Polonaise

from ‘Notebook for W.A. Mozart, 1762’

A Straightforward Mutable Number Analysis

The object of this analysis is to demonstrate the basic parameters of a mutable number interpretation of a piece of tonal music using a particularly straightforward and ‘transparent’ composition, hopefully thereby producing an equally straightforward and transparent analysis, which may be helpful to the reader in coming to grips with the notion of encapsulating harmonic progression by means of the nesting of one harmonic series within another – the mechanism that underlies mutable base numbers. Ideally the study of this analysis might follow on from a perusal of the introductory document *Music by Mutable Numbers*; in which the dominant–tonic exchange of the full cadence featured there also provides the main structural element found in this simple Polonaise. And it is hoped that by viewing this straightforward representation of the score, the nested harmonic series and the formal written mutable numbers – inter-leaved with comments; that the reader will easily become conversant with the workings of this type of analysis.

Introduction to the Analysis

Any piece of tonal music that can support a convincing Weberian Roman Numeral analysis – which is almost synonymous with any composition from the period of common practice – will also be amenable to a mutable base number interpretation. Which is to say, that the principal corpus of western art music, its very heart and soul, can be understood in terms of a particular number system and each individual composition construed as an example of number processing in that system. And thereby, each unique tonal composition describes through its unfolding performance an equally distinct set of magnitudes: always whole numbers and often quite large whole numbers. Further, the vast majority of western music that has survived from before this principal period of maximum applicability, plus most popular and commercial music of the modern era (much of which in reality still belongs to the common practice) and a traditional or conservative subset of ‘classical’ compositions written after the turn of the nineteenth century, also respond well to this form numerical analysis.

Below in Figure Y.1 the dominant-tonic exchange introduced on pages two through five of *Music by Mutable Numbers* is developed a little further. Because of the disposition of the notes within the chords of this real world example (middle e in the first chord) instead of twenty four the number delineated by this exchange is forty eight. Twelve digits/ratios built on a column base of four [H4] are exchanged for sixteen digits/ratios built on a column base of three [H3] and back again. The whole gamut of forty eight digits/ratios is displayed on the right of the figure. Most of these forty eight digits/ratios do not physically exist, however they are *implied* by the observable sound arising from the juxtaposition of the three chords and so available, and amenable, for the formation of mutable base numbers. The usefulness of these mutable numbers lies in the access and insight they provide into the ‘harmonic dynamics’ of a composition at its most fundamental level, perhaps in a manner somewhat akin to the mathematics of force and motion in physics. The observable notes and overtones generated by the music are marked by a tilde or asterisk, respectively; in addition to which there would also be a range of frequencies attributable to the formant of the instrument. These forty eight digit/ratios arise from the first harmonic exchange alone, when the harmonic progression of the whole piece is reviewed there will be considerably more to choose from – close to half a million.

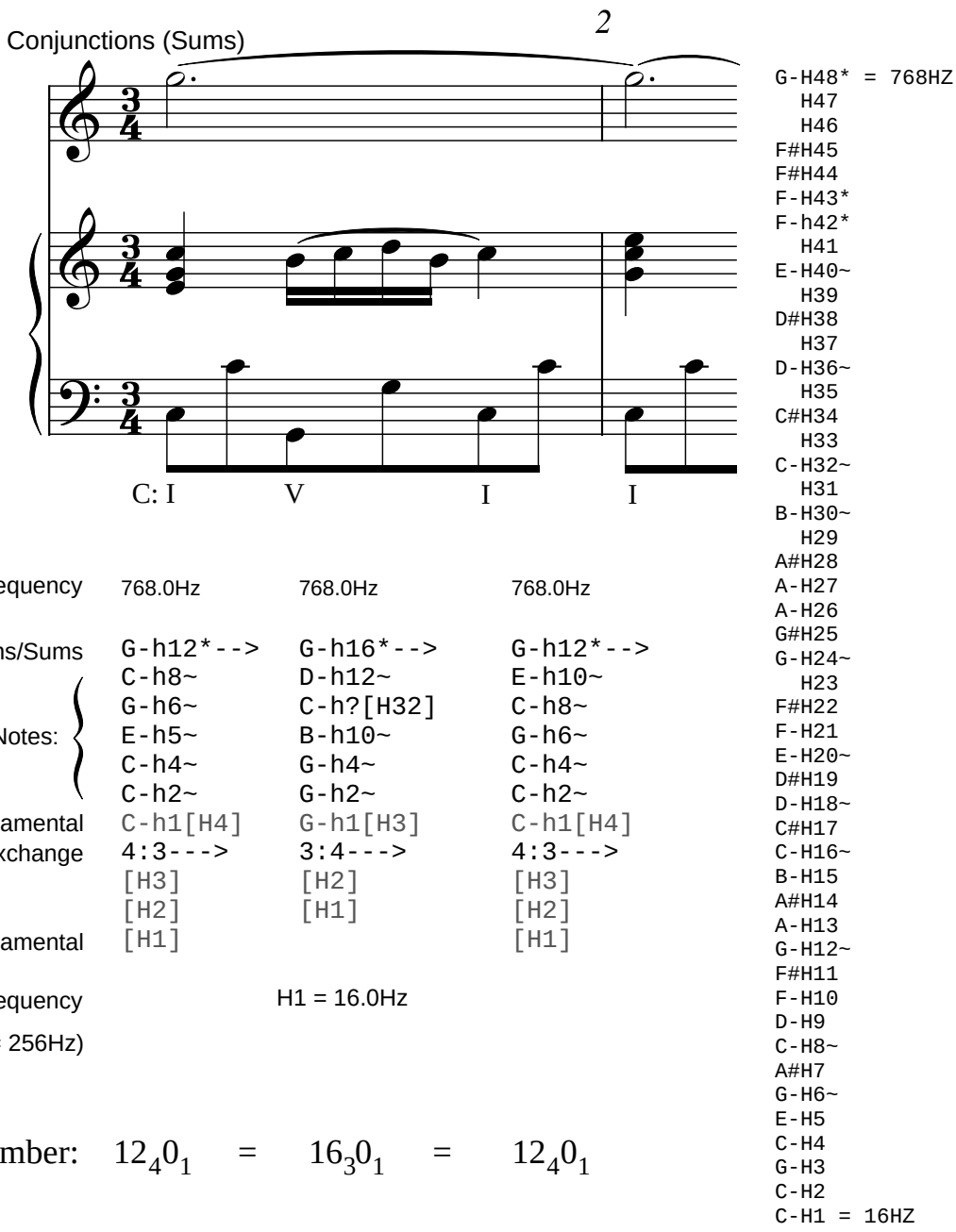


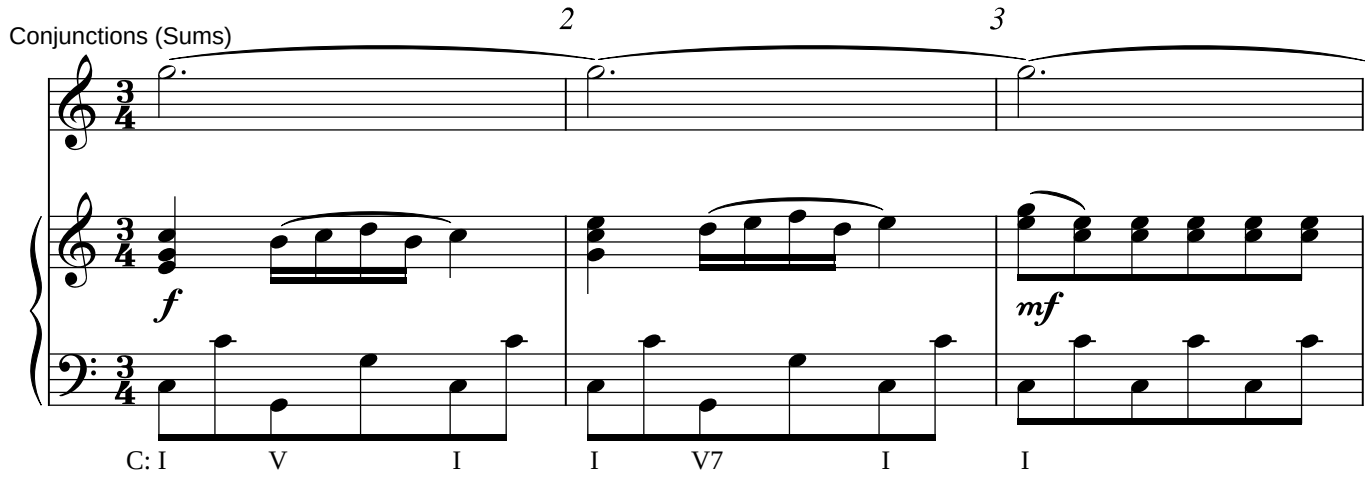
Figure Y.1 The first sum calculated in the Polonaise, taken in isolation, is forty eight: MBN $12_40_1 = 16_30_1 = 12_40_1$

Conjunctions, Sums

The choice of the conjunction/sum – the magnitude of the number – is to some degree arbitrary. The range of potential candidates is extensive considering the broad spectrum of scientifically observable harmonic frequencies produced by musical instruments. The general rule followed is to choose the lowest frequency equal to, or higher than, the highest written note. This makes the choice inclusive of the written score while also placing the conjunction, for the most part, in the range of the most energetic overtones – frequencies usually more powerful than the written notes themselves. The arbitrary element in the choice of conjunction frequency could be eliminated by simply using the lowest physically observable frequency common to the two chords involved in the process and commensurate with the proportion of the exchange; which for example, would render the conjunction between the first two chords of the Polonaise an octave lower at middle g 384Hz. However, the disadvantage of this practice is that chords with very different distributions of

notes across the stave(s) –e.g. clustered low down against spread from low to high pitch, would generate identical values; and so reduce the level of discrimination available to the analysis. While yet if some unalterable signature of the composition is required, this is still present (whether using a higher conjunction or not) in that the sequence of nested fundamental frequencies labelled by ‘h1’ at the bottom of the nested series crowned by the conjunction, will provide whole numbers unique to the piece. (A complete whole number analysis is given on page eight.) This sequence of whole numbers – the point of juncture between the absolute fundamental nesting series and the child series nested within it – is marked in the analysis as some note letter followed by ‘h1[H?]’ or where space allows the actual fundamental series ratio, for example ‘h1[H26244]’ as in the first chord, measure one, below.

Polonaise the Analysis



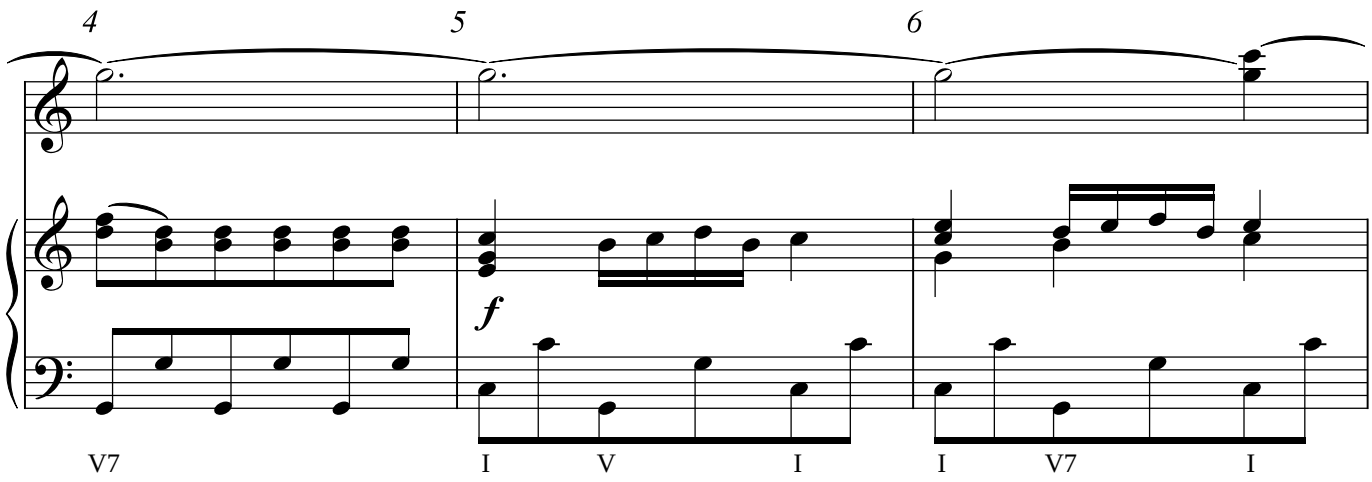
768.0Hz	768.0Hz	768.0Hz	768.0Hz	768.0Hz	768.0Hz
G-h12*-->	G-h16*-->	G-h12*-->	G-h16~-->	G-h12*-->	G-h12~-->
C-h8~	D-h12~	E-h10~	F-h14~	E-h10~	E-h10~
G-h6~	C-H?~	C-h8~	E-h13~	C-h4~	C-h8~
E-h5~	B-h10~	G-h6~	D-h12~	C-h2~	C-h4~
C-h4~	G-h4~	C-h4~	G-h4~	C-h1[H?]	C-h2~
C-h2~	G-h2~	C-h2~	G-h2~	1:1--->	C-h1[H26244]
C-h1[H26244]	G-h1[H19683]	C-h1[H26244]	G-h1[H?]	[H1]	4:3--->
4:3--->	3:4--->	4:3--->	3:4--->		[H1]
[H1]	[H1]	[H1]	[H1]		
64.0Hz	48.0Hz	64.0Hz	48.0Hz	64.0Hz	64.0Hz

Starting Pitch middle c = 256Hz

MBN: $12_{26244}0_1 = 16_{19683}0_1 = 12_{26244}0_1 = 16_{19683}0_1 = 12_{26244}0_1 = 12_{26244}0_1$ --->

(Decimal: 314928)

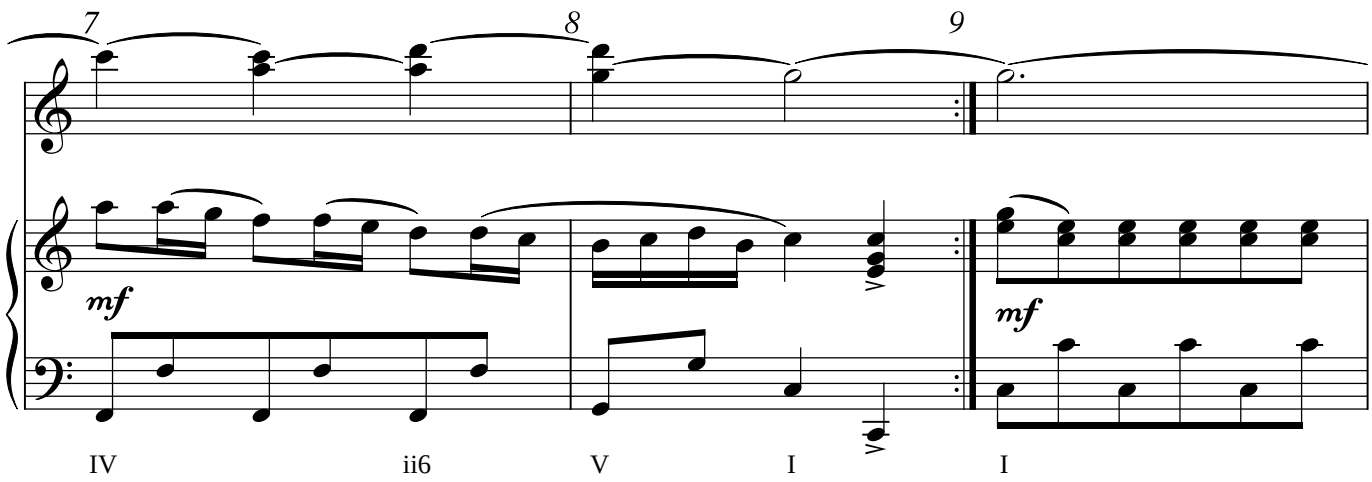
This Polonaise being straightforward and repetitive in harmonic structure generates relatively small values; at its most extensive less than half a million, whereas more complex and typical compositions will normally range above a billion and quite often much more besides.



	768.0Hz	768.0Hz	768.0Hz	768.0Hz	768.0Hz	1024.0Hz --> ---> 768.0Hz
---->	G-h16*--> G-h14~ D-h12~ B-h10~ G-h4~ G-h2~ G-h1[H19683] 3:4--->[H1]	G-h12*--> C-h8~ G-h6~ E-h5~ C-h4~ C-h2~ C-h1[H?] 4:3--->[H1]	G-h16*--> D-h12~ C-h?~[H209952] B-h10~ G-h4~ G-h2~ G-h1[H19683] 3:4--->[H1]	G-h12*--> E-h10~ C-h8~ G-h6~ C-h4~ C-h2~ C-h1[H?] 4:3--->[H1]	G-h16*--> F-h14~ E-h13~ D-h12~ B-h10~ G-h4~ G-h2~ G-h1[H?] 3:4--->[H1]	C-h16*---> G-h12* +4 E-h10~ C-h4~ C-h2~ C-h1[H?] 3:2--->[H1]
	48.0Hz	64.0Hz	48.0Hz	64.0Hz	48.0Hz	64.0Hz

---> $= 16_{19683}0_1 = 12_{26244}0_1 = 16_{19683}0_1 = 12_{26244}0_1 = 16_{19683}0_1 = 12_{26244}0_1$
$+ \ 4_{26244}0_1$
$= 16_{26244}0_1$ ->

The format of analysis given here is that of a ‘summary analysis’ consisting principally of the notes of the composition set or interpolated within a (low order) nested harmonic series, topped by a conjunction frequency and tailed by a notional nested fundamental (h1). Set below this is the proportion of exchange and an acknowledgement of the unit, the unchanging absolute fundamental frequency [H1]. Naturally, not all notes sit comfortably within the confines of the bottom end of an harmonic series and some find no place at all. The third chord from the left, above, is an example. In a G based harmonic series nothing approaching C appears until h21 and even then it is a poor fit, h43 is somewhat better. (In terms of equal temperament h171 is pretty good and h683 excellent.) However, the point of a summary analysis is to provide an easily read and comprehensible outline of what is going on in regard to the relationship between the harmonic progression and the mutable numbers being calculated and these short summary series do that job. Where notes are unobtainable, in the sense that they correspond to fractional harmonics, it is convenient to label them ‘h?’ and if necessary append the fundamental nesting ratio as in this example [H209952]. Indeed it would be possible to reconstruct these summary analyses in the full splendour of their underlying absolute nesting series but they could be a little hard on the eyes! The absolute detail of the actual mutable numbers is both succinctly held (yet thankfully hidden) within the subscript format of the formal written numeral, also above.

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----> 1024.0Hz              1137.777Hz  ----> 1137.777Hz
       853.333Hz  ------>    853.333Hz          758.518...Hz  ---> 758.518...Hz  -----> 758.518...Hz  ---->

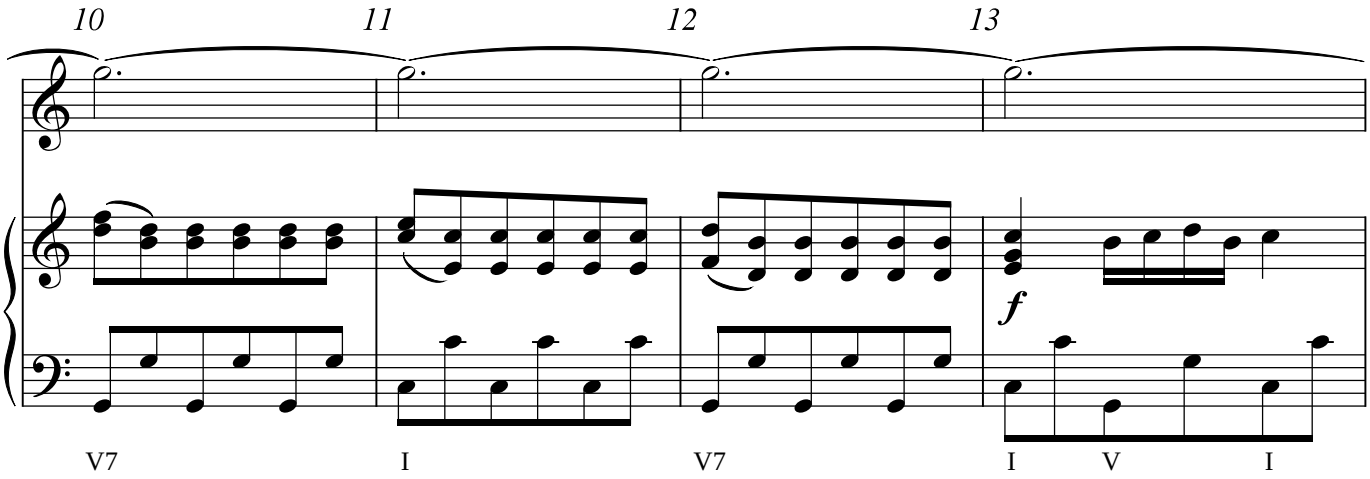
--->  C-h24* -4          D-h32*--->      D-h24* -8
      A-h20~--->         A-h24* +8       G-h16*--->       G-h12*-->       G-h12~-->
      G-h18~             D-h16~          D-h12~           C-h8~           E-h10~
      F-h16~             C-h14~          C-h?~[H207360]   G-h6~           C-h8~
      E-h15~             F-h?~[H69984]   B-h10~           E-h5~           C-h4~
      F-h4~              F-h?~[H34992]   G-h4~            C-h2~           C-h2~
      F-h2~              D-h1[H14580]    G-h2~            C-h1~[H25920]   C-h1[H25920]
      F-h1[H17496]       3:4--->         G-h1[H19440]     1:1--->         4:3--->
      6:5--->            [H1]            3:4-->           [H1]            [H1]
      [H1]                               [H1]

       42.666Hz          35.555Hz        47.407...Hz      63.20987654     63.20987654
```

$$\begin{aligned}
\text{---> } &= 24_{17496}0_1 \\
&- \ 4_{17496}0_1 \\
&= 20_{17496}0_1 \ \text{----> } = 24_{14580}0_1 \\
&\qquad + \ 8_{14580}0_1 \\
&\qquad = 32_{14580}0_1 \ = 24_{19440}0_1 \\
&\qquad\qquad - \ 8_{19440}0_1 \\
&\qquad\qquad = 16_{19440}0_1 \ = 12_{25920}0_1 \ = 12_{25920}0_1 \ \text{------->}
\end{aligned}$$

Here again in the chords on either side of barline eight there are notes without corresponding harmonics (h?). The minor third low down in a chord will always be wanting a ratio as in the two notes tenor F and bass F, their absolute ratios – justly tuned – are appended. The first chord, measure eight, repeats the case discussed on the previous page, measure five, however, notice that even over the short distance to measure eight the absolute ratio and therefore the pitch of the note has changed. In a mutable number analysis (as the name suggests) the system is in almost constant flux as it calculates its progress from one harmonic exchange to another in steps of simple, Just, proportions. The only certainties being that the ultimate unit [H1] is fixed and in ‘proper’ commensurable exchanges, conjunctions align. From the end of measure six the seesawing between tonic and dominant is replaced as the cadencial phrase moves out to the sub-dominant and back through the dominant-of-the-dominant to close in a full cadence, matched by an appropriate flurry of mutable number calculations illustrated above. Two unusual features in mutable number analyses occurs on either side of the measure nine barline, 1) the written observable notes of the composition touch ground,

briefly, at C-h1~[H25920] thus they are printed in black typeface (with tilde) rather than the grey typeface, without tilde, reserved for implicit notional digit/ratios; and 2) on the other side of the barline the piece brushes the conjunction at G-h12~ -->. For the written notes to touch the conjunction is less of a rarity; and as the conjunction is always a physically observable fact the typeface is necessarily black but here written with a tilde (note) rather than asterisk (overtone).



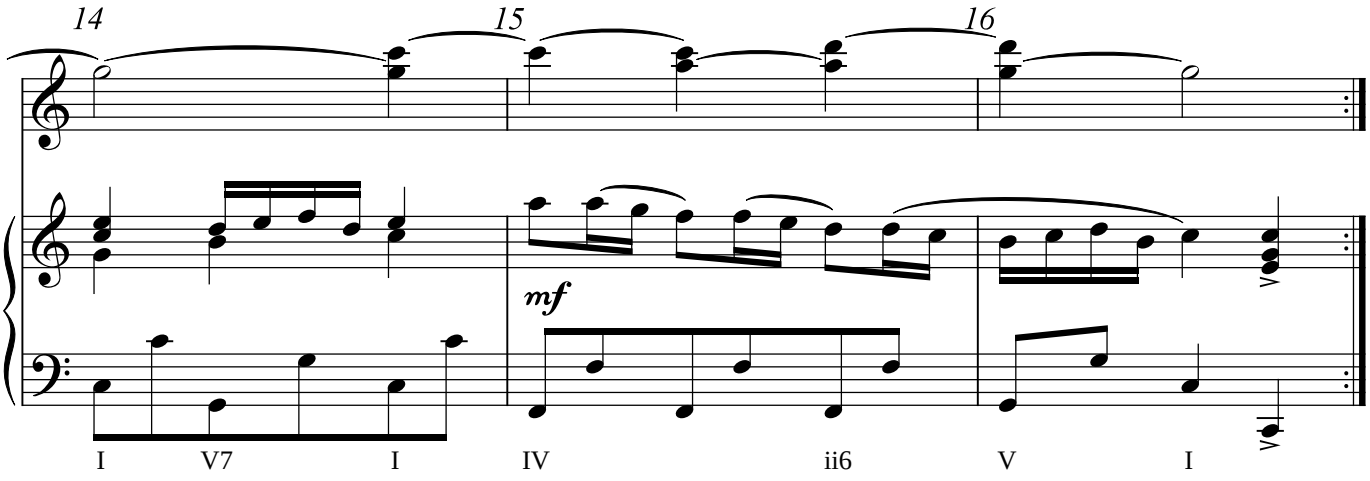
---->	758.516...Hz	758.516...Hz	758.516...Hz	758.516...Hz	758.516...Hz	758.516...Hz
---->	G-h16*-->	G-h12*-->	G-h16*-->	G-h24*-->	G-h32*-->	G-h24*-->
	F-h14~	E-h10~	D-h12~	C-h16~	D-h24~	E-h20~
	D-h12~	C-h8~	F-h7~	G-h12~	C-h21~	C-h16~
	G-h4~	E-h5~	D-h6~	E-h10~	B-h20~	G-h12~
	G-h2~	C-h4~	G-h4~	C-h8~	G-h8~	C-h8~
	G-h1[H19440]	C-h2~	G-h2~	C-h4~	G-h4~	C-h4~
	3:4--->	C-h1[H25920]	G-h1[H19440]	C-h1[H?]	G-h1[H?]	C-h1[H?]
	[H1]	4:3--->	3:2--->	4:3--->	3:4--->	1:1--->
		[H1]	[H1]	[H1]	[H1]	[H1]
	47.407...Hz	63.20987654Hz	47.407...Hz	31.60493827Hz	23.703...Hz	31.60493827Hz

--> = $16_{19440}0_1$ = $12_{25920}0_1$ = $16_{19440}0_1$ = $24_{12960}0_1$ = $32_{9720}0_1$ = $24_{12960}0_1$

From the repeat barline at measure nine the polonaise returns to seesawing between tonic and dominant, with the overall pattern of the second section a reprise of material from the first eight measures. Therefore in this second section the opportunity is taken to rearrange the mutable number digit sequences so as to eliminate the occurrence of notes not being able to find an appropriate digit/ratio in their nested series. This is achieved by pushing the position of the nested fundamental (h1) further down within the enfolding absolute series. The process begins at measure twelve where the proportion of exchange back to the tonic chord steps downward by 3:2 (whereas previously it had been an upward 3:4). Over the next few measures the exchanges follow this pattern of favouring downward steps of fourths 4:3 and fifths 3:2, a process that can be easily read from the column base subscripts in the mutable numbers, where they decline from 25920 in measure eleven to a low point of 3600 at the cadencial supertonic inversion in measure fifteen. The aim of pushing the nested fundamental (h1) down so far within the absolute series is to gain enough headroom at

this low point so as to allow the notes tenor F and bass F (minor thirds within the D minor chord) to be accommodated within the nested series at F-38~ and F-h19~ respectively. And a byproduct of this procedure is that C-h21~ is also made available to the dominant chord in measure thirteen – mirroring measure eight.

However, all of this is merely cosmetic, the actual magnitudes are unchanged; only the mutable number digit sequences by which the magnitudes are expressed have been altered: An example of the rich resource that mutable base numbers provide via their chameleon-like attribute of addressing unique values by multiple digit sequences. Whilst in the background, standing behind such ‘summary’ format analyses as here shown, with their more than occasionally ill-matched ratios, lies the *absolute series*, founded upon an unalterable *unit*, offering complete precision.



1011.358025Hz → 1011.358025Hz 1123.731139Hz → 1123.731139Hz

758.518Hz 758.518Hz → 758.518Hz 842.7983539Hz → 842.7983539Hz 749.1540924Hz → 749.1540924Hz

--> G-h24*-->	G-h32*-->	C-h64*-->	C-h96* -16	D-h128*-->	D-h96* -32	
E-h20~	F-h28~	G-h48*+16	A-h80~--->	A-h96* +32	G-h64*--->	G-h48*-->
C-h16~	E-h26~	E-h40~	G-h72~	D-h64~	D-h48~	C-h32~
G-h12~	D-h24~	C-h16~	F-h64~	C-h56~	C-h43~	G-h24~
C-h8~	B-h20~	C-h8~	E-h60~	F-h38~	B-h40~	E-h20~
C-h4~	G-h8~	C-h1[H?]	F-h16~	F-h19~	G-h16~	C-h4~
C-h1[H?]	G-h4~	3:2--->	F-h8~	D-h1[H?]	G-h8~	C-h1[H?]
4:3--->	G-h1[H?]	[H1]	F-h1[H?]	3:4--->	G-h1[H?]	1:4-->
[H1]	3:2--->		6:5--->	[H1]	3:4-->	[H1]
	[H1]		[H1]		[H1]	

31.60493827Hz 23.703...Hz 15.80246914Hz 10.53497942Hz 8.77914952Hz 11.70553269Hz 15.60737692Hz

$= 24_{12960}0_1 = 32_{9720}0_1 = 48_{6480}0_1$

$+ 16_{6480}0_1$

$= 64_{6480}0_1 = 96_{4320}0_1$

$- 16_{4320}0_1$

$= 80_{4320}0_1 = 96_{3600}0_1$

$+ 32_{3600}0_1$

$= 128_{3600}0_1 = 96_{4800}0_1$

$- 32_{4800}0_1$

$= 64_{4800}0_1 = 48_{6400}0_1$

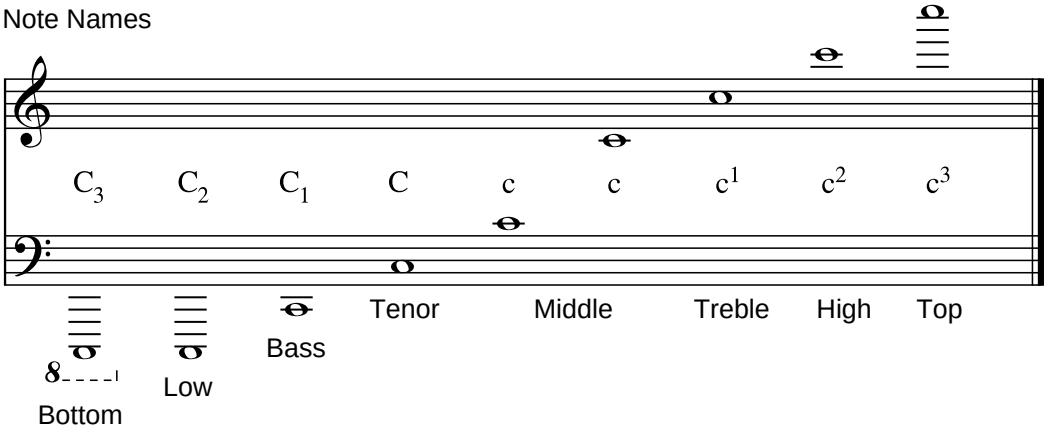
(Decimal 307200)

Whole Number Analysis

The table below provides an straightforward overview of the analysis showing one out-going exchange per line (the receiving side of the exchange is not listed). The conjunction sums remain broadly level, only rising up before the sectional cadences and falling back afterward. As is entirely typical of a composition from the period of common practice there is a small gradual decline in the conjunction/sums over the length of the piece (caused by a small imbalance in the proportions used) as the mutable numbers calculate their way progressively through the harmony by simple whole numbered relationships –i.e. Just Intonation. Also the pattern of growth in power of two factors matched by an equal decline in power of three factors through the composition as a whole is again perfectly typical of music from the zenith of tonal number processing.

Bar	Proportion	Fundamental Series × Nested Series	Conjunction Sum	Prime Factors
1.	FundamentalSeriesH1-> H?	26244 × 12	= 314928	2**4 3**9
	3/4 (4:3)	(h1) = 19683 × 16	= 314928	2**4 3**9
1/2.	4/3 (3:4)	(h1) = 26244 × 12	= 314928	2**4 3**9
	3/4 (4:3)	(h1) = 19683 × 16	= 314928	2**4 3**9
2/3.	4/3 (3:4)	(h1) = 26244 × 12	= 314928	2**4 3**9
4.	3/4 (4:3)	(h1) = 19683 × 16	= 314928	2**4 3**9
5.	4/3 (3:4)	(h1) = 26244 × 12	= 314928	2**4 3**9
	3/4 (4:3)	(h1) = 19683 × 16	= 314928	2**4 3**9
5/6.	4/3 (3:4)	(h1) = 26244 × 12	= 314928	2**4 3**9
	3/4 (4:3)	(h1) = 19683 × 16	= 314928	2**4 3**9
	4/3 (3:4)	(h1) = 26244 × 16	= 419904	2**6 3**8
7.	2/3 (3:2)	(h1) = 17496 × 20	= 349920	2**5 3**7 5
	5/6 (6:5)	(h1) = 14580 × 32	= 466560	2**7 3**6 5
8.	4/3 (3:4)	(h1) = 19440 × 16	= 311040	2**8 3**5 5
8/9.	4/3 (3:4)	(h1) = 25920 × 12	= 311040	2**8 3**5 5
10.	3/4 (4:3)	(h1) = 19440 × 16	= 311040	2**8 3**5 5
11.	4/3 (3:4)	(h1) = 25920 × 12	= 311040	2**8 3**5 5
12.	3/4 (4:3)	(h1) = 19440 × 16	= 311040	2**8 3**5 5
13.	2/3 (3:2)	(h1) = 12960 × 24	= 311040	2**8 3**5 5
	3/4 (4:3)	(h1) = 9720 × 32	= 311040	2**8 3**5 5
	4/3 (3:4)	(h1) = 12960 × 24	= 311040	2**8 3**5 5
14.	3/4 (4:3)	(h1) = 9720 × 32	= 311040	2**8 3**5 5
	2/3 (3:2)	(h1) = 6480 × 64	= 414720	2**10 3**4 5
15.	2/3 (3:2)	(h1) = 4320 × 80	= 345600	2**9 3**3 5**2
	5/6 (6:5)	(h1) = 3600 × 128	= 460800	2**11 3**2 5**2
16.	4/3 (3:4)	(h1) = 4800 × 64	= 307200	2**12 3 5**2
	4/3 (3:4)	(h1) = 6400 × 48	= 307200	2**12 3 5**2

Notes



The pitch nomenclature adopted in this document is shown above, one of the three schemes mentioned in the Harvard Dictionary of Music compounded with a verbal practice familiar to organ builders. The twelve ascending chromatic notes from bottom C_3 to bottom B_3 are spoken: bottom C, bottom C#, bottom D, etc... and written either as bottom C_3 or C_3 ; bottom $C\#_3$ or $C\#_3$, etc... This ascending octave based naming practice is applied throughout the compass of notes, and if required, may be extended further through the use of more super/subscripts. Also as amongst organ builders, notes are by preference named as sharps, for example A# rather than B-flat, but not exclusively so where the flattened form is more informative or convenient.