For my father,
Harold Ernest Perry,
who might well have written a book, had he not devoted so much of his time and energy to the care of his family: with gratitude, love and respect.

Preface

The collection of documents listed below might well be termed ‘the mutable number chronicles’ a written account of the evolution of my thoughts about the fundamental nature of music, a journey of many years duration. If you have opened this archive looking for bass recorder music there is just one piece at the end of Appendix XI.

Initially, and impetuously, I put a great deal of effort into articulating my ideas in a book-like form a task undertaken while the ideas themselves were fresh, yet still developing in my mind. This resulted in an almost unending cycle of revision and rewriting, leading to a rather amorphous outcome. Eventually, wearied, I decided in 2015 it would be better to bring the project to a halt and view it more as a record of the path taken than a destination secured. The dedication from *Journey to the Heart of Music* is reproduced above. Since then I have confined myself to writing separate documents addressing one or other aspect of the overall idea and/or the analysis of individual compositions.

Appendix I, *Music by Mutable Numbers* introduces the overall concept of tonal music viewed as a form of number processing; and hopefully, the deeply rooted traditional approach pursued here might compliment the more newly established mathematics centred upon atonal music and the modulus of twelve: Each view enhancing our understanding of the western musical canon. (A footnote to this introductory document has a section harkening back to the genesis of the whole idea itself in a speculative article entitled *Elements of Music?*) Building upon the document Music by Mutable Numbers, a straightforward example of a complete mutable number analysis is given for the *Polonaise from a Notebook for W.A. Mozart, 1762* (Appendix II).

However, in terms of drawing out and understanding the gist of ‘the thing’, my best endeavours at a foray into group theory has been for myself a catalyst in crystallizing precisely what it actually involves and is: Appendix III, *The Mathematics of Tonal Music and Mutable Base Numbers*. Following on from this, Appendix IV, *The Math of Exchange* is an analysis of a Bach chorale highlighting the group theory mechanics of mutable numbers developed in the former document.

Appendix V, *Bach's Calculation* is an analysis of the first prelude from the Well-tempered Clavier (my third attempt!) an iconic piece often cited as a delineation of the extent of a key or tonal-centre. Other analyses are: Appendix VI, *Chopin: Prelude 20* providing an example of how stylistic developments in the later nineteenth century were beginning to undermine the foundations of tonal music as understood through mutable numbers; Appendix VII, an investigation of the implications of the use of *Just Intonation in the MBN model*; Appendix VIII, an analysis of ‘impressionist’ harmonic language in *Piano Prelude No.1 by Claude Debussy*; and Appendix IX, a brief examination of the *‘Tristan Chord’* from the point of view of mutable numbers. Appendix X, *Spiral* provides a single page working, in mutable numbers, of one cycle in the spiral of fifths. Appendix XI, *The Divisors of 72* provides a worked example of computation in music alongside an equivalent computer program.

While naturally formed upon the basis of a lifetimes’ education, reading and experience, (and doubtlessly some degree of misapprehension, bias and error) I cannot say that the material presented in the above documents possesses any authority beyond that of being my own thoughts, opinions and conclusions on the question of what music fundamentally is.

Philip Perry, 2022.

Music by Mutable Numbers

Many years ago and on the other side of the world, a music class veered off-topic and into an inconclusive discussion of what music means. What is music – fundamentally? I now have no recollection of the intended topic of the day and little recollection of the off-topic student discussion; however, what has stuck in my mind is the comment with which our teacher gently tugged his class back to work: “I don’t know what music means, but it means a lot to me”. This neat sidestepping of the question disappointed me at the time. I rather hoped for a substantial answer. Later I wondered if our teacher was subtly pointing out that the meaning of music is unknowable, except in personal terms? Well, there certainly is much for philosophers to discuss regarding the meaning of music, and I appreciate that. There are many approaches and many different answers. Yet still more than fifty years on, that niggling disappointment at the lack of something more tangible, more precise, persists.

My series of articles *Elements of Music?* in 2003, was an attempt to articulate the idea that, looked at from a perspective informed by the harmonic series, it might be possible to discern some common structural features between a range of self-organising physical systems and the traditional harmonic language that has arisen in western tonal music. The prime example taken was that of electrons clustering around an atomic nucleus forming themselves into a structure reflected in the arrangement of the Periodic Table of Elements. This short article does not look back to that material (although there is a Parthian shot at the foot of this document) but forward, turning those speculative thoughts – in particular the idea of nested harmonic series – upon music itself. However, before getting on to the topic of music we must, as they say, ‘do the math’.

It is also electrons – their relative presence (1) or absence (0) – confined within the circuits of a silicon chip that provide the physical representation of the binary number system upon which our computers are (mostly) founded. And in handling binary we find some ‘power of two’ number bases useful: Octal, Hexadecimal and Base64. Apparently Charles Babbage, the nineteenth century mathematician and ‘computer’ pioneer, pondered binary base 2, as well as bases 3, 4, 5, 12, 16 and 100 before settling on the more familiar decimal, base 10, for his Difference and Analytical Engine designs. Historically other bases have been used, including Babylonian sexagesimal base 60, Maya vigesimal base 20 (but second position base 18 to reflect an approximate 360 day-year relationship), duodecimal base 12, and others. Often in both additive and positional systems there was a connection with fingers and thumb or the spaces between them – bases 10, 8 and 5. For example in Roman Numerals, I, II, III fingers and one hand or two, V & X.

There are other positional number systems that largely go unremarked – Imperial measures. For example, a length of 3 yards, 2 feet and 11 inches or the amount £1.15s.3d. These are implicit mixed base number systems which unlike familiar decimal (where each additional numeral position represents a further multiplication by 10) do not stick to a single fixed base quantity. Both Imperial systems have 12 units: 0–11 pence or inches. Thus the second position is base 12 – each foot comprises 12 inches and each shilling comprises 12 pence. In the third position each yard comprises 3 feet and each pound comprises 20 shillings – bases 3 and 20 respectively. We could conveniently represent the above quantities as:

Imperial Measures: $3_3\ 2_{12}\ 11_1$ inches and £$1_{20}\ 15s_{12}\ 3d_1$ pence

(In examples of non-decimal number systems presented in this document, multi-column decimal numbers are used to represent non-decimal magnitudes greater than nine – as was the practice for inches, pence and shillings in their day. This saves us inventing new numerals or using letters as in hexadecimal numbers.)

Thus far, we have considered single base positional number systems and introduced mixed base positional number systems, both of which have unalterable column bases. There is another possibility, a *Mutable Base Number System*. That is, a number system where not only do column magnitudes vary but also the column bases! For example, using the above notation and the acronym ‘MBN’ for mutable base number:

Decimal 24 (i.e. $2_{10}\,4_1$) = MBN $8_3\,0_1$ = MBN $6_4\,0_1$ = MBN $12_2\,0_1$

As this example illustrates, Mutable Base Numbers have the interesting characteristic that they can express individual magnitudes by a variety of different digit sequences. Indeed, with column digits and bases freed, a plethora:

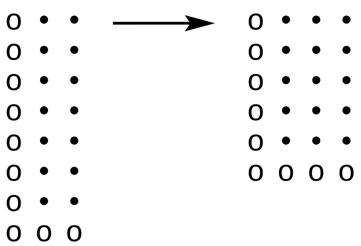
MBN $8_2\,8_1$ = MBN $6_3\,6_1$ = MBN $5_4\,4_1$ = MBN $3_5\,9_1$ = MBN $3_7\,3_1$ ... etc.

The possibilities are truly astronomic with larger values. Too many! The mutable base number system could be seen as a general amalgam of positional and additive features, able to express every possible combination of digit and base. Luckily, for reasons which come later, it is only the subset of MBN digit sequences that are composed of a most significant digit, followed by zero or more zero-digits, that are of particular interest with regard to music. All following example mutable base numbers fit this restricted pattern. Here is a larger number:

Decimal $3_{10}\,4_{10}\,5_{10}\,6_1$ = MBN $9_4\,0_{96}\,0_1$ = MBN $18_{192}0_1$ = MBN $36_2 0_6 0_8 0_1$

(Notice that the convention adopted here is to derive the column base value via multiplication of bases from the unit, through the succession of column bases, up to any particular column position – for example, in the decimal number 3456 above, the hundreds column holds 4×10×10×1 units.)

Although not digit sequences as such, a physical representation of magnitude can be made in the form of Number Patterns. Perhaps the most familiar of these visual patterns are formed by the square and cubic numbers, but in regard to mutable bases, it is rectangular or oblong numbers that have useful illustrative properties. For example, pebbles or some other counters might be arranged thus:



MBN: $8_3 0_1$ ————> $6_4 0_1$ = Decimal 24

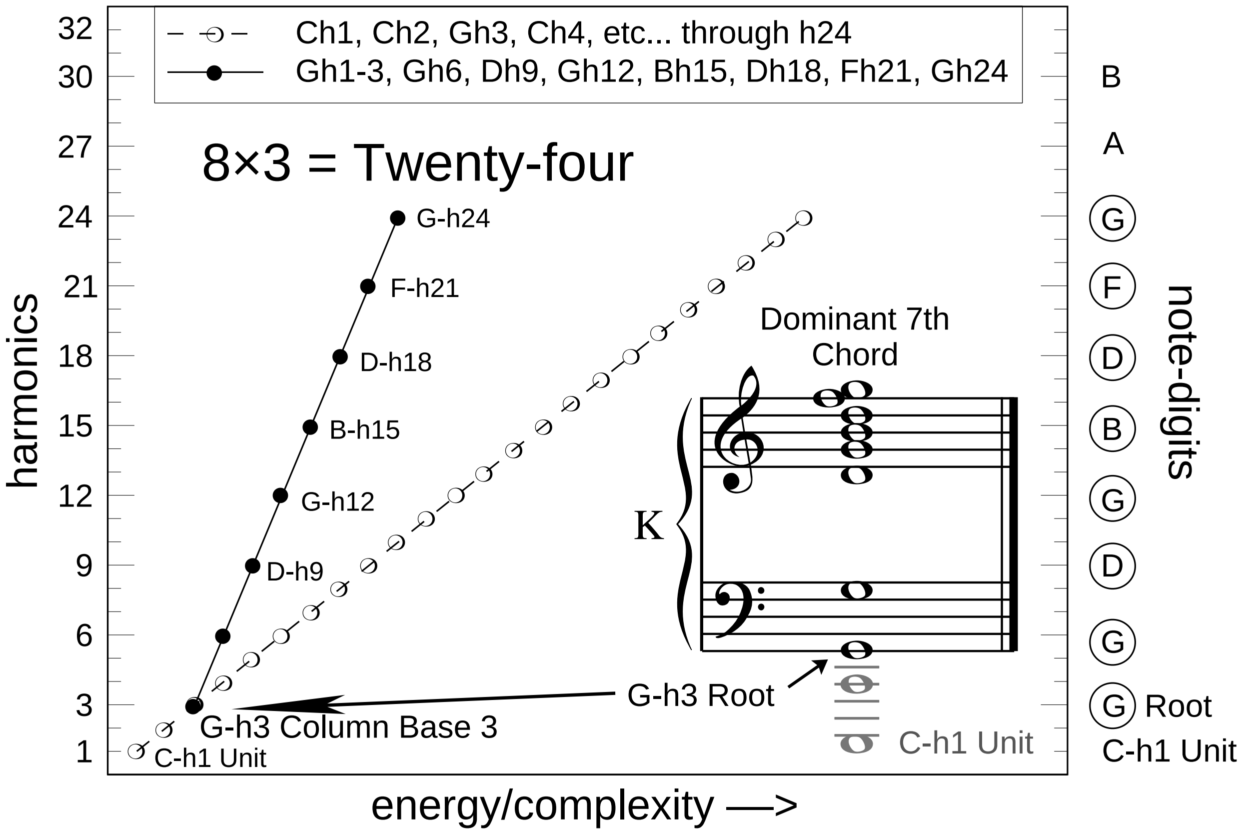
In the two number patterns above the re-ordering between the vertical and horizontal, mimics the exchange of two mutable number digit sequences both of which represent twenty-four. An exchange or transformation between these two rectangular patterns is possible because each pattern has the same number of counters

– there is a *conjunction* of sums. In contrast, the arrangement for MBN 23_1 or MBN $1_{23}\ 0_1$ (a row or column of twenty-three counters) cannot be reorganised to match some other filled rectangle. There is no conjunction of sums other than the limit case of a single column and a single row. This is true for all primes in the mutable base number system.

Although this abacus-like moving around of counters is rather low-tech in comparison to the operation of present day computers, in principle, it is much the same; and this would be true for whatever physical ‘counters’ were chosen to represent digits. Shuffling electrons or pebbles it makes no difference in principle, notwithstanding the considerable technical and practical considerations. So what about using musical sound to describe magnitudes, rather than electrons or pebbles, in particular the frequency relationships of the harmonic series – notes and their overtones? Is it possible? How could it be done?

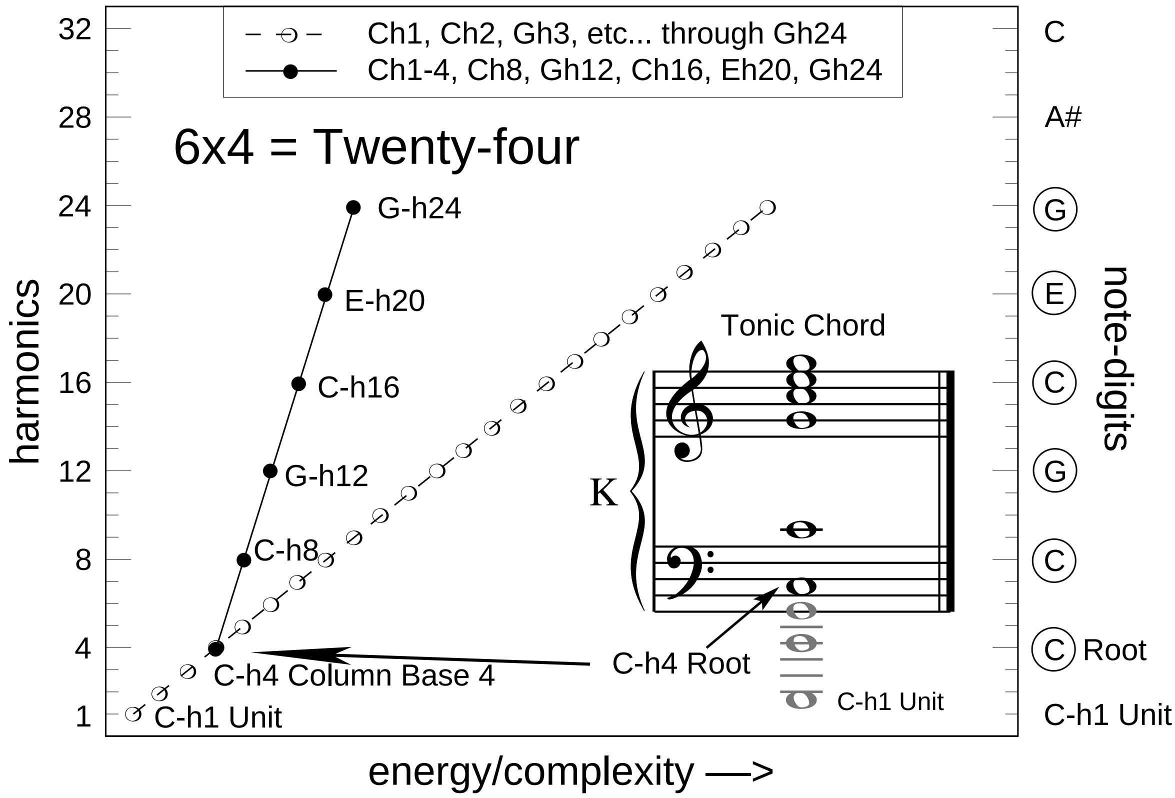
Sticking with the example sum of twenty-four, one needs to set out the note-digits in a musical equivalent of the pebble patterns above. The way to do this is to express the principle of multiplication integral to position-value number systems by way of nested harmonic series:

$$\text{MBN } 8 \times 3 + 0 \times 1 \;=\; \text{MBN } 6 \times 4 + 0 \times 1 \;=\; \text{Decimal } 2 \times 10 + 4 \times 1.$$



Written in note-digits, 8×3 can be expressed as eight ratios of the harmonic series nested within a more fundamental series, in groups of three fundamental ratios: h1-3, h6, h9, h12, h15, h18, h21, h24.

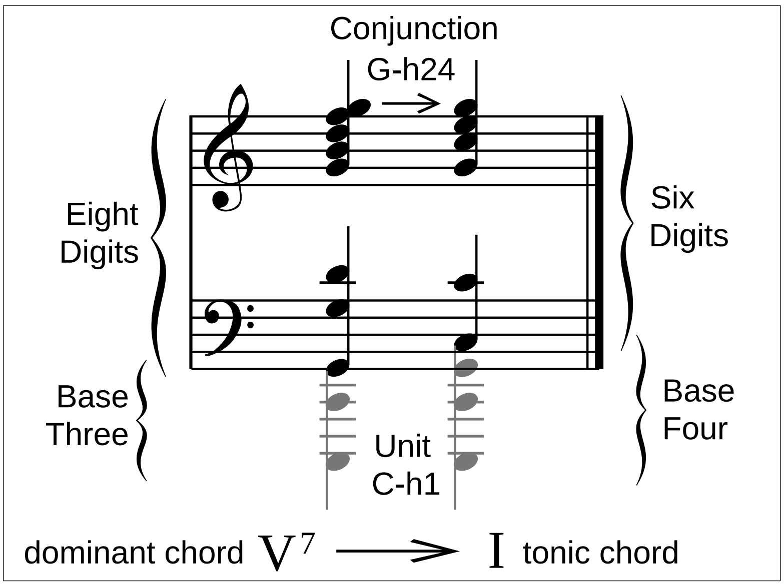
And as seen earlier in this document there is more than one way of expressing the magnitude twenty-four when *Mutable Base Numbers* are involved – illustrated in the following figure.



Similarly, 6×4 nests a harmonic series of six note-digits within a more fundamental series in groups of four fundamental ratios: h1-4, h8, h12, h16, h20, h24.

From these two examples it can be seen that the nesting of one harmonic series within another is a form of multiplication. Each successive layer of nesting is a new column in the physical representation of a mutable base number in sound. Any number of columns/nested levels is possible but we'll stick with just one for now.

Juxtaposing these two musical expressions of the value twenty-four together, yields a dominant-seventh chord (MBN $8_3 0_1$) followed by the tonic chord (MBN $6_4 0_1$). A full cadence.



The Full Cadence defining the magnitude twenty-four.

In the illustrations notice the grey 'note-counters' at the bottom of the two chords: three ratios C-h1, h2, (h3) and four ratios C-h1, h2, h3, (h4). They are the column bases of these two mutable number digit sequences expressed in musical sound – including the expression of unit value (0_1) in the common fundamental C-h1. These column base note-counters are implied by the objective chord progression with its conjunction G-h24, but are not physically present in the sound. Whether they have some existence or role in aural cognitive processes I do not know, but doubt. Some music theorists in the nineteenth century toyed with the concept of observable 'undertones' but ran up against a brick wall scientifically.

Taken together these two chords form a full cadence (chords V^7–I), a proto tonal composition. What makes them tonal music is that the sound, structured around simple frequency ratios, is intuitively understood by the ear while the equality of the sums expressed, twenty-four, G-h24, link the chords meaningfully together. Their sums are joined at their common highest note – treble g^1. This frequency conjunction, its sub-octave and probably a number of its multiples, are noticed by the ear, allowing it to interpret the chord progression as a comprehensible whole, a transmutation between two related sound states. Throughout the g^1 note-digit and associates remain constant, anchoring the harmony. To human aural cognition the chord progression is in essence a (very short) musical composition, a tonally organised composition.

Broadly defined, tonally organised music dominates the musical landscape worldwide, both today and historically; and therefore the pursuit of a full understanding of this almost universal human phenomenon is desirable. Further, I would argue that western tonal music in particular, and all tonally organised musics in general, are actually an external manifestation of the workings of the human hearing mechanism and aural cognition; in contrast experimental non-tonal/atonal music is essentially a direct outgrowth of the mind.

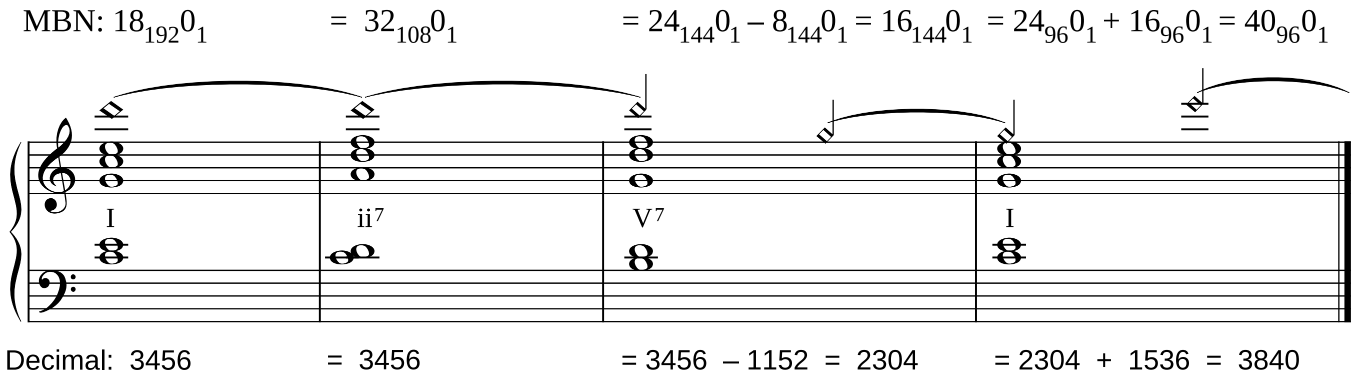
The ear is concerned with sound perception not mathematical grammar. So it is within the objective sound – including the overtones which the ear processes in order to create the pitch sensation – that the source of the implied, underlying column base values must be found: It is the *conjunction*. Where any two chords are joined by a common frequency equal to or greater than the highest note-digit frequency (i.e. common top note, or more often, common overtone) it is possible to construct a notional 'tail' of one or more nested harmonic series leading to a common fundamental frequency. The C-h1 in the two example chords, the unit. This implicit subterranean structure, which may consist of more than one level of nesting, forms the scaffolding of column bases found in mutable base numbers expressed in tonal musical sound.

The full cadence is the minimal example. To express magnitude by way of mutable base numbers formed from musical sound, requires a conjunction between two different chords to crystallize a value. All chord successions have frequency conjunctions, though some are more remote than others. Thus, in their essential nature, each harmonic progression is *a number*. However, there is no limit to the range of additional chords, beyond the first two, that may be strung together, each connected by their own frequency conjunctions and thus defining their own new values. Indeed, a tonal composition is, when viewed as mutable base digit sequences written in sound, simply a succession of magnitudes defined by a sequence of chord pairings – individual links in a chain of numbers. To relate all these magnitudes together will usually require an additional layer of nesting. Over an entire composition, by collecting together all the individual fundamental (h1) frequencies created through each successive harmonic progression, a deeper contiguous underlying harmonic series encompassing the whole piece can be constructed. Thus an entire tonal composition can become *a calculation* – a stream of related magnitudes.

Adding one or more underlying series will produce mutable numbers with scope for many more columns. Such multi-column numbers can express subordinate levels of nested structure in the form of less significant, zero-digit, columns. Thus the multiplication of column bases (containing zero-digits) is of particular interest, and from the myriad of possible mutable base numbers available, only those with zero-digit less significant columns, are employed. For example the decimal magnitude 3456 introduced earlier can be expressed by many different mutable number digit sequences, among which there is one that might be regarded as its ‘ground state’.

$$\text{Decimal } 3456 = \text{MBN } 2_2 0_2 0_2 0_2 0_2 0_2 0_2 0_3 0_3 0_3 0_1$$

While entwined around and within this abundant scaffold of mutable number digit sequences, there still remains wide scope for creativity, variation, unique composition. The underlying numbers (–i.e. the sums or conjunctions) make tonal music comprehensible, their embellishment makes it interesting.



The opening chords, from J.S. Bach’s Prelude No.1, The Well-tempered Clavier with the number processing and magnitudes they define. The conjunction sums (frequencies) are marked by diamond-headed ‘overtone notes’.

Decimal 3456 (MBN $18_{192}0_1$), the larger mutable base number example given earlier, is the magnitude defined by the opening two chords of the Bach Prelude. Unlike the full cadence example, in most ‘real world’ music, the conjunction ‘sums’ are found amongst the overtones rather than the written note frequencies. It is in this range of low order harmonics that most of the energy in musical sound is normally concentrated, and the human ear most sensitive.

When mutable base numbers are applied to non-trivial tonal compositions, there are choices to be taken regarding the details and conventions of implementation. In a strict mutable base number interpretation, conjunction sums (frequencies) are taken to be exactly aligned while the pitch of a composition adapts to the calculation of harmonic exchanges made in simple ratios. This flexing of pitch may be accounted for by small real number variations in the unit value. As also may any mismatching between the precise frequencies generated by the proportions of exchange (e.g. 3:2 or 6:5) and the ratios in the underlying fundamental series found at more granular levels. However if one really yearns for the ultimate, mutable base numbers ranging from MBN $18_{25509168}0_1$ through to MBN $12_{245760000}0_1$ (Decimal 459165024 – 294912000) will provide an absolute whole number solution to this Prelude based on a fixed unit from start to finish.

Yet in reality, ensembles of musicians, be they professional orchestras or amateur choirs, operate as self-stabilising groups in regard to pitch – each individual constantly listening and adjusting to the whole – and, in addition, the tolerance of the ear with its strong bias toward hearing and extracting simple ratio relationships, easily accommodates issues that may arise between the raw, plastic, objective sound; and the abstraction of mutable numbers. (More on Prelude No.1 in ExS3.pdf – *Bach's Calculation.*)

The high-watermark of 'tonal computation' in western art music broadly fell across the 'long' eighteenth century, encompassing the time from Corelli through to Beethoven and beyond into the Romantic era, this perhaps explains to some degree the enduring popularity of the Baroque and Classical idioms; and for example, why music of the highest complexity by J.S. Bach (1685–1750) is appreciated by a wide audience, while in contrast Josquin des Prez (circa 1440–1521) a contrapuntist of comparable stature, remains obscure.

As the nineteenth century wore on, the course of western art music fractured into many paths, most of which led away from the wholly explicit harmonic number processing of former times. Gradually the powerful dominant-tonic 3:2 and 3:4 relationships were diluted with more frequent use of less energetic and more enigmatic exchanges further up the harmonic series (e.g. Vivaldi compared to Debussy), until by the twentieth century some composers would give up tonal organisation altogether –i.e. atonalism. However any voids left by these developments were to be immediately and permanently filled by a mass proliferation in the many genres of popular music. A situation that persists down to the present day. Back then and now the vast majority, having discovered the delight of 'doing the math' in sound, vote with their ears in favour of aurally intelligible music. It is perhaps significant that toward the end of the nineteenth century, as western art music gradually left the centre ground of mutable number processing, interest in, and performance of, the historic tonal repertoire grew in matching proportion. Today, as much as ever, tonal computation dominates the western cultural sound environment taken en masse; and further afield, it is strongly invasive in other music cultures. This being the case, the interpretation of music offered by mutable numbers might provide fresh perspectives and new avenues of enquiry for musicology.

Yet the ear cares not for number systems and their niceties, human hearing has evolved solely to do its own business processing objective sound; though remarkably, what it achieves in the cognition of tonal music, when viewed from a perspective of nested harmonic series and the mutable base numbers they articulate, is effectively, *the computation of numbers*.

Link to Footnote Below

As far as I can see, at their heart, mutable numbers connect a sequence of whole numbered relationships (e.g. 2:3, 6:5, 43:61, ...) with the definition and computation of certain sums. Somewhere, near or far, upon the limitless expanse of numbers, any given sequence of these relationships will bind on to a particular collection of lowest whole numbers, and rising upward from each of these whole numbers, adjacent ladders of natural numbered multiples will find rungs of common height defining a succession of sums.

The simplest example is the relationships 1:1, 1:2, 2:3, 3:4, 4:5, 5:6, ... which binds on to 1, 2, 3, 4, 5, 6, ... and defines the sums 1, 2, 6, 12, 20, 30, ... processing the additions 1, 2+2, 3+3, 4+4, 5+5, ... in succession. This most basic sequence of relationships is used in the footnote below – where the musical examples have been chosen to illustrate that mutable numbers work equally well when applied to duration and metre.

Elements of Music? – a footnote

For many years I have harboured the speculative intuition that the patterns found in the Periodic Table of Elements (and therefore also the arrangement of electrons in atoms) might share some common characteristics with the structures and processes underlying western tonal music. (In the early 2000's I wrote, rather erratically, on this topic in *Archive* magazine.) This observation has some heritage. J.A.R. Newlands (1837–1898) presented a paper to the London Chemical Society in 1864 entitled *The Law of Octaves* which noted the recurring pattern of eight in the properties of the then known elements – coming very near to preempting Dimitri Mendeleyev’s (and J.L. Meyer’s) construction of the periodic table. Newlands’ ideas, which contained a hint of the electron shell structure later to be revealed, were ridiculed as unscientific and ignored at the time by his peers. What was missing for the most part from Newlands table were spaces for elements as yet undiscovered, ... as well as a suitably bland title.

Recently I came across the following table charting the relative energy levels of electron shells and sub-shells forming the outer atomic structure; and thus also the order in which electrons would normally attach themselves to the nucleus of atoms.

SubShell=>		sharp	principal	diffuse	fundamental	g	h
Period							
1.	K-Shell	1					
2.	L-Shell	2	3				
3.	M-Shell	4	5	7			
4.	N-Shell	6	8	10	13		
5.	O-Shell	9	11	14	17	21	
6.	P-Shell	12	15	18	22	26	31
7.	Q-Shell	16	19	23	27	32	37
8.		20	24	28	33	38	44
9.		25	29	34	39	45	51
10.		30	35	40	46	52	59

Electron Sub-shell Energy Levels Ranked from Lowest to Highest
(Sub-shells observed in nature underlined)

The Periods of the Table of Elements are listed in the left hand column from 1 to 10, of which only periods 1 to 7 are observed in nature. The now rarely used Shell designation by upper-case letters K to Q are also given. The sub-shell names/types are presented across the top of the table. The actual shells/sub-shells observed to exist in nature are underlined in the table, the other values provide a further extension of the system.

The relationship that caught my eye was that reading down the columns of energy levels for each sub-shell type, the differences between these values is precisely that yielded by the sequence of primary conjunctions found in a table of nested harmonic series (complete in the first column and then truncated at the head for subsequent columns). These values are: 1, 2, 2, 3, 3, 4, 4, 5, 5, ... etc.

The configuration of electrons clustering around the nucleus of atoms take on set patterns, formed of shells, sub-shells and orbitals. Each electron shell or sub-shell consists of one or more electron orbital and each electron orbital can contain a maximum of two electron. The relative energy level of individual electrons is broadly governed by which shell and sub-shell they inhabit, and their general preference to reside at the lowest energy level available to them. (Notwithstanding, within each orbital a fine structure pertains with pairs of electrons having slightly differing energy levels.) However, overall, the electrons in any particular sub-shell will possess similar energy levels. Thus the table above ranks the relative energy levels of the sub-shells which go into forming the electronic configuration of the atoms. And flowing from this ranking, the order and position in which electrons are placed within the shell, sub-shell and orbital structure with regard to increasing atomic size from Hydrogen (proton number Z1) to Uranium (Z92) and beyond.

```
Generation of Electron Sub-shell Differential Energy Levels------------------------>
from the Sequence of Primary Conjunction Exchanges (marked by '=')
         Frequency                                            Sub-shells
C  1:1   1                                      MBN 1_{1}               1s
C  1:2   X=2 .... sesquioctava 1:2 conjunction, MBN 2_{1}=1_{2}0_{1}      2s          1
G  1:3   X | 3                                                              (2p)
C  1:4   X X   4 ....                           MBN 2_{2}0_{1}          3s          2
E  1:5   X |     5                                                          (3p)
G  1:6   X X=X     6 .... sesquialtera 2:3 conjunction, MBN 3_{2}0_{1}=2_{3}0_{1}   4s   2
A# 1:7   X   |       7                                                      (3d)
C  1:8   X X | X       8                                                    (4p)
D  1:9   X   X           9 ....                 MBN 3_{3}0_{1}          5s          3
E  1:10  X X |   X         10                                               (4d)
F# 1:11  X   |             11                                               (5p)
G  1:12  X X X=X   X           12 .... sesquitertia 3:4 MBN 4_{3}0_{1}=3_{4}0_{1}   6s   3
A  1:13  X     |               13                                           (4f)
A# 1:14  X X   |     X           14                                         (5d)
B  1:15  X   X | X             15                                           (6p)
C  1:16  X X   X       X             16 ....  MBN 4_{4}0_{1}            7s          4
C# 1:17  X     |                         17                                 (5f)
D  1:18  X X X |   X     X                 18                               (6d)
D# 1:19  X     |                             19                             (7p)
E  1:20  X X   X=X         X                   20 sesquiquarta 4:5 MBN 5_{4}0_{1}=4_{5}0_{1}   4
F  1:21  X   X   |   X                           21
F# 1:22  X X     |       X                         22
   1:23  X       |                                   23
G  1:24  X X X X | X   X       X                       24
G# 1:25  X       X                                       25 .... MBN 5_{5}0_{1}     5
A  1:26  X X     |           X                             26
   1:27  X   X   |       X                                   27
A# 1:28  X X   X |   X             X                           28
   1:29  X       |                                               29
B  1:30  X X X   X=X       X         X sesquiquinta 5:6 conjunction30 MBN 6_{5}0_{1}=5_{6}0_{1}  5
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Table of Nested Harmonic Series

The Table of Nested (Acoustic) Harmonic Series above presents the systematic nesting of every possible harmonic series within an all-enfolding fundamental harmonic series. The values shown are those of frequency. At the top of the table toward the left hand side is Frequency 1 – H1. The column of Xs below it represent the fundamental harmonic series, as does the diagonal column of numbers stretching out and down toward the right.

The second column to the right of the fundamental column represents the first nested series, it is based on frequency 2 – h2. It is as if this column (and similarly all subsequent columns of nested series) has been drawn out of the fundamental series, constructed from every second rib. This nested series also has a diagonal arm, though lying at a more acute angle and filled with Xs rather than numbers. Also notice that the diagonal arm passes through the second column to hinge on frequency 2 in the fundamental column.

The third column to the right continues in the same vein, as do all subsequent columns. The sequence of nested harmonic series continues in whole number steps of frequency without limit, rather in the manner of a cellular automata.

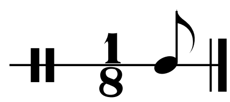
As this is a table of acoustic harmonic series, to the extreme left is a column of note letters labelling the whole number steps of the series; approximate, given the limitations of scale and temperament. Note letter C is arbitrarily chosen as frequency 1. To the right of the note letters is a column of ratios which also serves to label the fundamental harmonic series column of Xs.

Toward the right hand side of the table information about the electron sub-shell structure is coordinated, line by line, with progress through the succession of primary conjunction exchanges. There are three columns, the first gives the energy level positions of the sharp sub-shells being generated; the second, the positions of the other sub-shells (principal, diffuse, fundamental) and the third/last column the sequence of energy level differences of the sharp sub-shells, underlined, as employed above. (Also notice that the differential spacing of energy levels in the other sub-shell is correct and truncated as given in the ranking table.)

Drawn within the triangular web of the table of nested harmonic series, the sequence of primary conjunction exchanges moves through the system step by step. And in the illustrations and text below references to frequency and metre (period/wavelength) are rather freely and inconsistently interwoven – the frequency conjunction exchange and the metrical hemiola (generalised).

Beginning at frequency 1, the unit, at the top of the fundamental column:

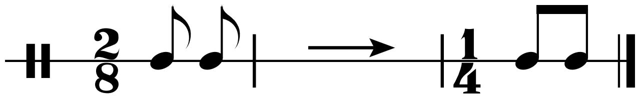
MBN: 1_1



Unity, the Fundamental Unit, H1.

By adding one to one, the system arrives at the first conjunction, frequency 2, marked by an equals sign in the table. (This isn't as clear as later conjunctions because the symbols are packed close together.) Expressing the values in mutable numbers and metrical form (below) the dynamic sesquioctava 1:2 exchange is rendered thus:

MBN: $2_1 = 1_2 0_1$



Primary Sesquioctava 1:2 Conjunction Exchange

In taking this first step away from the fundamental frequency (the 1s sub-shell energy level) in the table of nested harmonic series, the energy level of the 2s sub-shell is generated. Building upon this foundation, the next differential energy level for the sharp sub-shells is two steps further on, and now there is space in the table to mark this movement with a vertical line. Two steps or rather one step of two units (we are now working within the second column built on h2) takes the system to frequency 4. Thus generating the energy level of the 3s sub-shell.

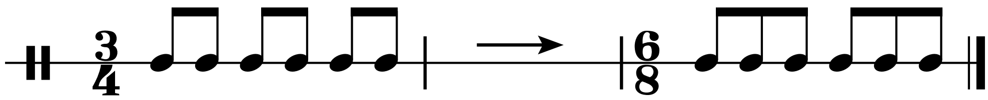
That is, MBN: $1_2 0_1 + 1_2 0_1 = 2_2 0_1$

and another, second, double step carries the system on to the energy level of the 4s sub-shell at frequency 6,

MBN: $2_2 0_1 + 1_2 0_1 = 3_2 0_1$

which is also the position of the next primary conjunction exchange.

MBN: $3_2 0_1 = 2_3 0_1$



Primary Sesquialtera 2:3 Conjunction Exchange

The conjunction exchange at frequency 6 changes the stride of the system again from groups of two units to groups of three – triple steps within the third column built on h3. So continuing on, the first triple step moves the system on from six to nine again marked by vertical lines in the table. This generates the energy level of the 5s sub-shell.

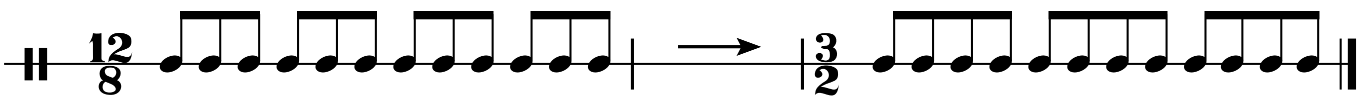
That is, MBN: $2_3 0_1 + 1_3 0_1 = 3_3 0_1$

and with another triple step the system is carried along to the energy level of the 6s sub-shell at frequency 12,

MBN: $3_3 0_1 + 1_3 0_1 = 4_3 0_1$

which is also the position of the next primary conjunction exchange.

MBN: $4_3 0_1 = 3_4 0_1$



Primary Sesquitertia 3:4 Conjunction Exchange

The pattern by now is becoming familiar. This conjunction exchange at frequency 12 changes the stride of the sequences from groups of three units to groups of four – quadruple steps within the fourth column built on h4. So continuing on, the first quadruple step moves the system on from twelve to sixteen, again marked by vertical lines in the table. This generates the energy level of the 7s sub-shell, at frequency 16, the last sharp sub-shell found in nature.

$$\text{MBN: } 3_4 0_1 + 1_4 0_1 = 4_4 0_1$$

This scheme for generating the energy levels of captive electrons continues on in the same fashion without limit, gradually spacing out the sharp sub-shells so as to allow the required slots for the principal, diffuse, fundamental, g, h, ... sub-shells to find their particular appropriate levels, at least in theory if not in nature. It is interesting, though perhaps no more than coincidence, that at the point in this process where the six electrons of the 2p sub-shell (in contrast to the simple pairs in the sharp sub-shells) would be drawn into the underlying series at the sesquiquarta 4:5 exchange (MBN $5_4 0_1 = 4_5 0_1$) the sequence falters.

I do not know if this simple model of nested harmonic series makes contact in any meaningful way with the material reality of the electronic configuration of the atom, or if it is merely a correspondence that appears to mimic what is scientifically observed and tabulated in the Periodic Table of Elements.