

Mutable Number Analysis of Prelude 20, Op. 28 Frédéric Chopin

Abstract

To sketch the overall concept: Any tonal composition is viewed as an independent system of harmonic and metrical relationships suspended within a two-dimensional pitch-duration space consisting of an all-encompassing fundamental nesting harmonic series. The gamut of possible relationships available to this nested system, the composition, are those contained within the positive, non-zero rational numbers. That is, all positive fractions and whole numbers, which, under the operation of multiplication, move the composition about the space that it inhabits. Additionally the view is taken that although something along the lines of equal temperament is the practical outcome under almost all performance scenarios, what the ear and mind actually extract from this approximation is the sense, and sensation, of rational Just relationships. For example, the relationship of $\frac{3}{2}$ between dominant chord V and tonic I, of 6:5 between the subdominant chord IV and supertonic II, etc. Thus to arrive at a truly meaningful understanding of tonal music, it is necessary to model the system, as does the ear, in relationships of whole numbers and fractions. Following this approach, embedding it within the physical fact of nature – the acoustic harmonic series – and informing it with the idea of nesting harmonic series within each other, leads on to the conception of tonal music being, effectively, a number system written in musical sound; and any individual piece, a computation of numbers.

Introduction

The most interesting feature of this composition highlighted by mutable number analysis is the ‘subversion’ of regular (mathematically symmetrical) conjunctions in measures five and six; and, not entirely unconnected with former point, the relative prevalence of root movement in steps of major and minor thirds – a stylistic feature more prominent in the later nineteenth century.

Essentially, the lesson of this piece is that the stylistic changes and developments that took place as the nineteenth century progressed, gradually drew western classical music away from the explicit, number processing driven techniques typical of the eighteenth century, toward a more opaque, vectorless harmonic style, casting the underlying computational procedures under a veil of equivocal chordal obfuscation. Generally a workable chassis of mutable number exchanges was retained, though somewhat muted in form, due to the increasing use of less ‘energetic’ exchanges such as the sesquiquarta 4:5 and sesquiquinta 5:6 exchanges, relative to the abundance of the more powerfully directional sesquialtera 2:3 and sesquitertia 3:4 progressions. To draw a literary parallel, the overt meaning of the words (the chords) gradually became less important than the effect of their sounds: western music was exploring the continuum between prose and poetry; a rich and fulfilling journey, but an excursion that ultimately lead toward tonal nihilism.

The basic mechanics of mutable number analysis are not dwelt upon below because they have been covered in other similar documents (ExS3, mathEx, mathMBN, etc.) and so here it is convenient to move straight into the detail: measure five and measure six.

Measures Five and Six

In measures five and six the chord sequence is C minor – Aflat major, Bdim/minor – G minor, A minor – D major, G major – G major seventh. In this two measure phrase each chord lasts one beat and the chords are paired by their sequence-like major-third root motion leading into a conclusive ii–V–I close on G major. The Aflat major and G minor chords are in first inversion and the Bdim/minor is without its diminished fifth. Mathematically commensurable conjunctions joining chords whose roots are separated by major (4:5) and minor (5:6) thirds lie where four/five elements are exchanged and five/six elements are exchanged between adjacent series. Equally these and all conjunction exchanges work both forward and backward. Here in measure five the major/minor third exchanges are arranged in reverse, running in the order 5:4, 5:6 and 5:4 again. Thereby the first pairing, C minor to Aflat major should exchange elements at a rate of four note-ratios of a C based series for five note-ratios of an Aflat based series. This rate of exchange is found at the conjunction note-ratios spaced, respectively, four and five elements apart in each series – illustrated in Figure 1. In the analysis the lowest, strongest, overtone conjunction inclusive of the written notes and conforming to this spacing (and thus symmetrical in terms of the exchange) is C-h128 → C-h160.

$$(19 \times 5) \div 4 = 23.75$$

	D#h19====>	D#h24	
	D-h18	D-h23	
	C#h17	D-h22	
		C#h21	
	C-h16---->	C-h20	
	B-h15	B-h19	
	A#h14	A#h18	
	A-h13	A-h17	
		G#h16	
	G-h12---->	G-h15	
	F#h11	F#h14	
	E-h10	F-h13	
	D-h9	D#h12	
		D-h11	
	C-h8----->	C-h10	
	A#h7	A#h9	
	G-h6	G#h8	
	E-h5	F#h7	
		D#h6	
	C-h4----->	C-h5	
	G-h3	G#h4	
	C-h2	D#h3	
	C-h1 (H5)	G#h2	
	24:19?!>	G#h1 (H4)	
	5:4--->		

--- C-based series ---->

--- Aflat-(G#)-based series ---->

Figure 1. The 24:19 (5:4) exchange between C and Aflat, measure five, beat one and two – four elements built on H5 are exchanged for five elements built on H4. The first four conjunctions are illustrated with arrows (--->) plus the improper approximate conjunction (==>) between the C based minor third (h19) and the Aflat based fifth (h24).

Measure Five: Beat One & Two

However, as illustrated in Figure 1, there are approximate conjunction frequencies lying between chords whose roots are separated by the proportions of four and five, –i.e. a major third. These conjunction frequencies link h19 and h24, and their whole numbered multiples. The conjunctions are not perfectly

aligned or symmetrical, but are close enough to allow the ear to accept them as valid. Indeed, as this is a piano piece, they are in performance (though not in theory) identical. Thus between the first and second chords in measure five the most effective aural link is Eflat while the correct conjunction C plays a lesser practical role. Both the proper, mathematically symmetrical interpretation and, what might be termed the ‘improper’ actualité are charted in the analysis below.

Measure Five: Beat Two & Three

Moving on to measure five, beats two and three, the tables are turned! In this 5:6 exchange between Aflat and B the formerly improper conjunction tone Eflat (D#) becomes an entirely proper and symmetrical link between these two chords – illustrated in Figure 2. However, the minor harmony of the B chord runs counter to this ‘correct’ conjunction as it is undermined by an emphatic right hand D octave and fifth harmonic of the left hand B octave. Consequently from the ear’s point of view it is doubtful that proportional symmetry counts for as much in this progression as the ‘improper’ link made by the repeated Aflat written note and its overtones – of which h3 is the entirely proper and symmetrical D# conjunction.

$$(192 \times 5) \div 6 = 160$$

	D#h192----->	D#h160	
	-----	-----	
	-----	-----	

	A#h18----->	A#h15	
	A-h17	A-h14	
	G#h16	G-h13	
	G-h15	F#h12	
	F#h14	F-h11	
	F-h13		
	D#h12----->	D#h10	
	D-h11	C#h9	
	C-h10	B-h8	
	A#h9	A-h7	
	G#h8	F#h6	
	F#h7		
	D#h6----->	D#h5	
	C-h5	B-h4	
	G#h4	F#h3	
	D#h3	B-h2	
	G#h2	B-h1 (H6)	
	G#h1 (H5)		
	5:6---->		

--- Aflat-(G#)-based series --->
--- B-based series ---^

Figure 2. The 5:6 exchange between Aflat and B, measure five, beat two and three – six elements built on H5 are exchanged for five elements built on H6. The first three conjunctions are illustrated with arrows (--->) plus the thirty-second conjunction used in this analysis.

Measure Five: Beat Three & Four

The procedure found in measure five, beats one and two is now repeated, more or less, between the last two chords B dim/minor and G minor on beats three and four with another 5:4 major third exchange. On the face of it the first chord of this pair is a diminished triad on G# in first inversion, however, given the strength of the B octave sounding in the left hand and the fact that it evolves into a B minor triad, eventually, it is

generally taken to be a chord whose principle allegiance is to B. And although this time the second chord of the pairing is minor rather than major, yet because the improper link is to the fifth (D) there is no difficulty. The mathematically symmetrical conjunction is found at B, the fourth element and fifth element respectively, mirroring the arrangement illustrated in Figure 1. However, actually hearing a B natural conjunction arising out of a G minor chord would be quite a stretch for any ear, though the conjunction does objectively exist, the fifth harmonic (high b^2) of the written note middle g. In reality, one must surmise, the ear adopts the same practical approach of straightforwardly accepting the self-evident link provided by the note-ratio D. So in measure five the effective conjunctions are: Eflat beats one and two, Aflat beats two and three (with perhaps a little residual Eflat) and D beats three and four. And looking ahead to measure six, they continue in descending motion: C beats one and two, concluding with B, plus D and G beats three and four.

Measure Five & Six: Chromatic Bass Line

When and where proper conjunctions fail to rule a run of exchanges, between chords in tonal music, the passages most often default to some form of ‘parallelism’, quite often involving step-wise and half-step-wise motion, or a sequence, or both. Perhaps this ‘glissando by discrete steps’ could be viewed as the ear’s default ‘Plan B’ in the absence of conjunct contrary motion typically found in the music of the common practice. (And historically in western music the practice of ‘parallelism’ as a more general organisational principle has been evident, for example in organum, fauxbourdon and in folk traditions like English discant and Norse influenced gymel.) Normally such parallel passages are short ‘purple patches’ book-ended by more solid tonal harmony – ‘snakes and ladders’ lying between more mundane and comprehensible harmony. Here in measures five and six the application of parallel motion techniques holds the piece together with a descending chromatic bass line, C through F, and an ‘half-rhyming’ sequence C–Aflat, B–G; leading into a third half-return A–D, which morphs into a ii–V–I of V cadence on G. While floating above this driving bass line the harmony dances a partial sequence with improper conjunctions drift downward in mirroring sympathy. Over the course of the nineteenth century the number and duration of such passages found in compositions grew steadily, until they came, eventually, to supplant outright tonal number processing as the predominant stylistic feature of western art music, early in the twentieth century.

Measure Five Beat Four & Measure Six Beat One

There is a choice of proper conjunctions for the exchange from measure five, beat four (G minor first inversion) to measure six, beat one (A minor in root position). Either 7:8 or 8:9 exchanges are candidates. A sesquiseptima 7:8 exchange would see the conjunction drawn at G while the sesquioctava 8:9 exchange requires A. Here again it would appear that the natural choice for the ear would be the rather less standard 7:8 exchange as the conjunction note-ratio treble g^1 arises as the second harmonic of the written note middle g, normally the most energetic partial. Whereas the standard Just whole step interval of 8:9 would require the ear to pick out the third harmonic of middle d and/or the third and sixth harmonic an octave higher. Either are possible, both are legitimate mathematically. So as to be able to compare these two conjunction series the more standard sesquioctava 8:9 series is written out, in position, at measure five, beat four in the analysis and the sesquiseptima 7:8 series is boxed just before measure six, beat one. This exchange carries the composition into measure six where a third pairing of chords first hints at another 5:4 exchange by employing yet another phony conjunction (C) before progressing to G major by way of a recognisably standard (ii–V of V) cadence.

Measure six begins with an A minor chord in root position with a top note C, followed by a D major seventh chord – somewhat disrupted by the Aflat octave in the left hand. The symmetrical exchange between these chords is 3:2 with a conjunction drawn at A. However, the chord progression will also allow another link to be made at C, one that would echo the pattern of the two chord pairings by 5:4 found in measure five. Contrary to the expectations built up by the pattern of chord progressions experienced in the previous measure, instead of A minor leading to an F minor first inversion chord, the composition swerves toward D major and on into a familiar tonal close. The improper conjunction at C is illustrated in Figure 3, and whether the ear makes use of it, or the proper conjunction at A, or both is open to debate. I would argue that it is both. Though the coarse mathematics and symmetry of a 28:19 exchange rubbing alongside a simple 3:2 might be unexpected, the D major harmony is somewhat obscured by the diminished fifth sounding emphatically in the left hand, and consequently, aural cognition must struggle to make sense of the progression. Under such circumstances the ability of the ear to draw out and infer meaning and relationship are considerable; and the effort to lend, perhaps even imposing, understanding upon such near-chaotic stimuli maybe adds to the aural enjoyment of the progression.

Figure 3. The approximate 28:19 (3:2) exchange between A and D, measure six, beat one and two – nineteen elements built on H28 are exchanged for twenty eight elements built on H19.

Measure Six: Beat Two, Three & Four

Finally, with the conjunction at D leading into the last two chords of measure six, the ear and mathematical propriety are reunited in the exchange leading to the cadence on G major. Where groups of four note-ratios are traded for three by means of a sequitertia 3:4 exchange from D major seventh to G major. After which the composition is propelled forward into measure seven by introducing the (G major) seventh through the continuing downward motion of the bass line.

Throughout the the analysis, and in these two measures in particular, the conjunction-sums are all chosen (and illustrated) so as to be inclusive of the written notes of the composition, and so are either top notes of chords or more often partials; but this in no way precludes other, lower pitched links from contributing to the ear's apprehension of relationships – connections between chord successions. Thus, for example, in measure five, beats one and two the lower written note frequencies: bass C¹, tenor C, middle c, middle d#, treble c¹ and treble d#¹, will all be contributing to the overall effect of connection between the C minor and Aflat major chords.

'Hybrid' Analysis

For this analysis an amalgam of the summary and absolute formats has been attempted, with the result that:

- 1) The unit [H1] is dissociated from any direct connection with the key or tonal-centre of the piece, and utterly remote; consequently,
- 2) the mutable base numbers are quite large, ranging between decimal 1440000 and 7372800, and
- 3) through the leeway granted by the above super-extended fundamental nesting series, there has been more than sufficient space to incorporate longer nested series, thus allowing for the minor third to be incorporated into these series as marked ratios (h19 and multiples) rather than leaving them as unmarked pitches. See discussion in *The Math of Exchange*, page five.

Conclusion

From the evidence of these two measures it might be concluded that the ear principally notices and makes use of linking frequencies between chords to navigate its way meaningfully through a tonal composition, irrespective of mathematical niceties: That the ear doesn't require a precise symmetry to existing between the proportion separating chord roots and the disposition of linking conjunctions, but will make do with an approximation. This is unsurprising as the acoustic stimuli, –i.e. the spectrum of all notes and partials, is only limited by the range and acuity of the ear. So why would the ear ignore any putative link? Any linking frequency is conveying special valuable information, as well as contributing to the overall inferences made by aural cognition. Presumably the ear scoops up all the stimuli it can apprehend and makes the best fist of the information to hand: which frequencies change, which remain unchanged, and what are their relationships – out to some degree of approximation. Yet still, the imposition of mathematical doctrine upon the practical workings of the ear may however be informative in illuminating some of the underlying principles involved in the perception of tonally organised music, both when the processes of the aural pathway conform to mathematical theory, and equally, when they don't!

Mutable Number Analysis of Prelude 20

Largo Frédéric Chopin

Conjunction
Values
Written
in
Sound

Piano *ff*

c: i 3:2 iv7 8:9 V7 3:2 i Ab: iii 5:4

C-h32*--->	G-h72*--->	G-h64*--->	G-h64* -32	C-h64*--->
G-h24~	C-h48* +24	G-h32~	G-h32*--->	G-h48* +16
D#h19~	G#h38~	D#h25~	F-h28~	D#h38~
C-h16~	D#h28~	B-h20~	D-h24~	C-h32~
G-h12~	C-h24~	G-h16~	B-h20~	G-h24~
C-h8~	G#h19~	F-h14~	G-h16~	D#h19~
C-h4~	F-h8~	G-h8~	F-h14~	C-h16~
G-h3	F-h4~	G-h4~	G-h8~	G-h12~
C-h2	C-h3	D-h3	G-h4~	C-h8~
C-h1[H60000]	F-h2	G-h2	D-h3	A#h7
(16Hz)	F-h1[H40000]	G-h1[H45000]	G-h2	G-h6
3:2-->	10.666...Hz	12Hz	G-h1[H45000]	E-h5
[H1]	8:9-->	1:1-->	12Hz	C-h4
	[H1]	[H1]	3:2-->	G-h3
			[H1]	C-h2
				C-h1[H30000]
				8Hz
				5:4-->
				[H1]

$$\text{MBN: } 32_{60000}0_1 \text{ --> } = 48_{40000}0_1$$

$$+ 24_{40000}0_1$$

$$= 72_{40000}0_1 \text{ --> } = 64_{45000}0_1 \text{ --> } - 32_{45000}0_1$$

$$= 32_{45000}0_1$$

$$= 48_{30000}0_1$$

$$+ 16_{30000}0_1$$

$$= 64_{30000}0_1 \text{ -->}$$

Measure

2

VI I 3:2 \flat II(N) IV 8:9 III7 V7 3:4 vi I 16:15

C-h64*--->	C-h80* -16	D#h144*--->	D#h128*--->	D#h128*--->	G-h120*--->
G-h48* +16	G#h64*--->	G#h96* +48	D#h64~	C#h54~	D#h96* +24
D#h38~	D#h48~	F-h80~	C-h51~	A#h48~	C-h40~
C-h32~	C-h40~	C#h64~	G-h40~	G-h40~	G#h32~
G-h24~	G#h32~	G#h48~	D#h32~	D#h32~	D#h24~
D#h19~	D#h24~	F-h40~	C#h27~	C#h27~	C-h20~
C-h16~	G#h16~	C#h16~	D#h16~	D#h16~	G#h16~
G-h12~	G#h8~	C#h8~	D#h8~	D#h8~	G#h8~
C-h8~	F#h7	A#h7	C#h7	C#h7	F#h7
A#h7	D#h6	G#h6	A#h6	A#h6	D#h6
G-h6	C-h5	F-h5	G-h5	G-h5	C-h5
E-h5	G#h4	C#h4	D#h4	D#h4	G#h4
C-h4	D#h3	G#h3	A#h3	A#h3	D#h3
G-h3	G#h2	C#h2	D#h2	D#h2	G#h2
C-h2	G#h1[H24000]	C#h1[H16000]	D#h1[H18000]	D#h1[H18000]	G#h1[H24000]
C-h1	6.4Hz	4.2666...Hz	4.8Hz	4.8Hz	6.4Hz
8Hz	3:2-->	8:9-->	1:1-->	3:4-->	16:15-->
5:4-->	[H1]	[H1]	[H1]	[H1]	[H1]
[H1]					

$$\begin{aligned}
 \text{-----} &= 80_{24000}0_1 \\
 &- 16_{24000}0_1 \\
 &= 64_{24000}0_1 \text{ --> } = 96_{16000}0_1 \\
 &\quad + 48_{16000}0_1 \\
 &= 144_{16000}0_1 = 128_{18000}0_1 = 96_{24000}0_1 + 24_{24000}0_1 = 120_{24000}0_1 \text{ ---->}
 \end{aligned}$$

Measure

3

V7 3:2 I7 3:4 iv(9) 2:3 I-(i?) 8:9

G-h120*--->	G-h128* -64	C-h128*--->	C-h96*--->	C-h96*--->	D-h72*--->
D#h96* +24	G-h64*--->	G-h96* +32	G-h72~	F-h64~	C-h64* +8
C-h40~	D-h48~	E-h80~	C-h48~	C-h48~	E-h40~
G#h32~	B-h40~	C-h64~	G#h38~	G#h38~	C-h32~
D#h24~	F-h27~	A#h54~	F-h16~	F-h16~	G-h24~
C-h20~	D-h24~	G-h48~	F-h8~	F-h8~	C-h16~
G#h16~	G-h16~	E-h40~	D#h7	D#h7	C-h8~
G#h8~	G-h8~	C-h16~	C-h6	C-h6	A#h7
F#h7	F-h7	C-h8~	A-h5	A-h5	G-h6
D#h6	D-h6	A#h7	F-h4	F-h4	E-h5
C-h5	B-h5	G-h6	C-h3	C-h3	C-h4
G#h4	G-h4	E-h5	F-h2	F-h2	G-h3
D#h3	D-h3	C-h4	F-h1[H20000]	F-h1[H20000]	C-h1[H30000]
G#h2	G-h2	G-h3	5.333...Hz	5.333...Hz	8Hz
G#h1	G-h1[H22500]	C-h2	1:1-->	2:3-->	8:9-->
6.4Hz	6Hz	C-h1[H15000]	[H1]	[H1]	[H1]
16:15-->	3:2-->	4Hz			
[H1]	[H1]	3:4-->			
		[H1]			

$$\begin{aligned}
 \text{----->} &= 128_{22500}^0 0_1 \\
 &- 64_{22500}^0 0_1 \\
 &= 64_{22500}^0 0_1 \text{ -->} = 96_{15000}^0 0_1 \\
 &\quad + 32_{15000}^0 0_1 \\
 &= 128_{15000}^0 0_1 \text{ ----->} = 96_{20000}^0 0_1 \text{ ----->} = 64_{30000}^0 0_1 \\
 &\quad + 8_{30000}^0 0_1 \\
 &= 72_{30000}^0 0_1 \text{ ---->}
 \end{aligned}$$

4

D-h72*---->	D-h64*---->	D-h96*---->	D-h128*---->	D-h128*---->	G-h128*---->
C-h64*	D-h32~	G-h64~	B-h102~	A-h96~	D-h96* +32
E-h40~	C-h27~	D-h48~	F#h80~	F#h80~	G-h64~
C-h32~	F#h20~	B-h40~	D-h64~	D-h64~	D-h48~
G-h24~	D-h16~	G-h32~	C-h54~	C-h54~	B-h40~
C-h16~	A-h12~	G-h16~	D-h16~	D-h16~	G-h16~
C-h8~	D-h8~	G-h8~	D-h8~	D-h8~	G-h8~
A#h7	C-h7	F-h7	C-h7	C-h7	F-h7
G-h6	A-h6	D-h6	A-h6	A-h6	D-h6
E-h5	F#h5	B-h5	F#h5	F#h5	B-h5
C-h4	D-h4	G-h4	D-h4	D-h4	G-h4
G-h3	A-h3	D-h3	A-h3	A-h3	D-h3
C-h1	D-h2	G-h2	D-h2	D-h2	G-h2
8Hz	D-h1[H33750]	G-h1[H22500]	D-h1[H16875]	D-h1[H16875]	G-h1[H22500]
8:9-->	9Hz	6Hz	4.5Hz	4.5Hz	6Hz
[H1]	3:2-->	4:3-->	1:1-->	3:4-->	3:4-->
	[H1]	[H1]	[H1]	[H1]	[H1]

$$\begin{aligned} \text{-----}> &= 64_{33750}0_1 \quad \text{----}> &= 96_{22500}0_1 \quad \text{-----}> &= 128_{16875}0_1 \quad \text{-----}> &= 96_{22500}0_1 \\ & & & & & & + 32_{22500}0_1 \\ & & & & & & \hline & & & & & & = 128_{22500}0_1 \quad \text{-----}> \end{aligned}$$

Measure

G-h128*---->
D-h96*
G-h64~
D-h48~
B-h40~
G-h16~
G-h8~
F-h7
D-h6
B-h5
G-h4
D-h3
G-h2
G-h1
6Hz
3:4-->
[H1]

C-h128*---->
G-h96* +32
D#h76~====>
G-h48~
D#h38~====>
C-h16~====>
C-h8~====>
A#h7
G-h6
E-h5
C-h4
G-h3
C-h2
C-h1[H30000]
8Hz
5:4-->
[H1]

D#h192*---->
C-h160* +32

D#h96~
G#h64~
D#h48~
C-h40~
C-h20~
B-h19
A#h18
A-h17
G#h16
G-h15
F#h14
F-h13
D#h12
D-h11
C-h10
A#h9
G#h8
F#h7
D#h6
C-h5
G#h4
D#h3
G#h2
G#h1[H24000]
6.4Hz
5:6-->
[H1]

D#h160*---->
D-h76~
G#h52~
D-h38~
B-h32~
B-h16~
A#h15
A-h14
G-h13
F#h12
F-h11
D#h10
C#h9
B-h8
A-h7
F#h6
D#h5
B-h4
F#h3
B-h2
B-h1[H28800]
7.68Hz
1:1-->
[H1]

B-h256*---->
D#h160* +96
D-h152*====>
B-h128*
D-h76~====>
F#h48~
D-h38~====>
B-h32~
B-h16~
A#h15
A-h14
G-h13
F#h12
F-h11
D#h10
C#h9
B-h8
A-h7
F#h6
D#h5
B-h4
F#h3
B-h2
B-h1[H28800]
7.68Hz
5:4-->
[H1]

B-h320* -176

D-h192*
A-h144*---->
D-h96~
G-h64~
D-h48~
A#h38~
A#h19~
A-h18
G#h17
G-h16
F#h15
F-h14
E-h13
D-h12
C#h11
B-h10
A-h9
G-h8
F-h7
D-h6
B-h5
G-h4
D-h3
G-h2
G-h1[H23040]
6.144Hz
8:9-->
[H1]

(Improper Conjunctions)

Measure

D-h192* -64	A-h128*--->	A-h192* -64	D-h96*--->	D-h96*--->	G-h128*--->
G-h128*====>	G-h112* +16	D-h128~--->	D-h96*--->	D-h96* +32	D-h96* +32
D-h96~	C-h76~	F#h80~	B-h80~	A-h72~	G-h64~
G-h64~	G-h56~	D-h64~	G-h64~	C-h43~	D-h48~
A#h38~	C-h38~====>	C-h56~	D-h48~	G-h32~	B-h40~
A#h19~	A-h32~	G#h44~	G-h32~	G-h16~	F-h28~
A-h18	A-h16~	G#h22~	G-h16~	F#h15	F-h14~
G#h17	G#h15	G-h21	F#h15	F-h14	E-h13
G-h16	G-h14	F#h20	F-h14	E-h13	D-h12
F#h15	F#h13	F-h19	E-h13	D-h12	C#h11
F-h14	E-h12	E-h18	D-h12	C#h11	B-h10
E-h13	D#h11	D#h17	C#h11	B-h10	A-h9
D-h12	C#h10	D-h16	B-h10	A-h9	G-h8
C#h11	B-h9	C#h15	A-h9	G-h8	F-h7
B-h10	A-h8	C-h14	G-h8	F-h7	D-h6
A-h9	G-h7	B-h13	F-h7	D-h6	F#h5
G-h8	E-h6	A-h12	D-h6	F#h5	G-h4
F-h7	C#h5	G#h11	F#h5	G-h4	D-h3
D-h6	A-h4	F#h10	G-h4	D-h3	G-h2
B-h5	E-h3	E-h9	D-h3	G-h2	G-h1[H23040]
G-h4	A-h2	D-h8	G-h2	G-h1[H23040]	6.24152381Hz
D-h3	A-h1[H25920]	C-h7	G-h1[H23040]	6.24152381Hz	3:2-->
G-h2	7.021714286Hz	A-h6	6.24152381Hz	1:1-->	[H1]
G-h1	3:2-->	E-h5	1:1-->	[H1]	
6.144Hz	[H1]	D-h4	[H1]		
7:8-->		A-h3			
[H1]		D-h2			
Improper		D-h1[H17280]			
Conjunction		4.681142857Hz			
Series		3:4-->			
		[H1]			

(Improper Conjunctions)

Measure

7

rit.

dim.

i 3:4 iv 8:9 V-7 3:4 i 5:4

G-h128*---->	G-h192* -64	G-h144*---->	G-h128*---->	G-h128*---->	G-h96* -32
D-h96* +32	C-h128~---->	C-h96* +48	G-h64~	F-h56~	C-h64*---->
G-h64~	G-h96~	G#h76~	D-h48~	D-h48~	D#h38~
D-h48~	C-h64~	C-h48~	G-h32~	G-h32~	C-h32~
B-h40~	D#h38~	G#h38~	B-h20~	B-h20~	G-h24~
F-h28~	D#h19~	F-h32~	B-h10~	B-h10~	C-h16~
F-h14~	D-h18	F-h16~	A-h9	A-h9	C-h8~
E-h13	C#h17	E-h15	G-h8	G-h8	A#h7
D-h12	C-h16	D#h14	F-h7	F-h7	G-h6
C#h11	B-h15	D-h13	D-h6	D-h6	E-h5
B-h10	A#h14	C-h12	B-h5	B-h5	C-h4
A-h9	A-h13	B-h11	G-h4	G-h4	G-h3
G-h8	G-h12	A-h10	D-h3	D-h3	C-h2
F-h7	F#h11	G-h9	G-h2	G-h2	C-h1[H30720]
D-h6	E-h10	F-h8	G-h1[H23040]	G-h1[H23040]	8.322031746Hz
F#h5	D-h9	D#h7	6.241523809Hz	6.241523809Hz	5:4-->
G-h4	C-h8	C-h6	1:1-->	3:4-->	[H1]
D-h3	A#h7	A-h5	[H1]	[H1]	
G-h2	G-h6	F-h4			
G-h1	E-h5	C-h3			
6.241Hz	C-h4	F-h2			
3:2-->	G-h3	F-h1[H20480]			
[H1]	C-h2	5.548021164Hz			
	C-h1[H15360]	8:9-->			
	4.161015873Hz	[H1]			
	3:4-->				
	[H1]				

$$----> = 192_{15360}0_1$$

$$- 64_{15360}0_1$$

$$= 128_{15360}0_1 \quad ----> = 96_{20480}0_1$$

$$+ 48_{20480}0_1$$

$$= 144_{20480}0_1 \quad ---->$$

$$= 128_{23040}0_1 \quad ----->$$

$$= 96_{30720}0_1$$

$$- 32_{30720}0_1$$

$$= 64_{30720}0_1 \quad ---->$$

Measure

8

VI 3:2 \flat II(N) 32:45 V7(\flat 6) 3:4 i

G-h96* -32

C-h64*---->

D#h38~

C-h32~

G-h24~

C-h16~

C-h8~

A#h7

G-h6

E-h5

C-h4

G-h3

C-h2

C-h1

8.322Hz

5:4-->

[H1]

C-h80* -16

G#h64*---->

D#h48~

C-h40~

G#h32~

D#h24~

G#h16~

G#h8~

F#h7

D#h6

C-h5

G#h4

D#h3

G#h2

G#h1[H24576]

6.657625397Hz

3:2-->

[H1]

G#h96* -16

G-h90*---->

F-h80~

C#h64~

G#h48~

F-h40~

C#h16

C#h8

B-h7

G#h6

F-h5

C#h4

G#h3

C#h2

C#h1[H16384]

4.438416931Hz

32:45-->

[H1]

G-h64*---->

D#h50~

B-h40~

G-h32~

F-h28~

G-h16~

G-h8~

F-h7

D-h6

B-h5

G-h4

D-h3

G-h2

G-h1[H23040]

6.24152381Hz

1:1-->

[H1]

G-h64*---->

D-h48~

B-h40~

G-h32~

F-h28~

G-h16~

G-h8~

F-h7

D-h6

B-h5

G-h4

D-h3

G-h2

G-h1[H23040]

6.24152381Hz

3:4-->

[H1]

G-h48*

C-h32~

G-h24~

D#h19~

C-h8~

C-h4~

G-h3

C-h2

C-h1[H30720]

8.322031746Hz

1:1

[H1]

$$\text{----->} = 80_{24576}^0 \text{ }_1$$

$$- 16_{24576}^0 \text{ }_1$$

$$= 64_{24576}^0 \text{ }_1 \text{ ---->} = 96_{16384}^0 \text{ }_1$$

$$- 16_{16384}^0 \text{ }_1$$

$$= 90_{16384}^0 \text{ }_1 \text{ ----->} = 64_{23040}^0 \text{ }_1 \text{ ----->} = 48_{30720}^0 \text{ }_1$$

9

pp *cresc.* *rit.* *dim.* *p*

Analysis in Whole (Decimal) Numbers

Measure	Proportion	Nested h1	Nested Series	Prime Factors
-----	-----	-----	& Conjunction	-----
1.	FundamentalSeriesH1->H?	60000 × 32 = 1920000	2**10	3 5**4
	2/3 (3:2) (h1) =	40000 × 72 = 2880000	2**9	3**2 5**4
	9/8 (8:9) (h1) =	45000 × 32 = 1575000	2**8	3**2 5**4
	2/3 (3:2) (h1) =	30000 × 64 = 1920000	2**10	3 5**4
2.	4/5 (5:4) (h1) =	24000 × 64 = 1536000	2**12	3 5**3
	2/3 (3:2) (h1) =	16000 × 144 = 2304000	2**11	3**2 5**3
	9/8 (8:9) (h1) =	18000 × 128 = 2304000	2**11	3**2 5**3
	4/3 (3:4) (h1) =	24000 × 120 = 2880000	2**9	3**2 5**4
3.	15/16 (16:15) (h1) =	22500 × 64 = 1440000	2**8	3**2 5**4
	2/3 (3:2) (h1) =	15000 × 128 = 1920000	2**10	3 5**4
	4/3 (3:4) (h1) =	20000 × 96 = 1920000	2**10	3 5**4
	3/2 (2:3) (h1) =	30000 × 72 = 2160000	2**7	3**3 5**4
4.	9/8 (8:9) (h1) =	33750 × 64 = 2160000	2**7	3**3 5**4
	2/3 (3:2) (h1) =	22500 × 96 = 2160000	2**7	3**3 5**4
	3/4 (4:3) (h1) =	16875 × 128 = 2160000	2**7	3**3 5**4
	4/3 (3:4) (h1) =	22500 × 128 = 2880000	2**9	3**2 5**4
5.	4/3 (3:4) (h1) =	30000 × 128 = 3840000	2**11	3 5**4
	4/5 (5:4) (h1) =	24000 × 192 = 4608000	2**12	3**2 5**3
	6/5 (5:6) (h1) =	28800 × 256 = 7372800	2**15	3**2 5**2
	4/5 (5:4) (h1) =	23040 × 128 = 2949120	2**16	3**2 5
6.	9/8 (8:9) (h1) =	25920 × 128 = 3317760	2**13	3**4 5
	2/3 (3:2) (h1) =	17280 × 128 = 2211840	2**14	3**3 5
	4/3 (3:4) (h1) =	23040 × 96 = 2211840	2**14	3**3 5
	1/1 (1:1) (h1) =	23040 × 128 = 2949120	2**16	3**2 5
7.	2/3 (3:2) (h1) =	15360 × 128 = 1966080	2**17	3 5
	4/3 (3:4) (h1) =	20480 × 144 = 2949120	2**16	3**2 5
	9/8 (8:9) (h1) =	23040 × 128 = 2949120	2**16	3**2 5
	4/3 (3:4) (h1) =	30720 × 64 = 1966080	2**17	3 5
8.	4/5 (5:4) (h1) =	24576 × 64 = 1572864	2**19	3
	2/3 (3:2) (h1) =	16384 × 90 = 1474560	2**15	3**2 5
	45/32 (32:45) (h1) =	23040 × 64 = 1474560	2**15	3**2 5
	2/3 (3:4) (h1) =	30720 × 48 = 1474560	2**15	3**2 5

Notes

Note Names

The diagram illustrates a pitch nomenclature system across two staves. The treble staff shows notes labeled C_3 , C_2 , C_1 , C , c , c , c^1 , c^2 , and c^3 . The bass staff shows notes labeled 8 , Low , Bass , Tenor , Middle , Treble , High , and Top . The notes are represented by various symbols including ledger lines, whole notes, and half notes.

The pitch nomenclature adopted in this document is shown above, one of the three schemes mentioned in the Harvard Dictionary of Music compounded with a verbal practice familiar to organ builders. The twelve ascending chromatic notes from bottom C_3 to bottom B_3 are spoken: bottom C, bottom C#, bottom D, etc... and written either as bottom C_3 or C_3 ; bottom $C\#_3$ or $C\#_3$, etc... This ascending octave based naming practice is applied throughout the compass of notes, and if required, may be extended further through the use of more super/subscripts. Also as amongst organ builders, notes are by preference named as sharps, for example A# rather than B-flat, but not exclusively so where the flattened form is more informative or convenient.