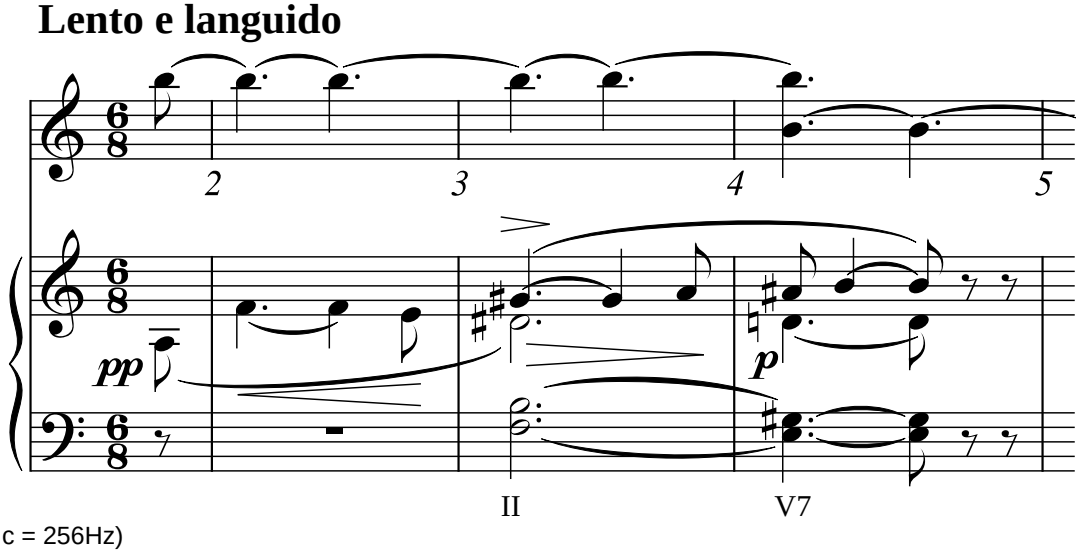
The ‘Tristan’ Chord

A Mutable Number Interpretation
(Example Z)

The first occurrence of the much discussed Tristan chord is found on the first beat of the third measure of the Prelude to the music drama *Tristan und Isolde* by Richard Wagner and consists of the notes tenor F, tenor B, middle d# and middle g#; and, the melodic motive beginning on this g# climbs up in half-steps to arrive at middle b by the close of the following dominant seventh chord. (See note nomenclature, page 14.)

But first, a digression: The treatment of melody, unaccompanied melody, in a mutable number analysis. There are two possible approaches, 1) to rigorously apply mutable numbers to the melody as a string of nested fundamental frequencies (h1) embedded within an absolute nesting series, or 2) select a sequence of nested fundamental frequencies to underlie the melody. Either approach could be seen to have advantages for this analysis, the former providing a rigorous application of the method, while the latter, would deliver a smoother integration of the short snatches of melody that intersperse the dramatic harmonic phrases. However, the latter method is somewhat more arbitrary in that it involves a degree of ‘selection’ and/or aggregation by the analyst as illustrated in Figure Z.1.



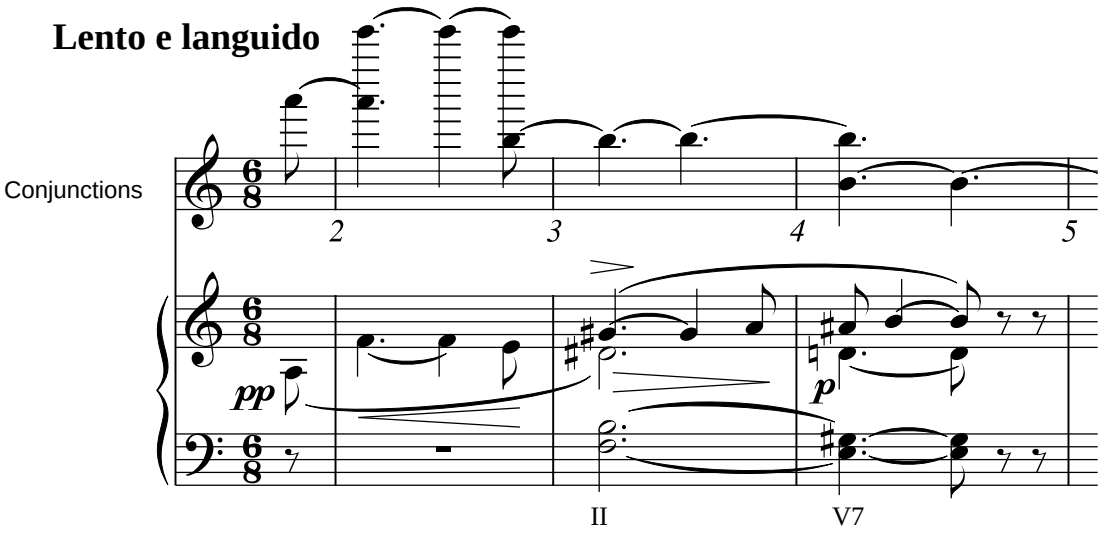
(Starting pitch middle c = 256Hz)

Conjunction Frequency	960.0Hz	960.0Hz	480.0Hz
Conjunctions/Sums	B-h72*---------->	B-h64*------>	B-h96*
Notes:	F-h25~	A-h28~	B-h48~--------->
	E-h24~	G#h27~	A#h45~
	A-h16~	D#h20~	D-h28~
Nested Fundamental	A-h1[H?]	B-h16~	G#h20~
Proportion of Exchange	8:9--->	F-h11~	E-h16~
Absolute Nesting Fundamental	[H1]	B-h1[H?]	E-h1[H?]
		3:2--->	2:3--->
		[H1]	[H1]
Nested Fundamental (h1) Frequency	13.333...Hz	15.0Hz	10.0Hz

Figure Z.1 The opening measures of the Prelude with the unaccompanied melody incorporated into a single nested harmonic series –i.e. a single mutable base number.

Notice in Figure Z.1 that the first and second melody notes, tenor A and middle f, do not make overtone contact with the conjunction (at treble b) as does the third note, middle e; and so tenor A and middle f are relegated to being auxiliary notes, arbitrarily. (An alternative reading using the proportion 7:8 with the conjunction placed at treble a, would favour tenor A rather than middle e, if selected by the analyst.) Notwithstanding, this approach does have some virtue, in that not all notes in a tonal melody are of equal structural importance and thus given judicious choices, sensible results may emerge.

However in this analysis (illustrated in Figure Z.2 below) the former method has been adopted, which involves the unaccompanied melody being treated as a sequence of notes governed (as are the chords) by the simple relationships of Just intervals, rigorously applied. For the first three notes the proportions of exchange are 5:8, 16:15 and then a jump, a tremendous leap into the harmony with 64:3 – i.e. 4:3 with multi-octave adjustment – and two measures later a jump back to unaccompanied melody with 1:24.



(Starting pitch middle c = 256Hz)					
Conjunction Frequency	1706.6Hz	5120.0Hz	960.0Hz	960.0Hz	480.0Hz
Conjunctions/Sums: asterisk *		E-h15*-->	E-h16*		
	A-h8*--->	A-h5*	B-h3*--->	B-h64*-->	B-h96*
Notes: tilde ~	A-h1~[H?]	F-h1~[H?]	E-h1[H?]	A-h28~	B-h48~-->
Proportion of Exchange	5:8--->	16:15--->	64:3	G#h27~	A#h45~
Absolute Nesting Fundamental	[H1]	[H1]	[H1]	D#h20~	D-h28~
				B-h16~	G#h20~
				F-h11~	E-h16~
				B-h1[H?]	E-h1[H?]
				3:2--->	1:24--->
				[H1]	[H1]
Nested Fundamental (h1) Frequency	213.333Hz	341.333Hz	320.0Hz	15.0Hz	10.0Hz

Figure Z.2 The opening measures of the Prelude with the unaccompanied melody treated as a string of individual notes each the h1 fundamental of its own nested series and each interval between these ‘note-series’ governed by Just Intonation. Apologies for the stratospheric quantities of ledger lines – the note letters can be easily read from the asterisk values in the harmonic series below.

The Tristan Chord

One of the features of mutable base numbers is that they requires an exchange based on the alignment of conjunction frequencies between two chords (or two notes in an unaccompanied melody) to define a magnitude – the sum. One chord alone can only be ascribed the not very useful value of infinity – the point at which all harmonic series come into agreement. Therefore when examining the ‘Tristan chord’ in the context of mutable numbers the question is what chord precedes it and what chord follows it; and most importantly, what notes and/or overtones do they share? What are the possible conjunctions?

Thus although the Tristan chord itself is ambiguous and open to myriad interpretations it is a relatively straightforward exercise to search among the objective notes and overtones for a frequency linking the Tristan chord to its preceding and succeeding chords. A procedure which is independent of whatever notes are favoured as structural above those deemed appoggiatura or passing note. In this way a mutable number interpretation may act in a manner akin to a data sifting algorithm that is able to draw out conclusions from what on the surface appears to be little more than ‘white noise’. Though perhaps it could be argued that such an ‘x-ray’ approach might fail to capture every subtle aspect and nuance of the chord.

In the case of the Tristan chord the lowest continuous objective tone linking the three harmonic entities straddling the ‘note cluster’ in question in measure three, as shown in Figure Z.2, is treble b; and consequentially flowing from this fact, the progressions supported by this conjunction frequency relative to the written notes are those of a downward sesquitertia 4:3 (64:3) exchange and sesquialtera 3:2 exchange; which in turn infer the Tristan chord to be (an altered) supertonic chord II, in the nominal key, A minor.

However, given the density of overtones arising from the amorphous intervals of the Tristan chord crowding around the conjunction, it is doubtful whether the ear could receive any sustained signal concerning the nature of the exchange from this quarter alone. Initially, the third harmonic (treble b) of the melodic note middle e would be apprehended, but thereafter the complex cluster of overtone frequencies generated by the Tristan chord would severely limit its continued clear perception. Yet somewhat counterbalancing this aural miasma, throughout measure three the note tenor B and its octave/superoctave overtones are sounding and in measure four there is the note middle b plus its second harmonic treble b, with the additional continuous support of the bass note tenor E's third and sixth harmonic.

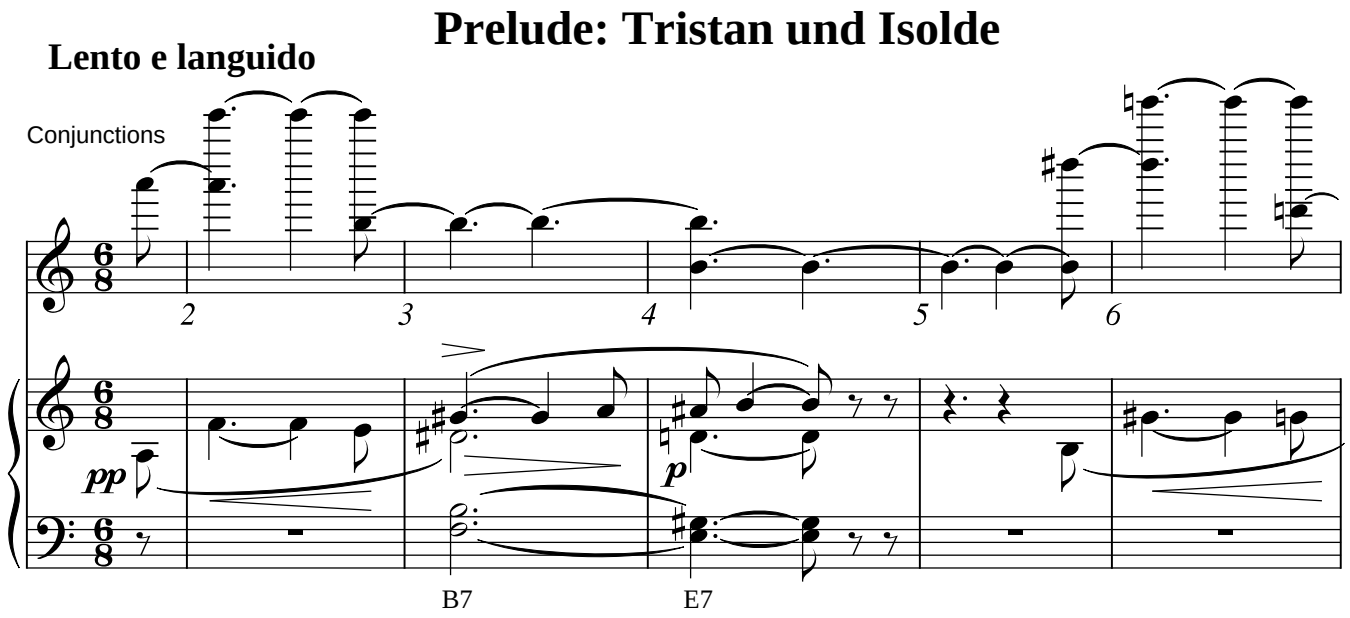
With such continuous support amongst the melee of other shifting tones, the mutable number interpretation of the Tristan chord points toward a possible aural impression, on balance and particularly in retrospect from the security of measure four's resolution to the dominant seventh in A minor, of a veiled, valedictory, downward sesquialtera 3:2 exchange between chords II and V – a meta-level perfect cadence.

The Analysis

The extent of this mutable number analysis runs for only twenty five measures of the Prelude's total of one hundred and ten; just enough to get a feeling for how the technique handles the later romantic style introduced by Richard Wagner. The matching whole number analysis given at the end of this document is similarly limited but provided in two levels; the first a standard analysis of the overall sums computed and the second a breakdown into prime factors of the whole number nested (h1) positions in the absolute series. This latter table is given to illustrate that the reversal of the typical common practice pattern of prime factors shown by the whole number analysis derives from the proportions of exchange between chords and not the conjunction sums.

Due to the many contending harmonic interpretations of the Tristan chord or for that matter the harmony of the whole work – a confirmation of the ambiguous tonal nature of this style – the chords have been simply letter-labelled. Neither to tie a Roman Numeral interpretation in to A minor solely, or alternatively, shift key relentlessly through the measures would be particularly helpful given the nature of this piece. And thankfully, the harmonic series that constitute mutable numbers step through for the most part the complications of enharmonic relations and note spelling by transforming the chords and passing tones into systems of simple whole numbered relationships, both internally and externally. (In measures 14-15 and 24-25 enharmonic notes have been name-lettered as such in their nested series, for example E# rather than F in an F-rooted series, for ease of reading. And in general, enharmonic differences are incorporated within mutable numbers as the pitch/frequency of the system is in near constant flux throughout its length.)

Nevertheless all is not plain sailing for mutable numbers. In some exchanges there is only distant or negligible objective aural support for the conjunction frequencies mandated by the proportions lying between chords. For example in measure eleven (apart from the last eighth note in the bar) the principle notes do not produce approximate conjunction level B natural overtones until their eleventh, thirteenth, fifteenth and nineteenth harmonics – an ill-focused tone near to 2 kilohertz. There is little likelihood that the ear would apprehend such a connection to the following chord, except perhaps in retrospect, with the momentary appearance of middle e's third harmonic at the end of the measure and then the onset of the B major harmony. All of which is shown on the conjunction stave attached to the score. However, as the harmonic successions are chosen precisely for their opaque and ambiguous qualities it not surprising that difficulties arise in finding a sufficient quantity of clear acoustic handles and signposts to sustain a well defined interpretation of harmonic progression – as is generally the case for earlier tonal styles. Nonetheless mutable numbers cope: where conjunctions are vague the proportion between chords may give guidance as to what at least the conjunction would under normal circumstances be; and equally, where the chords are indeterminate, the presence of a connecting frequency – a conjunction – may elucidate the proportion. The Tristan chord is an example of the latter, while the former technique is used with the chord exchange across measures 22–23 (described in the analysis below, pages 10–11).



Conjunction Frequency

A-1706.6Hz	E-5120.0Hz	B-960.0Hz	960.0Hz	B-480.0Hz	D#2400.0Hz	G-6000.0Hz	D-1125.0Hz
	E-h15*-->	E-h16*-13				G-h15*-->	G-h16*-13
A-h8*--->	A-h5* +10	B-h3*---->	B-h64*--------->	B-h96* -48	D#h10*-->	D#h6* +9	D-h3*--->
A-h1~[H?]	F-h1~[H?]	E-h1[H?]	A-h28~	B-h48~------>	B-h2* +6	G#h1~[H?]	G-h1[H?]
5:8-->	16:15-->	64:3-->	G#h27~	A#h45~	B-h1~[H?]	16:15--->	64:3--->
[H1]	[H1]	[H1]	D#h20~	D-h28~	3:5--->	[H1]	[H1]
			B-h16~	G#h20~	[H1]		
			F-h11~	E-h16~			
			B-h1[H754974720]	E-h1[H503316480]			
			3:2--->	1:24--->			
Nested h1 Frequency			[H1]	[H1]			
213.333Hz	341.333Hz	320.0Hz	15.0Hz	10.0Hz	240.0Hz	400.0Hz	375.0Hz

The opening ’cello solo passages have been dealt with rigorously with regard to the notes’ conjunction frequencies being spelt out individually with each note-exchange producing its own calculation and value:

MBN: $8_{10737418240}0_1 = 5_{17179869184}0_1$

$\rightarrow \quad 5_{17179869184}0_1 + 10_{17179869184}0_1 = 15_{17179869184}0_1$

$\rightarrow 15_{17179869184}0_1 = 16_{16106127360}0_1$

$\rightarrow \quad 16_{16106127360}0_1 - 13_{16106127360}0_1 = 3_{16106127360}0_1$

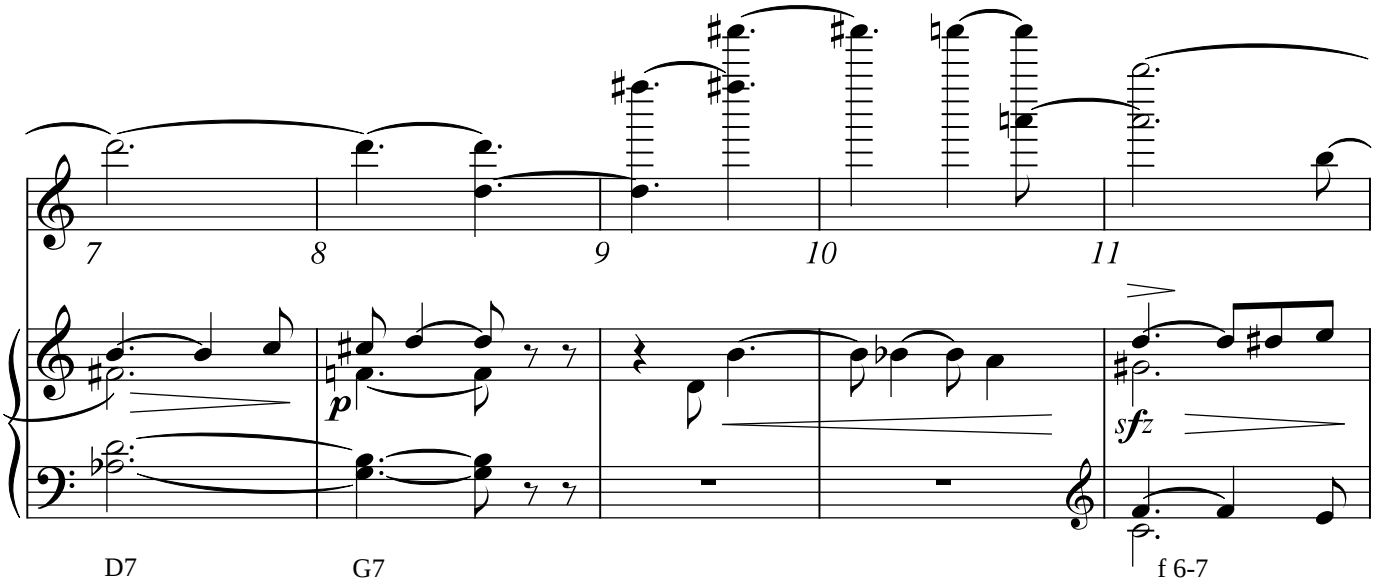
$\rightarrow \quad 3_{16106127360}0_1 = 64_{754974720}0_1$

$\rightarrow \quad 64_{754974720}0_1 = 96_{503316480}0_1$

The last two digit sequences above are the Tristan chord itself and its resolution to an E^7 major chord.

Please notice that the melodic nested (h1) fundamental tones are written in a black typeface in the analysis, rather than grey, as they represent objective frequencies. Making the analysis in this rigorous way has inflated the values and led to a flurry of ledger lines, but at least the mathematics are un-fudged. Upon reflection however, the ‘fudged’ method may actually be closer to what the ear actually does when it brings

to bear its many resources in interpreting an unaccompanied melody: In that pitch memory, particularly for the prime and fifth, must help with orientation and note recognition not to mention the added effects of experience and expectation. Thus a ‘well-fudged’ nested series containing a number of adjacent, related, notes could mimic some of these extra resources which mutable base numbers can know nothing of.



D7 G7 f 6-7

D-1125.0Hz D-562.5Hz F#2812.5Hz A#7031.3Hz A-6591.8Hz A-1647.9Hz B-927Hz

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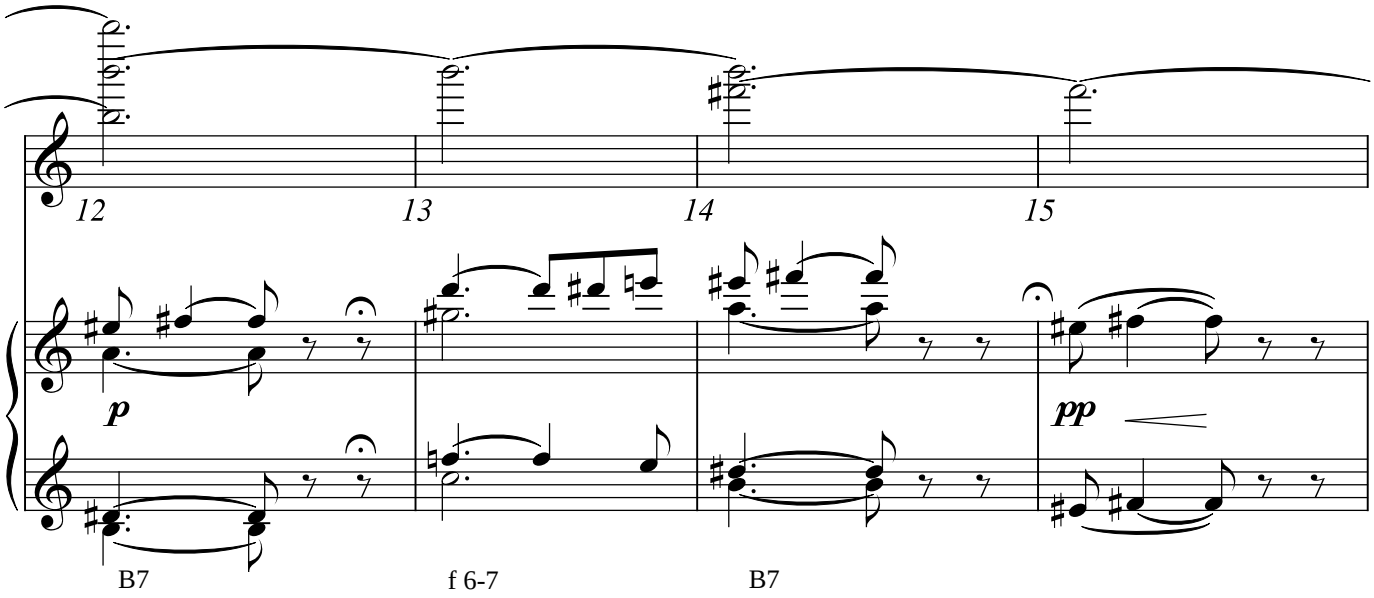
                                          A#h15*--> A#h16*-1
---> D-h64*--------> D-h96* -48      F#h10*--> F#h6* +9   A-h15*--> A-h16*-12  B-h192*----->
     C-h28~          D-h48~-------->  D-h2* +8   B-h1~[H?] A#h1~[H?] A-h4*----> A-h80*+35
     B-h27~          C#h45~           D-h1~[H?] 16:15-->   16:15-->   A-h1~[H?]  B-h45*------>
     F#h20~          F-h28~           3:5-->    [H1]       [H1]       20:1--->    E-h30~
     D-h16~          B-h20~           [H1]                                [H1]     D#h28~
     G#h11~          G-h16~                                                       D-h27~
     D-h1[H?]        G-h1[H?]                                                     G#h19~
     3:2--->         1:24--->                                                     F-h16~
     [H1]            [H1]                                                         E-h15~
                                                                                  C-h12~
                                                                                  F-h1[H?]
     17.578125Hz     11.71875Hz       281.25Hz  468.75Hz  439.453125Hz           64:45--->
                                                                                  [H1]
                                                                411.9873047Hz
                                                                                  20.59936523Hz
```

The Tristan chord itself in measure three and its transposed repetition in measure seven are from the mutable number perspective fairly clearly defined by the presence of a continuous, objective, connecting frequency – the conjunctions at B or D. Prior to the Tristan chord the third harmonic generated by the last note of the solo melody prepares the ground – a frequency likely to possess more energy than the fundamental note itself on ’cellos. Then throughout the Tristan chord this tone is maintained and bolstered as the fourth harmonic of the written notes tenor B (or middle d) respectively. Followed by, firstly a continuous presence as the sixth harmonic of the written bass note of the succession chord E (or G) and after an eighth note appoggiatura, its reinforcement as the second harmonic of a sounded written note one octave below.

Thus from the perspective of a mutable number interpretation, the objective presence of the conjunction frequency has illuminated the nature of the Tristan chord and linked it to its precursor and successor by the calculation of a common sum[1] – MBN $3_{16106127360}0_1 = 64_{754974720}0_1 = 96_{503316480}0_1$

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      B-1853.9Hz              A-1853.9Hz              F#1390.5Hz              F#1390.5Hz

---> B-h256*
     B-h128*---------->      B-h45*---------->       B-h64* -16
---> B-h64* +64              E-h30~                  F#h48~---------->       F#h64*-------->
     F#h48~                  D#h28~                  E#h45~                  F#h32~
     E#h45~                  D-h27~                  A-h28~                  E#h30~
     A-h28~                  G#h19~                  D#h20~                  F#h16~
     D#h20~                  F-h16~                  B-h16~                  E#h15~
     B-h16~                  E-h15~                  B-h1[H?]                F#h1[H?]
     B-h1[H?]                C-h12~                  4:3--->                 1:1--->
     45:128--->              F-h1[H?]                [H1]                    [H1]
     [H1]                    64:45--->
                             [H1]

     14.48392868Hz           41.19873046Hz           28.96785735Hz           21.72589302Hz
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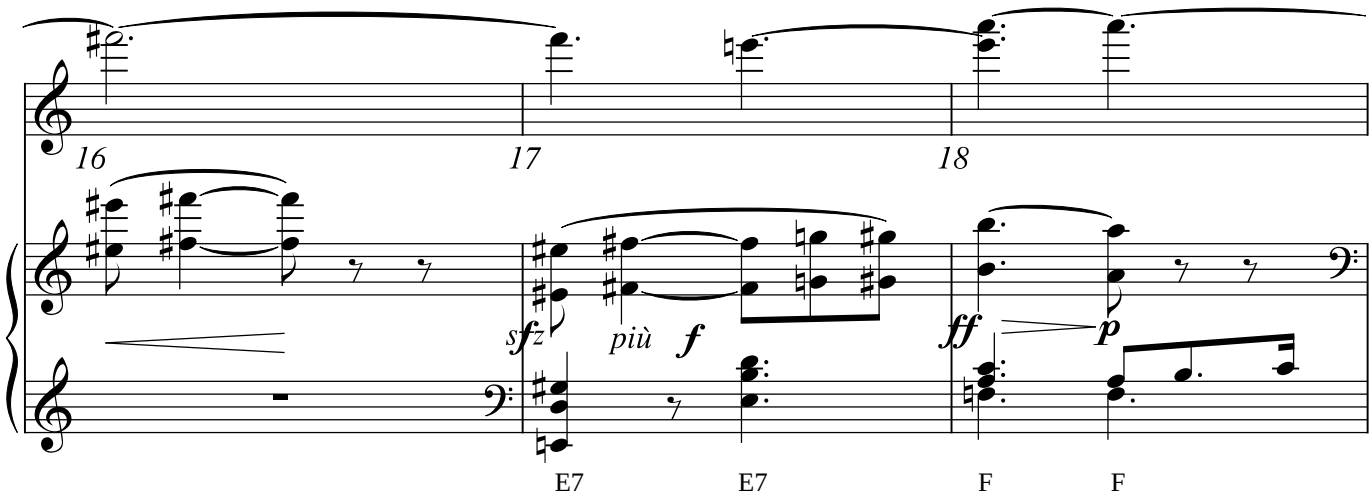
Whereas the mutable number approach to the Tristan chord cuts through the Wagnerian veil to yield the bare bones of a familiar ‘V-of-V to V’ downward sesquialtera 3:2 chord progression; in two places in the analysis, the root movement of the chords is in tritone steps: here in measures 11–14 and in measures 22–23. Originally I analyzed these chords using the septimal tritone proportions of 5 and 7, and thus employed a conjunction at ‘A’ that linked the fifth harmonic of the root F to the seventh harmonic of the root B, presaged by the written solo melody note middle a. However, while this strategy is plausible and perhaps closer to the ear’s actual journey, in that the melody note middle a and its second harmonic treble a, is so clear and irrefutable in preparation; yet I have used the established Just intonation proportions of 45 and 64 above for the particular reason of excluding the septimal 5 and 7 proportions to be sure they are not implicated in provoking the unusual pattern of prime factors found in this analysis. Consequently the tritone related chord exchanges use rather less well prepared conjunctions at treble b (and high b) discussed earlier on page 4.

Further below at measures 22–23, three alternative proportions for the tritone interval are illustrated. This change in strategy away from the septimal tritone was prompted by an unusual pattern in the whole number analysis (given at the end of the document) which upturned the normal distribution of prime factors as found in typical common practice pieces. The change away from the ‘non-standard’ septimal proportions was an

attempt to eliminate them as the cause of the unusual distribution. However as it turned out their presence or absence had little effect on the ‘inverted’ distribution of, power of two and three, prime factors in the sums (conjunctions) computed. Indeed as could have easily be foreseen the small effect generated by replacing the factors 5 and 7 came about from the additional powers of 2 and 3 contained in the Just tritone proportions – most of which cancel each other out.



E7 E7 F F

F#1390.5Hz E-1236Hz E-1236Hz A-1647.9Hz A-1647.9Hz

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----> F#h64~------------------>  F#h72* -8                                A-h80*----->  A-h80*------>
      E#h60~                     E-h64*----->  E-h64*------->  G#h40~     E-h60* +20    A-h40~
      F#h32~                     F#h36~        G#h40~          G-h38~     B-h44~        A-h20~
      E#h30~                     E#h34~        G-h38~          F#h36~     B-h22~        C-h12~
      F#h1[H?]                   F#h18~        F#h36~          G#h20~     C-h12~        B-h11~
      9:8--->                    E#h17~        G#h20~          G-h19~     A-h10~        A-h10~
      [H1]                       G#h10~        G-h19~          F#h18~     F-h8~         F-h8~
                                 D-h7~         F#h18~          D-h14~     F-h1[H1]      F-h1[H1]
                                 E-h4~         D-h14~          B-h12~     1:1-->        6:5--->
                                 E-h1[H1]      B-h12~          E-h8~      [H1]          [H1]
                                 1:1--->       E-h8~           E-h1[H1]
                                 [H1]          E-h1[H1]        15:16--->
                                               15:16--->       [H1]
                                               [H1]
```
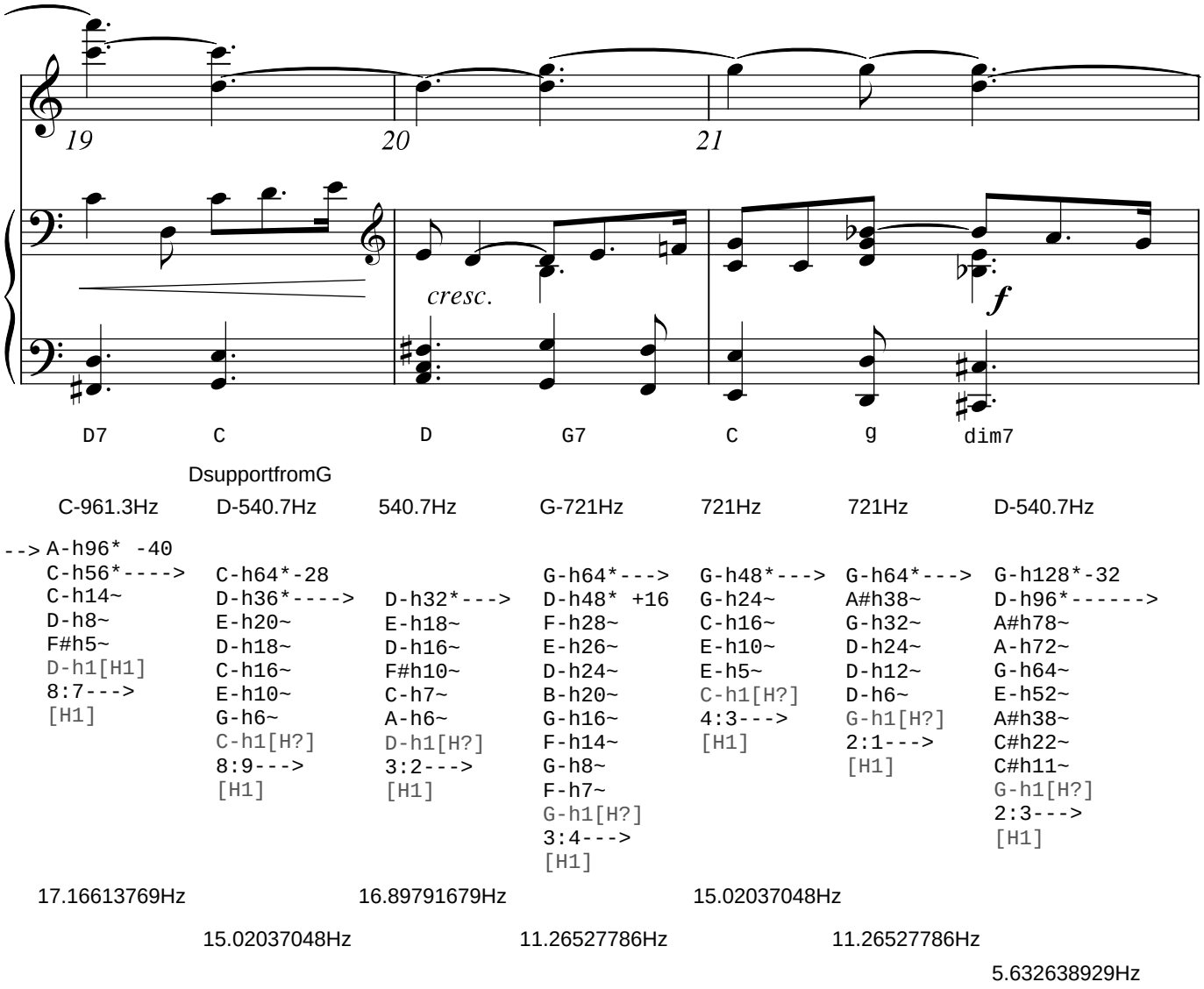

21.72589302Hz 19.3119049Hz 20.59936523Hz

These prime factors that accumulate in the conjunction sums are directly related to the proportions of exchange between chords. In ‘standard’ common practice music the defining structural entity is the cadence. Phrase after phrase voyaging away and returning back to the tonic or some meta-tonic via a $^{2}/_{3}$ or $^{4}/_{3}$ chord exchange inevitably favours these proportion above all others. Notwithstanding that a good number of these factors of three will be neutralised by counterbalancing numerators in such proportions as the upward stepping minor third $^{6}/_{5}$ or $^{15}/_{16}$ downward minor second. But not all of the surfeit of denominator threeness will be absorbed in this way. Thus, in a ‘standard’ common practice piece of music the whole number analysis will begin with a magnitude stuffed full with an equal amount of denominator threeness to match that which had failed to be neutralised; and as the analysis proceeds through the measures this store of threeness will be gradually eaten away by those remaining denominator factors of three – only to be replaced by mounting quantities of factors of two sucked into their void. A steady, progressive, decline in threeness in mutable numbers is the telltale signature of a ‘standard’ common practice composition.



If all the proportions in this example analysis are strung together from start to finish and where possible numerators and denominators are cancelled out, the resulting aggregate proportion is:

$$\frac{8}{5} \times \frac{15}{16} \times \frac{3}{64} \times \frac{2}{3} \times \ldots \quad = \quad \frac{3^6 \times 5^5 \times 7}{2^{28}}$$

However, if the same procedure is applied to the Mozart Polonaise from Example Y – a supremely straightforward example of common practice harmony – the resulting aggregate proportion is:

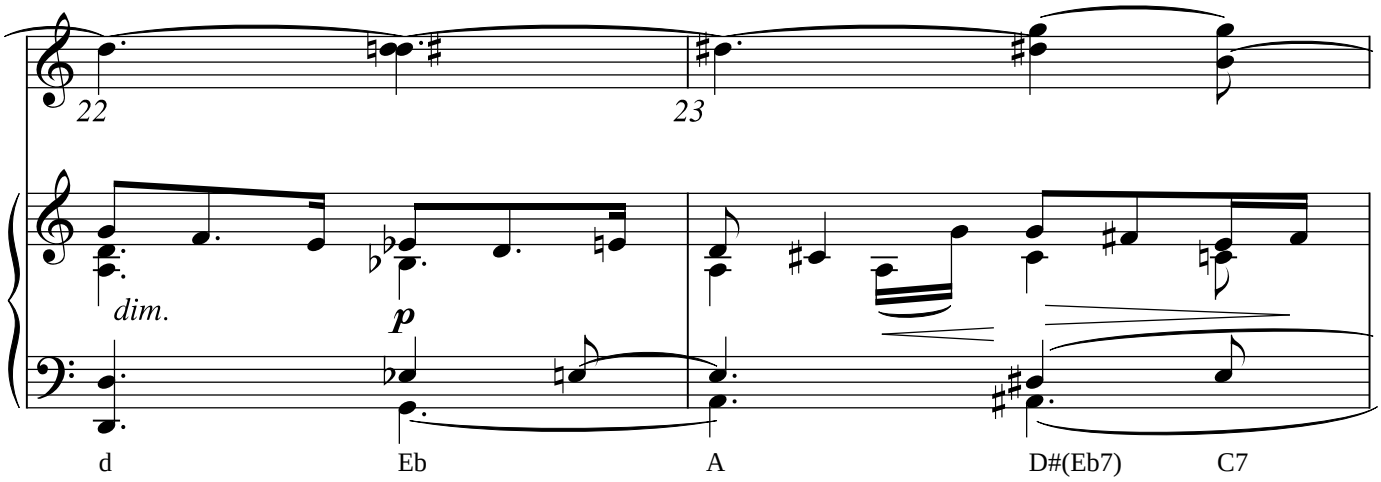
$$\frac{2^6 \times 5^2}{3^8}$$

The contrast is stark. The positions of the factors of two and three are reversed between the Wagnerian and Mozartian examples. The latter fraction with the power of three concentrated in the denominator is entirely typical of music from the common practice period. Richard Wagner's harmonic usage is fundamentally

different. This loss of the ‘normal’ (power of three) denominator anchor in the Prelude's aggregate proportion condemns the piece to roam havenless as the *Flying Dutchman*, across the tonal main. Yet of course it is no accident, it is a choice lying at the heart of the Wagnerian style. A choice to un-tether harmony from the constraint of defining a specific key – the core element in common practice structural organisation – and instead to rove freely amongst many tonal centres, unconfined.

$$10737418240 \quad \times \quad \frac{3^6 \times 5^5 \times 7}{2^{28}} \quad = \quad 63787500$$

However it should be noticed that the aggregate of all proportions does not quite give the entire story (though a useful shorthand) because it takes no account of the order in which the individual proportions are encountered successively through the composition. This makes a difference. The aggregate of proportions will not produce the actual whole number sequence of nested (h1) positions in the underlying absolute series, and even less the sums progressively computed, but if the first nested (h1) position is known, multiplying this whole number by the aggregate proportion will deliver the final nested (h1) position. (Illustrated above)



D-540.7Hz

---> D-h64*----------> G-h42~ F-h38~ E-h36~ D-h32~ A-h24~ D-h16~ D-h8~ D-h1[H?] 15:16---> [H1]

8.448958394Hz

D#576.8Hz
D-540.7Hz
C#504.7Hz

D#h64*----------> D-h60*+4 --> C#h56*-----> E-h34~ D#h32~ D-h30~ A#h24~ E-h17~ D#h16~ G-h10~ D#h1[H?] 45:64--> 21:30--> 5:7----> [H1]

9.012222287Hz

D#576.8Hz
D-540.7Hz
C#504.7Hz

D#h45*----------> D-h42*--> C#h40*--> D-h21~ C#h20~ A-h16~ E-h12~ A-h8~ A-h1[H?] 32:45--> 15:21--> 7:10----> [H1]

12.81738281Hz
12.87460327Hz
12.6171112Hz

G-721.0Hz

G-h40*-----> D#h32* +8 D-h30* C#h28* G-h20~ F#h19~ C#h14~ D#h8~ A#h6~ D#h1[H?] 6:5---> [H1]

18.02444457Hz

B-450.6Hz

G-h48* -18 B-h30*---> F#h22~ E-h20~ C-h16~ E-h10~ A#h7~ C-h1[H?] 16:15---> [H1]

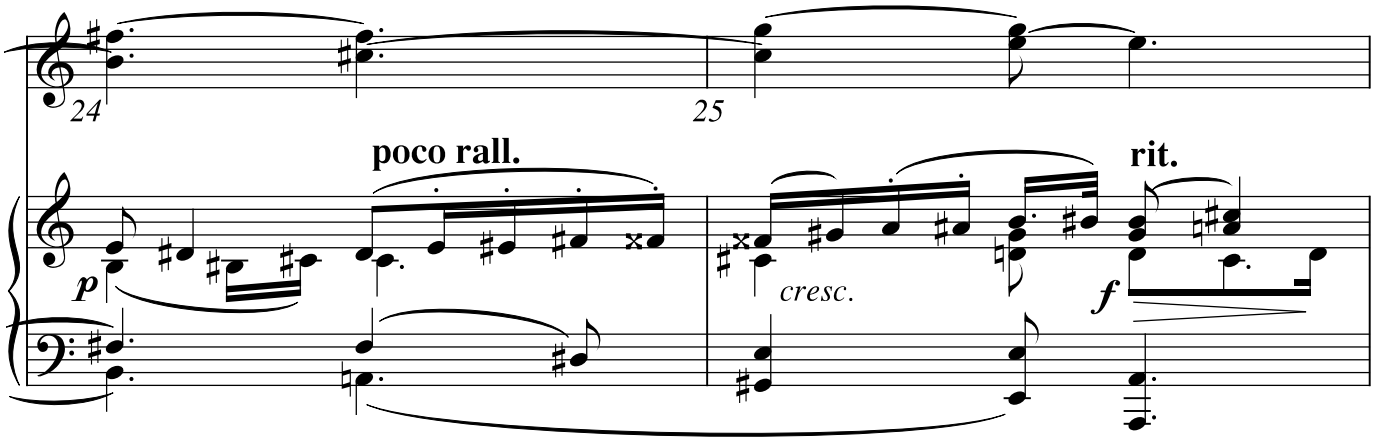
15.02037048Hz

Crossing over between measures 22 and 23 a range of alternative tritone exchange proportions are illustrated.

As explained earlier, after first using the septimal 5:7 tritone exchange I reverted to orthodox Just Intonation and the conjunctions associated with these ‘correct’ proportions are shown with extended arrows. None of the choices of conjunction is much more convincing then the other alternatives. The D conjunction for measure 22 beat one has firm support from the written notes and equally the D# conjunction in beat two, but from thereon no clear path can be drawn through the broad cluster of conjunctions illustrated. Each of the illustrated conjunctions have strengths and weaknesses. C# is favoured by the A major chord in measure 23 beat one; and D by its appearance in measure 22 beat two and as an appoggiatura in measure 23 beat one; while the D# conjunction actually used above sits very awkwardly astride the A major chord.

Ultimately in such circumstances (and opposite to the approach pursued in the interpretation of the Tristan chord) the proportion of exchange between the chords must be used to determine which of the candidate conjunctions to select. The choice of Just proportions of 45:64 and 32:45 mandate that D# should be used.

In this passage Wagner has succeeded pretty well in obscuring any reference to a single over-arching key or tonal centre by which the chords may sensibly be interpreted and it has been difficult to assign mutable numbers to the music with any conviction that they are capturing its essence. It is not that the mutable numbers fail, they still work perfectly well, but they lose credence, credibility.



B	f#	c#	E7	A
F#675.9Hz	C#506.9Hz	G#760.4Hz	E-608.3Hz	C#1013.9Hz
F#h48*---------->	F#h128*	G#h96*-------->	G#h80*	C#h80*--?-->
---> B-h32*	C#h96*---------->	C#h64*	E-h64*------>	E-h48*
E-h21~	Fxh68~	Fxh34~	B#h50~	C#h40~
D#h20~	F#h64~	A#h52~	B-h48~	B-h36~
B-h16~	E#h60~	A-h50~	G#h40~	A-h32~
F#h12~	E-h56~	G#h48~	D-h28~	G-h28~
B-h8~	D#h52~	Fxh44~	E-h16~	D-h21~
B-h1[H?]	C#h48~	C#h24~	E-h8~	C#h20~
8:3--->	F#h32~	C#h32~	E-h1[H?]	A-h8~
[H1]	D#h26~	E-h19~	3:4--->	A-h4~
	A-h19~	G#h12~	[H1]	A-h1[H?]
	F#h1[H?]	C#h1[H?]		6:5--?-->
	2:3--->	5:6--->		[H1]
	[H1]	[H1]		
14.08159733Hz	5.280598997Hz	7.920898495Hz	9.505078194Hz	12.67343759Hz

Whole Number Analysis

The whole numbers are large and would be a good deal larger if the complete Prelude were analysed. The principle point of interest though is the upending of the ‘normal’ common practice distribution of the factors of two and three – where twoness steadily increases and threeness steadily erodes from beginning through to end. Here Wagner has contrived otherwise, and the lack of tonal direction experienced by the listener in his restless roving texture finds its counterpart in this reversal of factors.

Bar	Proportion	Fundamental Series up to nested h1	Nested Series	Sum Computed	Prime Factors			
1	FundamentalSeriesH1->H?	10737418240 ×	8 =	85899345920	2^34		5	
	8/5 (5:8)	(h1) = 17179869184 ×	15 =	257698037760	2^34	3	5	
2	15/16 (16:15)	(h1) = 16106127360 ×	3 =	48318382080	2^30	3^2	5	
	3/64 (64:3)	(h1) = 754974720 ×	64 =	48318382080	2^30	3^2	5	
3	2/3 (3:2)	(h1) = 503316480 ×	48 =	24159191040	2^29	3^2	5	
4	24/1 (1:24)	(h1) = 12079595520 ×	10 =	120795955200	2^29	3^2	5^2	
5	5/3 (3:5)	(h1) = 20132659200 ×	15 =	301989888000	2^28	3^2	5^3	
6	15/16 (16:15)	(h1) = 18874368000 ×	3 =	56623104000	2^24	3^3	5^3	
	3/64 (64:3)	(h1) = 884736000 ×	64 =	56623104000	2^24	3^3	5^3	
7	2/3 (3:2)	(h1) = 589824000 ×	48 =	28311552000	2^23	3^3	5^3	
8	24/1 (1:24)	(h1) = 14155776000 ×	10 =	141557760000	2^23	3^3	5^4	
9	5/3 (3:5)	(h1) = 23592960000 ×	15 =	353894400000	2^22	3^3	5^5	
	15/16 (16:15)	(h1) = 22118400000 ×	15 =	331776000000	2^18	3^4	5^6	
10	15/16 (16:15)	(h1) = 20736000000 ×	4 =	82944000000	2^16	3^4	5^6	
	1/20 (20:1)	(h1) = 1036800000 ×	45 =	46656000000	2^12	3^6	5^6	
11	45/64 (64:45)	(h1) = 729000000 ×	128 =	93312000000	2^13	3^6	5^6	
12	128/45 (45:128)	(h1) = 2073600000 ×	45 =	93312000000	2^13	3^6	5^6	
13	45/64 (64:45)	(h1) = 1458000000 ×	48 =	69984000000	2^11	3^7	5^6	
14	3/4 (4:3)	(h1) = 1093500000 ×	64 =	69984000000	2^11	3^7	5^6	
16	8/9 (9:8)	(h1) = 972000000 ×	64 =	62208000000	2^14	3^5	5^6	
17	16/15 (15:16)	(h1) = 1036800000 ×	64 =	66355200000	2^18	3^4	5^5	
18	5/6 (6:5)	(h1) = 864000000 ×	80 =	69120000000	2^15	3^3	5^7	
19	7/8 (8:7)	(h1) = 756000000 ×	56 =	42336000000	2^11	3^3	5^6	7^2
	9/8 (8:9)	(h1) = 850500000 ×	36 =	30618000000	2^7	3^7	5^6	7
20	2/3 (3:2)	(h1) = 567000000 ×	32 =	18144000000	2^11	3^4	5^6	7
	4/3 (3:4)	(h1) = 756000000 ×	64 =	48384000000	2^14	3^3	5^6	7
21	3/4 (4:3)	(h1) = 567000000 ×	48 =	27216000000	2^10	3^5	5^6	7
	1/2 (2:1)	(h1) = 283500000 ×	64 =	18144000000	2^11	3^4	5^6	7
	3/2 (2:3)	(h1) = 425250000 ×	96 =	40824000000	2^9	3^6	5^6	7
22	16/15 (15:16)	(h1) = 453600000 ×	64 =	29030400000	2^14	3^4	5^5	7
	64/45 (45:64)	(h1) = 645120000 ×	64 =	41287680000	2^20	3^2	5^4	7
23	45/32 (32:45)	(h1) = 907200000 ×	45 =	40824000000	2^9	3^6	5^6	7
	5/6 (6:5)	(h1) = 756000000 ×	40 =	30240000000	2^11	3^3	5^7	7
	15/16 (16:15)	(h1) = 708750000 ×	30 =	21262500000	2^5	3^5	5^8	7
24	3/8 (8:3)	(h1) = 265781250 ×	48 =	12757500000	2^5	3^6	5^7	7
	3/2 (2:3)	(h1) = 398671875 ×	96 =	38272500000	2^5	3^7	5^7	7
25	6/5 (5:6)	(h1) = 478406250 ×	96 =	45927000000	2^6	3^8	5^6	7
	4/3 (3:4)	(h1) = 637875000 ×	64 =	40824000000	2^9	3^6	5^6	7

Because the style allows for few boundaries policed by clear energetic 3:2 or 3:4 chord exchanges to occur, the ear has difficulty acquiring much purchase upon the music's harmonic structure and this lack of 'grab holds' is recorded in the reverse distribution of factors. A factor analysis of the nested (h1) positions in the absolute underlying series is included below to show that this feature of the Wagnerian style arises from the proportions between chords and not from the placement of conjunction frequencies.

Whole Number Analysis of Nested (h1) Fundamentals

Bar	Proportion	h1 position in the Fundamental Series	Prime Factors
1	Fundamental Series H1->H?	10737418240	2^31 5
	8/5 (5:8) (h1) =	17179869184	2^34
2	15/16 (16:15) (h1) =	16106127360	2^30 3 5
	3/64 (64:3) (h1) =	754974720	2^24 3^2 5
3	2/3 (3:2) (h1) =	503316480	2^25 3 5
4	24/1 (1:24) (h1) =	12079595520	2^28 3^2 5
5	5/3 (3:5) (h1) =	20132659200	2^28 3 5^2
6	15/16 (16:15) (h1) =	18874368000	2^24 3^2 5^3
	3/64 (64:3) (h1) =	884736000	2^18 3^3 5^3
7	2/3 (3:2) (h1) =	589824000	2^19 3^2 5^3
8	24/1 (1:24) (h1) =	14155776000	2^22 3^3 5^3
9	5/3 (3:5) (h1) =	23592960000	2^22 3^2 5^4
	15/16 (16:15) (h1) =	22118400000	2^18 3^3 5^5
10	15/16 (16:15) (h1) =	20736000000	2^14 3^4 5^6
	1/20 (20:1) (h1) =	1036800000	2^12 3^4 5^5
11	45/64 (64:45) (h1) =	729000000	2^6 3^6 5^6
12	128/45 (45:128) (h1) =	2073600000	2^13 3^4 5^5
13	45/64 (64:45) (h1) =	1458000000	2^7 3^6 5^6
14	3/4 (4:3) (h1) =	1093500000	2^5 3^7 5^6
16	8/9 (9:8) (h1) =	972000000	2^8 3^5 5^6
17	16/15 (15:16) (h1) =	1036800000	2^12 3^4 5^5
18	5/6 (6:5) (h1) =	864000000	2^11 3^3 5^6
19	7/8 (8:7) (h1) =	756000000	2^8 3^3 5^6 7
	9/8 (8:9) (h1) =	850500000	2^5 3^5 5^6 7
20	2/3 (3:2) (h1) =	567000000	2^6 3^4 5^6 7
	4/3 (3:4) (h1) =	756000000	2^8 3^3 5^6 7
21	3/4 (4:3) (h1) =	567000000	2^6 3^4 5^6 7
	1/2 (2:1) (h1) =	283500000	2^5 3^4 5^6 7
	3/2 (2:3) (h1) =	425250000	2^4 3^5 5^6 7
22	16/15 (15:16) (h1) =	453600000	2^8 3^4 5^5 7
	64/45 (45:64) (h1) =	645120000	2^14 3^2 5^4 7
23	45/32 (32:45) (h1) =	907200000	2^9 3^4 5^5 7
	5/6 (6:5) (h1) =	756000000	2^8 3^3 5^6 7
	15/16 (16:15) (h1) =	708750000	2^4 3^4 5^7 7
24	3/8 (8:3) (h1) =	265781250	2 3^5 5^7 7
	3/2 (2:3) (h1) =	398671875	3^6 5^7 7
25	6/5 (5:6) (h1) =	478406250	2 3^7 5^6 7
	4/3 (3:4) (h1) =	637875000	2^3 3^6 5^6 7

Finally to close the analysis, it is to be noted that there is a greater degree of 'judder' in the distribution of the factors of two and three here than is found in traditional common practice music – and somewhat more than was found in the analysis of the Debussy Piano Prelude too. The source of this instability also derives from the sequence of proportions between adjacent chords, a flow more anarchic than found in former times. This contrast points toward not only the predominance of the 3:2 and 3:4 exchanges during the common practice period, but their constancy of use, and their generally consistent distribution throughout the measures.

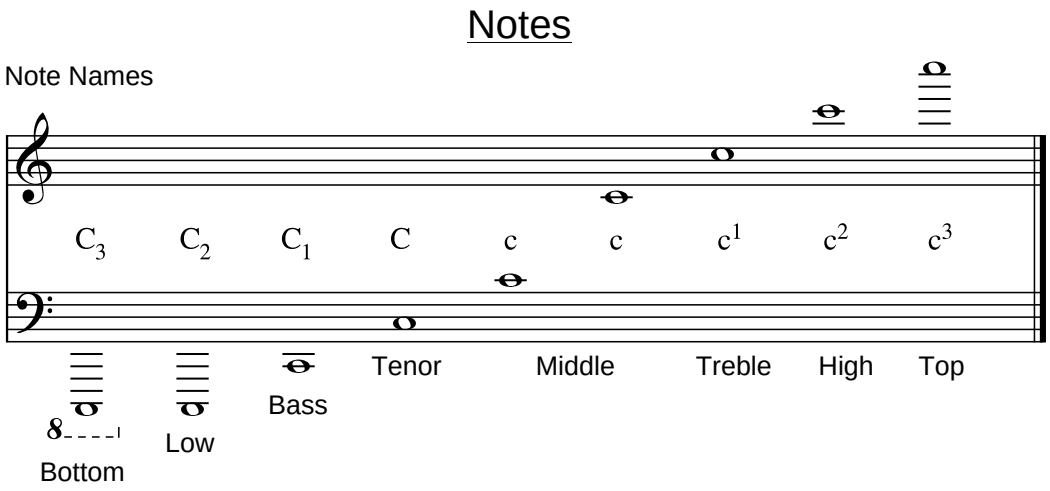
The Demise of Tonality?

Tristan und Isolde was first performed on the 10th June, 1865. It was not quite the day tonal music died, after all there were the national movements yet to come and some of the greatest landmarks of romanticism, but it was perhaps the day that tonalism first glimpsed its demise. The Wagnerian style took music to the edge of tonal reference; and, skilfully, kept it tittering on the brink with an almost unending stream of seventh, ninth, diminished and augmented chords of every persuasion following one upon another in a grand amorphous cavalcade. Points of cadence became rare and phrases blurred in an unbounded restless texture. Tonality was not so much abolished as expanded to its very limit, with tonal centres becoming no more than shifting ambiguous shadows, flitting by the ear. Meanwhile other means of articulating structure were forced into prominence: melodic/motivic development, dynamics, orchestration and dramatic representation. The tonal balloon, inflated to near bursting point with Romantic hyperbole, surprisingly, survived fifty more years as many reckon it; and when the moment eventually did arrived Arnold Schoernberg, who amongst others witnessed the event, believed he was only further inflating it in the direction of infinity!

Yet still today, more than one hundred and fifty years later, tonally organised music is everywhere heard? It floods out of music playing devices, radios, televisions, cinemas; is taught privately and publicly in schools and conservatories, overwhelms the programmes of concert auditoria and is probably the predominant element in music culture throughout the world. Tonal music clearly did not *die out* during the closing decades of the nineteenth century, rather it *spread out*.

As the vanguard of western art music pushed on across the boundaries, tonality took wings upon the changing dynamics of western societies to establish itself within the new niches being created amongst the mass audiences of the present industrial world. During this period of transition the already established art music audience turned increasing to the historic tonal canon for succour, while the newly forming mass audiences found popular and commercial idioms springing up to suit their myriad tastes. The principle of tonal organisation in music was not *deceased*, it was *democratised*.

Today, arguably, tonal music forms the basis of the overwhelming majority of all music heard throughout the western influenced world. It's presence is ubiquitous. Further, western tonal forms have proved highly invasive in most other music cultures; either by supplanting them almost entirely or by the creation of parallel local and western usages, or sometimes through degrees of hybridization in suitably amenable foreign music cultures – or perhaps some mixture of all three. Therefore the continuing study of tonally organised music, the quest to understand its ultimate foundations and most fundamental mechanisms, remains I believe, a worthwhile pursuit.



The pitch nomenclature adopted in this document is shown above, one of the three schemes mentioned in the Harvard Dictionary of Music compounded with a verbal practice familiar to organ builders. The twelve ascending chromatic notes from bottom C_3 to bottom B_3 are spoken: bottom C, bottom C#, bottom D, etc... and written either as bottom C_3 or C_3 ; bottom $C\#_3$ or $C\#_3$, etc... This ascending octave based naming practice is applied throughout the compass of notes, and if required, may be extended further through the use of more super/subscripts. Also as amongst organ builders, notes are by preference named as sharps, for example A# rather than B-flat, but not exclusively so where the flattened form is more informative or convenient.

Note 1. This common sum (Decimal 48318382080) has been calculated on the basis of the first twenty five measures of the Prelude taken in isolation. If the Prelude were analysed in full it would probably be considerably larger.