant (plus their larger compounds e.g. 10th, 12th, 15th, etc.). Indeed, when composed of only one frequency the ‘devil in music’, the augmented fourth, is smooth to the ear. The Pythagorean scale yielded an uncomfortable aug. fourth ratio of 512:729 far wide of the natural interval ratio in the harmonic series, 5:7; where it lies but one beyond Zarlino’s reformed senario of one through six. (The Just temperament aug. fourth 32:45 is between the two.) However, in normal circumstances the notes of music are not simple single frequencies but complex spectra and the ear sweeps them all in. Thus when individual notes are composed of an array of harmonic frequencies and these notes are sounding simultaneously their higher harmonics may clash. For example in the augmented fourth interval C–F#, h3-G of the C series will sound against h2-F# from the F# series and h4-C against h3-C#, and so forth. Both sharply dissonant minor seconds creating turbulence and interference in the ear. And it has already been remarked that most of the sound energy lies in these low order harmonic frequencies, typically h2, h3 and h4. Thereby the smooth single frequency augmented fourth morphs into a dissonant devil in music under normal performance conditions.

Augmented Fourth **F#h1**........F#h2....C#h3.........F#h4...
C-h1...C-h2.....G-h3....C-h4..E-h5.G-h6.

There are many variables in addition. The differing formant of individual instruments, the acoustics of the room or outdoors, etc. The bottom heavy harmonic spectrum of a flute in comparison to the hollowed-out spectrum of a clarinet or the flaring harmonic array of a trumpet are all factors affecting the perception of consonance/dissonance. Also register is a factor. The hair cell receptors in the inner ear lie closer together at lower frequencies, thus the minor third interval already near the limit of perceived consonance is pushed ever closer to the edge in lower registers. Intervals of a minor third are seldom written below tenor C, except perhaps on the harpsichord.

In contrast to the aug. fourth the notes of the minor triad A-C-E produce a more clearly defined lower spectrum with harmonic reinforcement:

E-h1...........**E-h2**......B-h3......**E-h4**...
Minor Triad **C-h1**.......C-h2....G-h3....C-h4....**E-h5**...
A-h1...A-h2....**E-h3**...A-h4...C#h5..**E-h6**...

And similarly for the major triad A-C#-E:

E-h1...........**E-h2**......B-h3......**E-h4**...
Major Triad **C#h1**.......C#h2....G#h3......**C#h4**..**E-h5**...
A-h1...A-h2....**E-h3**...A-h4...**C#h5**..**E-h6**...

In root position the repetition rate of the major triad, ratios 4:5:6, is $^{1}/_{4}$ of its root, while that of the root position minor triad, ratios 10:12:15, is $^{1}/_{10}$ of its perceived root. Inevitably the tone color of both triads will be heavily influenced by the mix of harmonics arising from their constituent notes. The character of these relationships remain when chords are spread out in various dispositions and inversions; thus the identity of the chord type is conserved. (Though the actual repetition rate may be projected to some greater fraction.)

The repetition rates of intervals and chords provides a basis for the ear's ability to recognise and classify them individually. For example, a first inversion major chord is both recognised aurally as being 'a major chord' and as having its notes arrayed in a particular disposition – a detail of aural acuity which suggests the possible perception of nested repetition rates. (–i.e. $^1/_4$ nested within $^1/_8$ or $^1/_{16}$ for a first inversion chord, depending on the note dispositions.) For reference a repetition rate of $^1/_{10}$ of middle C would represent a frequency around the lower threshold of human hearing.

The diminished triad A-C-D# (Eb) is also instructive in that in contrast to the minor triad its repetition rate is $^1/_5$ of its lower note and half that of its cousin the minor triad. The ear will generally interpret this triad as being a common seventh chord missing its root. Which perhaps indicates some perception of the $^1/_5$ repetition rate is feeding into the sensation of a root not represented in the chord itself. Equally, one could argue that this root expectation has been learned. Unlike the minor triad the diminished triad contains no countervailing overt fifth interval and its perceived rootedness (such as it is) mirrors its repetition rate – even though this repetition rate is hinting at a root not represented in the notes of the diminished chord itself.

Extending this line of thought further to the diminished seventh chord A-C-D#-F# (Gb) produces a match with the minor triad of a repetition rate of $^1/_{10}$ of its lowest note. Yet also a contrast in that the defining characteristic of the diminished seventh chord is its rootlessness. (This rootlessness also supports the contention that the diminished triad's rootedness is intrinsic rather than learnt, because we do not learn to root the diminished seventh chord, even after years of exposure.) The diminished seventh has no perceptible root when heard in isolation while the minor triad does. The minor triad is perceived as being rooted. Taking all these factors into consideration together suggests that while the major and diminished triads probably derive their rootedness from their broader repetition patterns, the extended repetition rate of $^1/_{10}$ in the minor triad and diminished seventh chord provides an insufficiently strong signal to induce a perception of rootedness in the listener. Therefore the rooted perception of the minor triad, which is a real and visceral sensation, must arise from its internal structure of 'root' note with fifth and thereby it is functioning in effect as an *altered major triad*. A 'doctored' major triad even. Sadly (for music theorists) the minor triad has no magic. It is a hack!

The human hearing system functions by processing 'real world' ascending acoustic relationships. The ear processes the minor triad frequencies exactly as it would any other sound stimuli, blind to any descending relationships however elegant and attractive they may be to music theorists. Musicians and composers on the other hand, use their ears and make their choices accordingly. The frequencies of the minor third in the minor triad fall just wide enough apart upon the basilar membrane to be perceived as consonant though the character and tone color of the pitch sensation produced is distinctly different from that of the major triad. Sometimes the answer to a problem is not simple, nor elegant, but an uncomfortable compromise. Gioseffo Zarlino was on the right track in his suspicion that the minor triad was inferior to the major triad. His view of the minor third in the minor triad departing from the mathematical mean mirrored the ear's own ascending perspective. Later, Hauptmann and von Oettingen introduced the dualist abstraction of descending undertone series, rehabilitating the status of the minor third and triad, at least from a mathematical point of view. However their beautiful theory remained in the realms of philosophy because science finds no evidence for objective undertones in the physical world. The evolved human hearing mechanism processes ascending frequency relationships. And so, thus far, the paradox of the minor triad has been unravelled. The age old mystery was not solved by music theorists, their numbers and ratios ever ready to hand, but by physiologists! What more wonders will they, and their neuroscience colleagues, reveal in years to come?

Equally the minor third and minor triad proved an awkward bedfellow for the mutable number model of tonal music to handle. The repetition rates of intervals and chords are redolent with hints and suggestions of the form taken by mutable base numbers, however it would be going too far to say that the processes of the ear and aural understanding actually involve their explicit use. The ear appears to recognise the ‘rightness’ of succeeding chords where common frequenices occur in their respective harmonic spectra and the hearer grasps implicitly the simple ratios of exchange between their harmonies. Yet the hearing mechanism has evolved in a particular way to achieve its own necessary ends, and though the parallels appear deep and remarkable, the situation might be better stated, that nature’s gift, the *corps sonore*, to use Rameau’s elegant terminology for the harmonic series, imbues a similarity amongst the many systems it touches. The mutable number model *does* require both the ascending overtone series and its mirror image, the descending undertone series, to provide a complete mathematical system. Thus as has occurred many times before, mathematics not only encapsulates the workings of a physical system but casts its light further, beyond hacks and happenstance, to illuminate a deeper vein of knowledge; and at last, allow the theorist to smile.

In the late Renaissance Zarlino approached music theory from a leading position within the central core of the western intellectual tradition, and his student Vincenzo Galilei would father the birth science in more ways than one. By the time of Rameau, in the eighteenth century, the scientific revolution had left music theory in its wake, and he was forced to import scientific ideas and techniques to accomplish his work. The great strides taken in the nineteenth century were to be made by musically literate men of science: Helmholtz and Oettingen. In the twentieth century the separation had become practically total, with music theory ceding what had once been its central ground to the science of acoustics. Ironically, this final loss occurred just as science was beginning to discover the quantum world of whole-numbered relationships that suffuse the very bedrock of harmonic theory. Is it too much to imagine that one day, music theory might recapture a little territory, with music theorists perhaps even making something of a common cause with mathematicians and scientists, and in so doing, return the discipline to the position it once held in former days, at the vanguard of human knowledge and understanding?

Notes

1. Apel, W., *Harvard Dictionary of Music,* (HUP, Cambridge, Massachusetts 1966) p.163

2. Tyson, A., (Ed), *Selected Letters of Beethoven*, (Macmillan, London, 1967).

3. Grout, D.J., *A History of Western Music*, (J.M. Dent, London, 1962) p.68 & p.154

4. Helmholtz, H. von, *On the Sensations of Tone...*, (4th Edn, Vieweg, Brunswick, 1877; Eng Tr. Ellis, A.J., 1885; Dover, New York, 1954).

5. Oettingen, A. von, *Das duale Harmoniesystem*, (Linnemann, Leipzig 1913).

6. Hauptmann, M., *Die Natur der Harmonik und der Metrik zur Theorie der Musik,* (Leipzig 1853; 2nd Edn 1873; Eng. trans. 1888).

7. Beament, J., *How We Hear Music,* (The Boydell Press, Woodbridge, 2005)